PRODUCT CYCLES AND MARKET PENETRATION*

BY AMY JOCelyn GLASS

Ohio State University, U.S.A.

This paper constructs a quality ladders product cycle model with multiple quality levels. A lower quality level of each product sells due to differences in willingness to pay for quality across consumers. The model determines how far Southern firms penetrate high-technology product markets. Expanded resources or weakened protection of intellectual property rights in the South relative to the North lead to increased Southern market penetration as observed with East Asian countries. Either cause of increased Southern market penetration implies a reduction in the wage in the North relative to the South and a reduction in the rate of innovation.

1. INTRODUCTION

Newly industrialized countries (NICs) have penetrated high-technology product markets previously dominated by developed countries (DCs). Although DCs maintain exclusive ability to invent higher quality levels of products, NICs soon imitate what DCs have invented. Market penetration occurs first at low-quality levels and then at high-quality levels.

The U.S.A. has suffered increased penetration of its high-technology product markets by East Asian NICs (South Korea, Taiwan, Hong Kong, and Singapore). The U.S. Department of Commerce (1984) notes “East Asian NICs are...quickly expanding their high-tech exports and making inroads on the world export shares of most developed country suppliers” (p. 16). Specifically, “While U.S. high-tech exports to these countries rose by 41 percent between 1980 and 1984, U.S. high-tech imports rose by 139 percent” (p. 23). Why? “This growth in high-tech exports by the East Asian NICs reflected their ability to use technology...from the United States and elsewhere” (p. 24).

This paper develops a dynamic general equilibrium model of the product cycle to examine the extent that the South penetrates quality differentiated markets through imitating products invented by the North. The model determines the percentage of product markets that are North dominated, North–South split and South dominated

* Manuscript received March 1994; revised August 1995.
† E-mail: glass.29@osu.edu
‡ I thank Laurel Adams, Michael Blackman, Maureen Blair, Tasneem Chipty, Elias Dinopoulos, Mario Epelbaum, Eric Fisher, Anne Kerttula, Jim Peck, Sanjaya Panth, Kamal Saggi, Paul Segerstrom, Nikolaos Vettas, seminar participants at the University of Pennsylvania, Ohio State University, McGill University, University of Missouri, the Midwest International Economics Meetings, and the Chinese Economic Association of North America, U.S.A., and two referees. I especially thank Wilfred Ethier, Gene Grossman, Patrick Kehoe, Andrew Postlewaite and Rafael Rob for advice and guidance.
based on the labor supplies and labor requirements in research and development (R&D) for each country. Analysis using the model provides two potential explanations for the increased market penetration of East Asian countries: weakened protection of intellectual property rights (IPR) and increased resources in the South relative to the North.

In each country, a fixed supply of resources is allocated between R&D and production. An increase in resources enables the South to expand its labor allocation to both imitation and production, yielding more R&D activity and a larger share of world production. Even if the South does not accumulate resources, it can further its market penetration by weakening its IPR protection. Weak IPR protection lowers imitation costs (similar to improved imitation technology) and thus loosens the Southern resource constraint. Developing countries (including NICs) generally have weaker IPR protection than developed countries. Developing countries may be using weak IPR protection to augment resource accumulation as a means of furthering their penetration of high-technology markets.

In contrast, R&D subsidies neither increase the supply of resources nor increase the efficiency of resources. R&D subsidies merely increase the incentives for undertaking R&D. By paying some of R&D expenses, the government encourages firms to undertake more R&D activity. To expand R&D activity, firms must employ more resources in R&D. In a general equilibrium setting, assuming a common resource is employed in R&D and production of high-technology products, resources allocated to R&D must be removed from production. Resources removed from production lead to a decreased share of world production. Thus, Southern governments that subsidize imitation are apt to hurt rather than help their firms' efforts to penetrate high-technology product markets. Increased market penetration stems from resource accumulation or increased efficiency, not across-the-board R&D subsidies.

Vernon (1966) founded the theory of the product cycle, emphasizing that goods are initially produced in high income countries, close to the product market, then later in low income countries, where costs are lower. Krugman (1979) created the first formal product cycle model, but assumed technology leaks to the developing country at an exogenous rate. Jensen and Thursby (1986) determined both the rate of innovation and the rate of imitation, and hence the North–South production split, but a social planner chooses the rate of technology transfer in the South and products are only horizontally differentiated.

