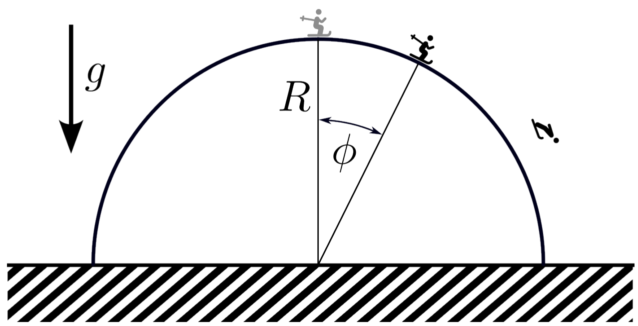
EXAM 1. Friday October 13, 9:10am – Monday, October 16, 9:10am

Problem 1. *Skier on sphere*
A skier starts from the top of a semi-sphere of radius R with negligible initial velocity. There is no friction or air resistance.

1. At what angle ϕ_0 measured as shown on the figure the skier will lift off?
2. What will be this angle if the skier is on the Moon?

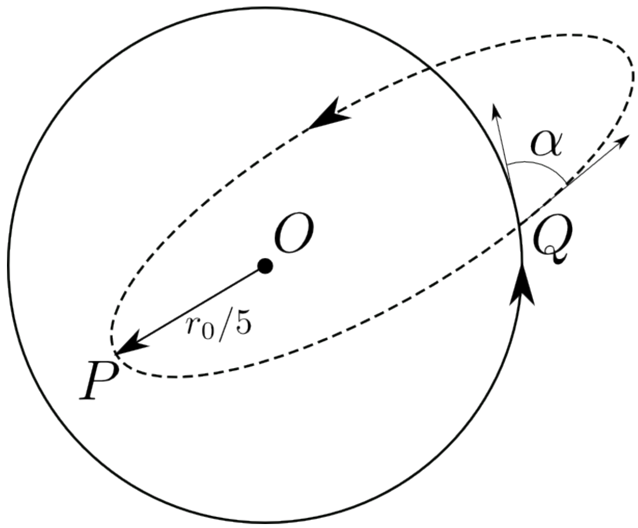


Problem 2. *1991-Spring-CM-U-3.*
A uniform sphere with a mass M and radius R is set into rotation with a horizontal angular velocity ω_0. At $t = 0$, the sphere is placed without bouncing onto a horizontal surface as shown. There is friction between the sphere and the surface. Initially, the sphere slips, but after an unknown time T, it rolls without slipping.

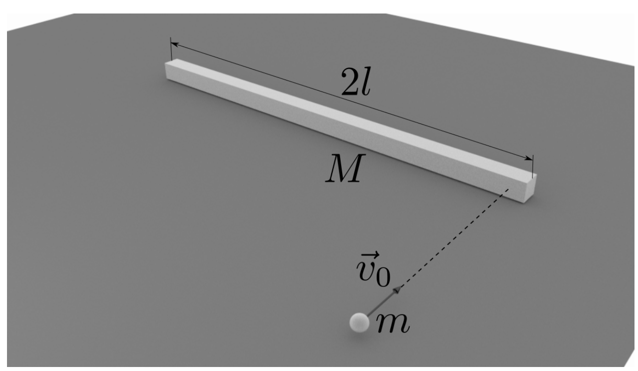
1. What is the angular speed of rotation when the sphere finally rolls without slipping at time T?
2. How much energy is lost by the sphere between $t = 0$ and $t = T$?
3. Show that amount of energy lost is equal to the work done against friction causing the sphere to roll without slipping?

Problem 3. *1993-Fall-CM-U-1*
A satellite of mass m is traveling at speed v_0 in a circular orbit of radius r_0 under the gravitational force of a fixed mass M at point O. At a certain point Q in the orbit (see the figure below) the direction of motion of the satellite is suddenly changed by an angle α without any change in the magnitude of the velocity. As a result the satellite goes into an elliptic orbit. Its distance of the closest approach to O (at point P) is $r_0/5$.

1. What is the speed of the satellite at P, expressed as a multiple of v_0?
2. Find the angle α.



Problem 4. *1995-Spring-CM-U-3*
A uniform line-like bar of mass M, and length $2l$ rests on a frictionless, horizontal table. A point-like particle of mass $m \ll M$ slides along the table with velocity v_0 perpendicular to the bar and strikes the bar very near one end, as illustrated below. Assume that the force between the bar and the particle during the collision is in the plane of the table and perpendicular to the bar. If the interaction is elastic (*i.e.*, if energy is conserved) and lasts an infinitesimal amount of time, then determine the rod's center-of-mass velocity V and angular velocity ω, and the particle's velocity v after the collision.