Jaskold Gabszewicz et al. (1981) constructed a simple partial equilibrium model of international trade in different quality levels of a product where quality levels are exogenously fixed. Flam and Helpman (1987) created a static product cycle model with products differentiated by quality. Demand for different qualities stems from the differences in consumer income: higher income consumers demand higher quality levels of the product. As the North introduces higher quality levels, the South stops producing the lowest quality levels and takes over production of some quality levels previously produced by the North. Stokey (1991) provided necessary conditions for the product cycle equilibrium to occur in a similar model.

Segerstrom et al. (1990) developed a dynamic model where the time between innovations is deterministic and R&D is applied sequentially to a finite number of
products. The North–South production split is fixed due to an exogenous period of patent protection. Grossman and Helpman (1991c) determined the North–South production split through optimal investments in R&D by firms in both countries. In their model the time between innovations is stochastic and R&D is applied simultaneously to a continuum of products. Only the highest quality level of each product sells in equilibrium as consumers share the same valuation of quality. In existing dynamic product cycle models, including variety-based models such as Jensen and Thursby (1987), and Grossman and Helpman (1991b), Southern market penetration for each product is either complete or zero.

The present paper determines the extent of Southern market penetration for each product. The model resembles Grossman and Helpman (1991c), but introduces differences in willingness to pay for quality improvements across consumers to permit multiple quality levels to sell in equilibrium, similar to Flam and Helpman (1987). Partial Southern market penetration arises when the South does not imitate the highest quality level of each product unless some Southern firm already possesses the knowledge necessary to produce the quality level below the highest. The imitation process then exhibits properties of product specific national learning-by-doing: by initially producing a low quality level, a country obtains the background necessary to eventually produce a higher quality level.

Having multiple quality levels sell and requiring the South to imitate one step at a time generates the vital feature of the model: the South gradually works its way into markets as Southern firms imitate higher quality levels of existing products. Southern firms do not capture entire markets through single imitation success, but gradually work their way into markets through repeated imitation success. Starting from zero Southern market penetration, initial imitation occurs at quality levels below the highest existing level, leading to partial Southern market penetration. Further imitation occurs at the highest existing quality level, permitting the South to catch up to the North on the quality frontier and leading to complete Southern market penetration. The model determines the fraction of product markets where Southern market penetration is complete, partial, or nonexistent and what forces lead to increased Southern market penetration.

Section 2 describes the optimal decisions made by consumers choosing which quality level of each product to purchase. Section 3 describes the optimal decisions of firms choosing prices and R&D intensities. Section 4 finds the solution for each possible steady-state equilibrium. Section 5 provides an analysis of how expanded relative resource availability or weakened relative IPR protection (but not R&D subsidies) can facilitate the South’s pursuit of the technology frontier. Section 6 concludes.

2. CONSUMERS

The model examines quality upgrading in continuous time in an economy composed of two types of consumers and many firms located in two countries. Difference in willingness to pay for quality improvements across consumers permits two types of consumers to consume two different quality levels of each product. Firms devote resources to R&D to improve the continuum of products and produce
quality levels invented. Firms in the North innovate while firms in the South imitate quality levels the North has invented. In equilibrium, expected profits from the product market just compensate firms for their R&D costs, and the labor supply is fully employed in either R&D or production in each country.

Consumers live in one of two countries (North and South), $i \in \{N, S\}$, and are each one of two types (low and high), $\omega \in \{A, B\}$, based on willingness to pay for quality improvements. Willingness to pay for quality improvements, $\lambda^{\omega}$, differs across consumers as high-type consumers are willing to pay more for quality improvements: $\lambda^{B} > \lambda^{A}$. This heterogeneity across consumers permits more than one quality level of each product to sell in equilibrium. Additionally $\lambda^{A} > 1$, so all consumers agree that new designs achieve a quality improvement, although they disagree on the magnitude of quality improvement. Let $f(\omega)$ describe the percentage of world income distributed to each type of consumer: $f(B)$ of world income is distributed to high-type consumers, while the remaining $f(A) = 1 - f(B)$ of world income is distributed to low-type consumers. For simplicity, the same distribution of income applies to both countries.

Consumers choose from a continuum of products $j \in [0, 1]$, available in a discrete number of quality levels $m$. A consumer of type $\omega$ in country $i$ has lifetime utility

$$U_i^{\omega} = \int_0^\infty e^{-\rho t} \log u_i^{\omega}(t) \, dt$$

where $\rho$ is the discount factor. Instantaneous utility is

$$\log u_i^{\omega}(t) = \int_0^1 \log \left( \sum_m (\lambda^{\omega})^m x_m^{\omega}(j, t) \right) dj$$

where $(\lambda^{\omega})^m$ is the assessment by type $\omega$ consumers of quality level $m$, and $x_m^{\omega}(j, t)$ is consumption by type $\omega$ consumers in country $i$ of quality level $m$ of product $j$ at time $t$. Each consumer maximizes lifetime utility subject to an intertemporal budget constraint. Since preferences are homothetic, aggregate demands for each group of consumers are found by maximizing lifetime utility subject to the aggregate intertemporal budget constraint

$$\int_0^\infty e^{-R(t)} E^{\omega}(t) \, dt \leq A^{\omega}(0) + \int_0^\infty e^{-R(t)} Y^{\omega}(t) \, dt$$

where $R(t) = \int_0^t r(s) \, ds$ is the cumulative interest rate up to time $t$, and $A^{\omega}(0) = f(\omega)A(0)$ is the aggregate value of initial asset holdings by type $\omega$ consumers. Individuals hold assets in the form of ownership in firms, but with a diversified portfolio any capital losses are offset by equal capital gains elsewhere, so only initial asset value remains. Aggregate income of type $\omega$ consumers is

$$Y^{\omega}(t) = \sum_i f(\omega) L_i w_i(t)$$

where $w_i(t)$ is the wage in country $i$ at time $t$ and $L_i$ is the labor supply in country $i$. Thus $L_i w_i(t)$ is total labor income in country $i$ at time $t$, and $f(\omega) L_i w_i(t)$ is the
share that goes to type $\omega$ consumers. Normalize by the Southern wage $w_S = 1$, and define the relative wage as $w \equiv w_N / w_S$. Aggregate spending by type $\omega$ consumers is

$$E^\omega(t) = \int_0^1 \left[ \sum_m p_m(j,t) x_m^\omega(j,t) \right] dj$$

where $p_m(j,t)$ is the price of quality level $m$ of product $j$ at time $t$. Define aggregate spending by all consumers as $E = E^A + E^B$.

The consumer's maximization problem can be broken into three stages: allocation of lifetime wealth across time, allocation of expenditure at each instant across products, and allocation of expenditure at each instant for each product across available quality levels. In the first stage, each consumer evenly spreads lifetime spending for each product across time, $E^\omega_i(j,t) = E^\omega_i(j)$. In the second stage, each consumer evenly spreads spending at each instant across products, $E^\omega_i(j) = E^\omega_i$. In the final stage, each consumer allocates spending for each product at each instant to the quality level with the lowest quality-adjusted price, $p_m/(\lambda^\omega)^m$. Accordingly, each consumer type $\omega$ has an indifference price ratio, a ratio of prices where consumers of type $\omega$ are just indifferent between buying a quality level or the quality level immediately below it. The indifference price ratio for each type equals the willingness to pay for quality improvement of that type, $p_m/p_{m-1} = \lambda^\omega$. Consumers settle indifference in favor of the higher quality level.

The consumer's problem and solution matches Grossman and Helpman (1991c), except not all consumers share the same assessment of quality, so not all consumers necessarily purchase the same quality level of each product, as in Glass (1996). While all consumers agree that quality level $m$ is indeed better than quality level $m-1$, the two types of consumers disagree over how much better. Thus, high-type consumers are willing to pay more for quality improvements than low-type consumers. If the relative price lies in the region $\lambda^B \leq p_m/p_{m-1} > \lambda^A$, high-type consumers select quality level $m$, while low-type consumers select quality level $m-1$. As a result, a separating equilibrium can occur where consumers of different types purchase different quality levels of each product. Heterogeneity in the strength of consumers preference for quality can thus support multiple quality levels selling in equilibrium, with one quality level for each type of consumer.