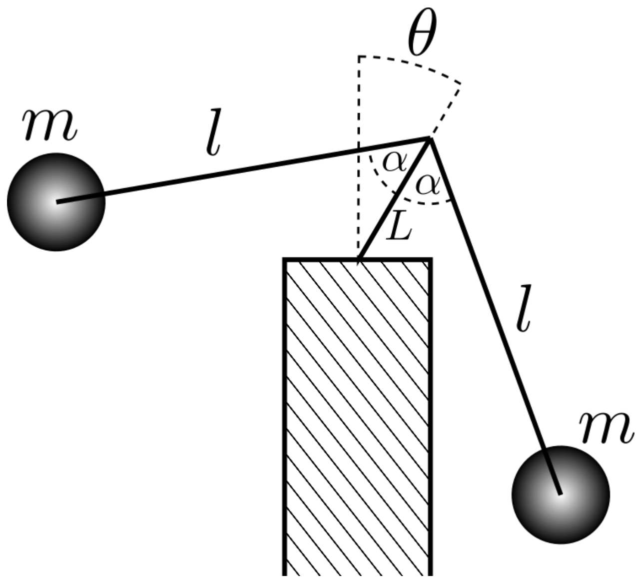
EXAM 2. Final. Friday, December 8, 2023, 8-10 am

Problem 1. *1992-Fall-CM-U-2.*
A toy consists of two equal masses (m) which hang from straight massless arms (length l) from a massless pin. The pin (length L) and the arms are in plane at angle α. Consider only motion in this plane.

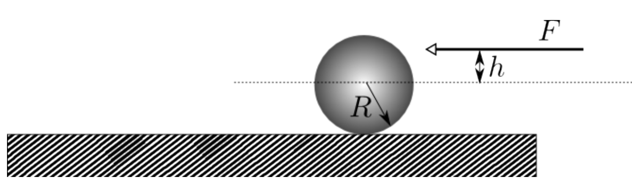
1. Find the potential and kinetic energies of the masses as a function of θ, the angle between the vertical and the pin, and the time derivatives of θ. (Assume the toy is rocking back and forth about the pivot point.)

2. Find the condition in terms of L, l, and α such that this device is stable.

3. Find the period of oscillation if θ is restricted to very small values.



Problem 2. *1996-Spring-CM-U-1*
A billiard ball initially at rest, is given a sharp impulse by a cue stick. The cue stick is held horizontally a distance h above the centerline as in the figure below. The ball leaves the cue with a horizontal speed v_0 and, due to its spin, eventually reaches a final speed of $9v_0/7$. Find h in terms of R, where R is the radius of the ball. You may assume that the impulsive force F is much larger than the frictional force during the short time that the impulse is acting.



Problem 3. *1997-Spring-CM-U-2*
A particle of mass m moves in a region where its potential energy is given by

$$V = Cr^4,$$

where C is a real, positive constant. Consider the case where the particle moves in a circular orbit of radius R.

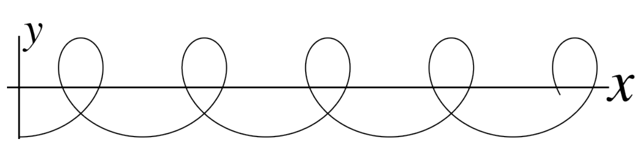
1. Express its total energy E and angular momentum L as a function of R.
2. Determine the period τ_{orb}, of this circular orbit as a function of R.
3. What is its period τ_{rad} for small radial oscillations if the orbit is slightly perturbed? Express τ_{rad} as a factor times τ_{orb}.

Problem 4. *1997-Spring-CM-G-4.jpg*
The curve illustrated below is a parametric two dimensional curve (not a three dimensional helix). Its coordinates $x(\tau)$ and $y(\tau)$ are

$$x = a\sin(\tau) + b\tau$$
$$y = -a\cos(\tau),$$

where a and b are constant, with $a > b$. A particle of mass m slides without friction on the curve. Assume that gravity acts vertically, giving the particle the potential energy $V = mgy$.

1. Write down the Lagrangian for the particle on the curve in terms of the single generalized coordinate τ.
2. From the Lagrangian, find p_τ, the generalized momentum corresponding to the parameter τ.
3. Find the Hamiltonian in terms of the generalized coordinate and momentum.
4. Find the two Hamiltonian equations of motion for the particle from your Hamiltonian.



THE END!