Having more than one quality level sell in equilibrium generates the possibility that firms from the North and the South sell different quality levels of the same product for some products, as commonly observed. If all consumers were to buy the same quality level of a product regardless of their valuation of quality, the equilibrium would resemble Grossman and Helpman (1991c). Since only the highest quality level of each product would sell, partial Southern market penetration would never occur. To permit analysis of partial Southern market penetration, some markets must be structured with a Northern firm selling the high quality level and a Southern firm selling the low quality level. Thus, more than one quality level of some products needs to be sold in equilibrium. I turn to firm behavior to describe the conditions needed for multiple quality levels of some products to sell in equilibrium.
3. FIRMS

To produce a quality level of a product, a firm must first design it. Firms are willing to endure the costs of developing higher quality levels because they earn profits in the product market if successful. The potential for quality improvement is unbounded. Each quality level \( m \) of product \( j \) is better than the previous quality level, but consumers differ in their assessment of how much better. While Northern firms push forward the quality frontier of existing products through innovation, Southern firms pursue the quality frontier through imitation. Both innovation and imitation must proceed one quality level at a time.

Assume R&D races occur simultaneously for all products, with all firms able to target the quality level above the current highest quality level successfully developed in that country. Southern firms can imitate a quality level of a product only if some Southern firm already has imitated the quality level below it. The South is limited to imitating quality levels one step ahead of the present Southern capability, similar to Northern firms being limited to innovating quality levels one step ahead of the present Northern capability. This assumption could be replaced by the underlying assumption that innovating or imitating more than one quality level at once (a larger quality increment) is sufficiently more costly, similar to the endogenous quality increment extension in Grossman and Helpman (1991a).

This restriction limits the South to appropriate technologies: in a situation where the South is far behind the technology frontier, the South cannot catch up with one imitation success. Instead the South needs rapid and repeated imitation success, outpacing Northern advancement of the technology frontier through innovation. The ability to benefit from knowledge flows is conditional on having sufficient current knowledge to absorb the flows. Knowledge flows from the North to the South, but such knowledge is only useful to Southern firms if they already have a sufficiently advanced knowledge base from previous imitation success.

To keep the South always able to imitate some products, assume that the South is at worst just out of the market: some Southern firm always has the technological capability to produce quality levels no longer being produced in equilibrium. Such a situation occurs if knowledge of the production design of quality levels no longer produced in the North, discarded technology, leaks out to Southern firms. A technology becomes discarded when the Northern firm that invented that quality level no longer earns a profit from producing it and therefore has no incentive to protect its design.

Without the assumption that discarded technology is freely available to Southern firms, if Northern firms experienced repeated innovation successes without an intervening imitation success by a Southern firm, no Southern firm could imitate even a low-quality level, and the South would then be permanently pushed out of that market. Since a sequence of uninterrupted innovative successes occurs with positive probability, eventually the South would lose hold of all markets without access to discarded technology. Discarded technology serves as a means of supporting base technological capabilities and thus keeps the South from falling too far behind the technological frontier.
Assume R&D is a memoryless stochastic process. Normalize the labor requirement in production to one. For a firm of status $k$ in country $i$, undertaking R&D intensity $\xi^k_i$ for a time interval $dt$ requires $a^k_i \xi^k_i dt$ units of labor at a cost of $w_i a^k_i \xi^k_i dt$ and leads to success with probability $\xi^k_i dt$. A firm in country $i$ is either a leader for some product or a follower, $k \in \{L, F\}$, relative to other firms in that country at any point in time. For each country $i$ and product $j$, the firm in that country that developed the current highest quality level for that product is the leader; all other firms in that country are followers. For each product, the two leaders mark the quality frontier for the two countries. The terms leader and follower refer to the position of the firm relative to other firms in the same country; hence the Southern leader may be one (or more) quality level below the Northern leader for any product. When a firm achieves R&D success, that firm becomes the leader for that product in that country until subsequent R&D success in the same country pushes the firm back into the pool of followers.

Assume free entry into R&D, so that the pool of potential followers is infinite. Let leaders have a lower labor requirement in R&D than followers in each country, $a^L_i < a^F$. Leaders have a cost advantage over followers in R&D because they designed the most advanced quality level of that product developed in that country. The leader's advantage exists not only for innovative activity but for imitative activity as well. The leader's advantage in imitative activity persists despite availability of discarded technology: the most recent successful imitator for a product still maintains a cost advantage over other potential imitators stemming from its experience imitating the most recent generation to have been imitated.

Although leaders have a cost advantage in R&D, generally not all R&D is undertaken by leaders. If a leader were to engage in further R&D targeting its own product for improvement, the firm would gain only the amount the value of the new design exceeds the value of its current design if its subsequent R&D is successful. On the other hand, a follower not producing the product at that time would gain the full value of the new design without sacrificing any current value. Thus, provided followers are not too disadvantaged, leaders do not undertake R&D as long as they are still producing in the market (see Appendix). Assume the labor requirements in R&D satisfy the sufficient condition $a^F_i / a^L_i < \lambda^F$, so that firms already producing a quality level of a product do not engage in further R&D on that product. Therefore, a firm's problem can be broken into two stages. First, when undertaking R&D, the firm chooses its intensity of R&D to maximize its expected value, given the R&D intensities of other firms. Once successful in R&D, the firm then chooses the price of its product to maximize its value, given prices and R&D intensities of other firms.

Two firms with the same costs selling the same quality level of the same product would each price at cost and earn zero profits. Thus once a quality level has been invented, another Northern firm never invents that same quality level. By the same argument, once a quality level has been imitated, another Southern firm never imitates that same quality level. A Southern firm and a Northern firm may end up at the same quality level because a Southern firm can price below the Northern firm and thus earn profits sufficient to compensate for imitation costs.
At any point in time, at most two different quality levels of any product sell due to the two types of consumers. Label the two quality levels of a product produced in equilibrium at any point in time high and low. The labeling of a given quality level changes over time: high when first invented, low after subsequent innovation. The two quality levels are the two highest existing quality levels, since all quality levels cost the same to produce but consumers value higher quality levels more.

The three possible market structures reflect the number of quality levels the Southern technology frontier trails the Northern technology frontier. If the North has a two-quality-level lead over the South in a product market, the North produces both the high- and low-quality levels; call this market structure North dominance ($N$). If the North has a one-quality-level lead over the South in a product market (the South has imitated only the low-quality level), the North produces the high-quality level and the South produces the low-quality level; call this market structure North–South split ($T$). Finally, if the North has no quality level lead over the South in a product market (the South has imitated the high-quality level), the South produces the high-quality level (the only quality level that sells in that market); call this market structure South dominance ($S$). Let $n_N$, $n_T$ and $n_S$ denote the measures of these markets, which must sum to one. The measures of North dominance, North–South split and South dominance indicate the extent of Southern market penetration: nonexistent, partial and complete. As Southern firms penetrate product markets, market structures shift from North dominance to North–South split to South dominance.

3.1. Innovation and Imitation. The property that producing firms do not conduct further R&D indicates the pattern of which market structures are targeted by each form of R&D. Northern leaders target only South-dominated markets and Southern leaders target only North-dominated markets. Northern and Southern followers target North–South split markets. Northern followers target North-dominated and North–South split markets since innovation success leads to the same value, the value of a top firm in a North-dominated market, either way.

Each nonproducing firm chooses its intensity of R&D to maximize its expected value net of R&D costs, given the R&D intensities of the other firms. Costs must not exceed gains for R&D intensity to be positive; gains must not exceed costs for R&D intensity to be finite. The gain from R&D for a nonproducing firm is the value of the producing firm it becomes if successful in R&D (subscripts on value $v$ denote market structure, superscripts denote $1 = \text{top}$ or $2 = \text{trailing position}$). A leader successful at innovation gains the value of a top Northern firm in a North–South split market.

\[(3.1) \quad v_T^L \leq wa_N^L, \quad v_N^L > 0 \iff v_T^L = wa_N^L\]

A follower successful at innovation gains the value of a top Northern firm in a North-dominated market.

\[(3.2) \quad v_N^L \leq wa_N^L, \quad v_N^L > 0 \iff v_N^L = wa_N^L\]
A leader successful at imitation gains the value of a trailing Southern firm in a North–South split market.

\[ v^l_1 \leq a^L_S, \ i^L_S > 0 \iff v^l_1 = a^L_S \]

A follower successful at imitation gains the value of a top Southern firm in a South-dominated market.

\[ v_S \leq a^F_S, \ i^F_S > 0 \iff v_S = a^F_S \]

These R&D conditions ensure that R&D yields no excess returns. The reward to R&D, the value of a producing firm, is the present discounted expected value of the stream of instantaneous profits stemming from the design for a higher quality level of a product.

3.2. Production. In the production stage, each producing firm chooses the price of its product to maximize its value, given the prices and R&D intensities of other firms. Imitation by Southern leaders and innovation by Northern followers target North-dominated markets. For the top firm in a North-dominated market, innovation by Northern followers leads to the trailing position in a North-dominated market, while imitation by Southern leaders leads to the top position in a North–South split market.

\[ v^l_N = \frac{\pi^l_N + i^F_N v^2_N + i^L_S v^1_T}{\rho + i^F_N + i^L_S} \]

For the trailing firm in a North-dominated market, innovation by Northern followers or imitation by Southern leaders leads to no value.

\[ v^2_N = \frac{\pi^2_N}{\rho + i^F_N + i^L_S} \]

Innovation by Northern followers and imitation by Southern followers targets North–South split markets. For the top firm in a North–South split market, innovation by Northern followers leads to the trailing position in a North-dominated market, while imitation by Southern followers leads to no value.

\[ v^l_T = \frac{\pi^l_T + i^F_N v^2_N}{\rho + i^F_N + i^L_S} \]

For the trailing firm in a North–South split market, innovation by Northern followers or imitation by Southern followers leads to no value.

\[ v^2_T = \frac{\pi^2_T}{\rho + i^F_N + i^L_S} \]
Innovation by Northern leaders targets South-dominated markets. For the firm in a South-dominated market, innovation by Northern leaders leads to the trailing position in a North–South split market.

\[
u_{S} = \frac{\pi_{S} + \nu_{K}^{2}}{\rho + \nu_{K}}
\]

The value terms on the right-hand side are eliminated by repeated substitution, leaving the value of each firm as its discounted stream of expected instantaneous profits.

Firms know only the two strengths of preference for quality, \(\lambda^{B}\) and \(\lambda^{A}\), present in the population, and the percentage of income in the hands of consumers of each type, \(f(\omega)\). Firms cannot perfectly discriminate by charging each consumer their full valuation because firms do not know each consumer’s type. Instead, firms must offer prices that induce consumer types to separate if they want to reap a higher profit from high-type consumers. Provided the distribution of income is sufficiently skewed toward high-type consumers, firms offer prices that make these consumers choose a higher quality level of each product than low-type consumers (see Appendix). By separating consumer types, firms receive a higher profit margin from selling to high-type consumers, but delayed sales to low-type consumers. A sufficient share of income in the hands of high-type consumers ensures that the higher profit margin applies to enough sales to offset the delay in receipts from low-type consumers.

In North-dominated markets, the top firm charges price \(p_{N}^{1} = \lambda^{B} \lambda^{A}\) and makes sales \(x_{N}^{1} = E^{B}/(\lambda^{B} \lambda^{A})\), yielding instantaneous profits.

\[
\pi_{N}^{1} = E^{B} \left(1 - \frac{w}{\lambda^{B} \lambda^{A}}\right)
\]

The trailing firm charges price \(p_{N}^{2} = \lambda^{A}\) and makes sales \(x_{N}^{2} = E^{A}/\lambda^{A}\), yielding instantaneous profits.

\[
\pi_{N}^{2} = E^{A} \left(1 - \frac{w}{\lambda^{A}}\right)
\]

The top firm engages in limit pricing against the trailing firm by charging a premium reflecting the higher willingness to pay for quality improvements. The trailing firm engages in limit pricing against the Southern firm(s) that are able to produce the quality level just out of the market (discarded technology) at cost 1, by charging a premium reflecting the lower willingness to pay for quality improvements.

In North–South split markets, the top firm charges price \(p_{T}^{1} = \lambda^{B} w\) and makes sales \(x_{T}^{1} = E^{B}/(\lambda^{B} w)\), yielding instantaneous profits.

\[
\pi_{T}^{1} = E^{B} \left(1 - \frac{1}{\lambda^{B}}\right)
\]

The trailing firm charges price \(p_{T}^{2} = w\) and makes sales \(x_{T}^{2} = E^{A}/w\), yielding instantaneous profits.
\[ \pi_I^2 = E \left( 1 - \frac{1}{w} \right) \]

(3.13)

The top firm once again engages in limit pricing against the trailing firm. The trailing firm now engages in limit pricing against the Northern firm at the same quality level by charging a price equal to the Northern firm’s cost. In South-dominated markets, the top firm charges price \( p_S = w \) and makes sales \( x_S = E/w \), yielding instantaneous profits.

\[ \pi_S = E \left( 1 - \frac{1}{w} \right) \]

(3.14)

Here the top Southern firm captures the whole market: pricing at the cost of the Northern firm at the same quality level implies a quality-adjusted price below the cost of the trailing Southern firm, even adjusting for quality based on the lower valuation \( (w/\lambda^A < 1) \).

These pricing outcomes share the common trait that each firm engages in limit pricing against its closest rival. Northern firms mark their prices up to reflect their quality advantage; Southern firms undercut the price charged by Northern firms at the same quality level and still earn positive profits due to lower production costs. These traits resemble firm pricing behavior in Grossman and Helpman (1991c); however, supporting this limit pricing behavior when more than one quality level of a product sells in equilibrium requires more complicated firm strategies than those used in their model. Here, in a one-shot game, given the limit price chosen by the top firm in a separating equilibrium, the trailing firm could lower its own price by \( \epsilon \) and capture the entire market, thus increasing instantaneous profits. The firm’s objective is to maximize its value, not its instantaneous profits. The trailing firm does not undercut the top firm’s price for fear of retaliation: the higher instantaneous profits would lead to zero profits once the top firm retaliates in the next instant. Hence the top firm can engage in limit pricing against the trailing firm even though the trailing firm prices above cost.

This outcome is supported by each firm playing a trigger strategy: each firm picks the separating price indicated above, unless its rival deviates from its separating price. Once a deviation occurs, both firms revert to pooling prices during a permanent punishment phase: \( (p_N^1)^p = \lambda^A w \) and \( (p_N^2)^p = w \) in North-dominated markets, \( (p_T^1)^p = \lambda^A \) and \( (p_T^2)^p = 1 \) in North–South split markets. The top firm’s punishment price is sufficiently low to prevent the trailing firm from earning any profits during a punishment phase. Punishment, by forcing the trailing firm’s future profits down to zero, offsets the momentary gain in instantaneous profits so that the trailing firm is not tempted to undercut the top firm’s price. The trailing firm (for finite discount rate) never defects from its separating price, so the punishment never occurs and prices are indeed constant and separating in equilibrium. Having a quality advantage with no higher production cost enables the top firm to dictate the form of equilibrium to maximize its value. The trailing firm earns profits only because splitting the market leads to higher value for the top firm.

Firms active in North-dominated markets cannot increase their values by raising prices further because all Southern firms can produce quality levels two steps
behind the quality frontier (due to discarded technology) and would capture the market were prices pushed higher than $p^*_N = \lambda^A$ and $p^*_N = \lambda^B$. Firms active in North–South split markets cannot increase their values by raising prices further because the trailing Northern firm (whose quality level the Southern firm imitated) can produce the low-quality level at cost $w$ and would capture the market were prices pushed higher than $p^*_T = w$ and $p^*_T = \lambda^B w$. Firms active in South-dominated markets cannot increase their value by raising prices further because the top Northern firm (whose quality level the Southern firm imitated) can produce the high quality level at cost $w$ and would capture the market were prices pushed higher than $p^*_S = w$. The pricing equilibrium chosen seems more plausible than allowing firms producing the same quality level of the same product to collude, as the price of products falls once imitation occurs. The key property needed for the results regarding market penetration is that North–South split markets use less Northern resources and more Southern resources than North-dominated markets, and South-dominated markets also use less Northern resources and more Southern resources than North–South split markets, as seen in the resource constraints.

3.3. Resource Constraints. In the labor markets, the fixed supply of labor is allocated between R&D and production. For equilibrium in the labor market, the demand for labor must equal the supply of labor in each country. In the North, labor is allocated to innovation targeting all markets and production in all but South-dominated markets.

\begin{equation}
(3.15) \quad a^N n_N + a^N n_T (n_N + n_T) + n_N \left[ \frac{E^B}{\lambda^B} + \frac{E^A}{\lambda^A} \right] + n_T \left[ \frac{E^B}{\lambda^B w} \right] = L_N
\end{equation}

In the South, labor is allocated to imitation targeting all but South-dominated markets and production in all but North-dominated markets.

\begin{equation}
(3.16) \quad a^S n_N + a^S n_T + n_S \left[ \frac{E^A}{w} \right] + n_T \left[ \frac{E^A}{w} \right] = L_S
\end{equation}

Increased production comes at the expense of decreased R&D in each country due to the fixed labor supply.

4. SOLUTION

High- and low-type consumers in the North and South maximize their intertemporal utility subject to the intertemporal budget constraint, leader and follower firms in the North and South maximize their value given prices and R&D intensities of other firms, and labor markets in the North and South clear. The solution is a combination of aggregate spending, the relative wage, and innovation and imitation intensities such that labor demand equals labor supply in each country and R&D yields no excess returns, given the labor supplies and labor requirements in R&D. In a steady-state equilibrium, the relative wage, the rate of innovation

\begin{equation}
(4.1) \quad \nu_N = \nu_N n_S + \nu_N (1 - n_S)
\end{equation}
and the rate of imitation

\[ \mu = \nu_S^T n_N + \nu_S^T n_T \]

are constant, as are the measures of market structures.

The equilibria of interest possess positive rates of both innovation and imitation. Three steady-state equilibria have positive rates of both innovation and imitation: inefficient Northern followers, inefficient Southern followers, and all followers efficient. Which form of equilibrium prevails depends on the labor requirements in R&D for followers relative to leaders: if the labor requirement in R&D is too much larger for followers than for leaders, then followers become inefficient and only leaders are active in R&D.

Technological barriers to entry in innovation lead to the inefficient Northern followers equilibrium, where North dominance is not achievable; technological barriers to entry in imitation lead to the inefficient Southern followers equilibrium, where South dominance is not achievable. Southern market penetration is at most partial if followers are severely disadvantaged in imitation; Southern market penetration is at least partial if followers are severely disadvantaged in innovation. If followers are efficient in both innovation and imitation, then Southern market penetration for any product can be complete, partial or nonexistent. Each equilibrium is described below with examples generated for \( f(B) = 1/2 \), \( \rho = 0 \), \( \lambda^A = 2 \) and \( \lambda^B = 3 \).

4.1. All Followers Efficient. Under all followers efficient, leaders have a small enough advantage over followers in both innovation and imitation that both leaders and followers undertake innovation and imitation. Starting from North dominance, imitation by Southern leaders moves the market from North dominance to North–South split; imitation by Southern followers moves the market from North–South split to South dominance. Starting from South dominance, innovation by Northern leaders moves the market from South dominance to North–South split; innovation by Northern followers moves the market from North–South split to North dominance.

The flows into each market structure must equal the flows out and the market measures must sum to one: \( \nu_S^T n_T = \nu_S^T n_N, \nu_S^T n_T = \nu_S^T n_S, n_N + n_T + n_S = 1 \). These steady-state conditions combine to give expressions for the market measures in terms of the innovation and imitation intensities. The key equations reduce to a system of six equations. The first two equations are the resource constraints. The four remaining equations are profits inserted into the producing firm valuation equations and the resulting values inserted into the R&D conditions.

The system determines aggregate spending, the relative wage, the rate of innovation, the rate of imitation, the measure of North–South split markets and the measure of South-dominated markets, based on the labor supply in the North, the labor supply in the South, the labor requirement in leader innovation, the labor requirement in follower innovation, the labor requirement in leader imitation and the labor requirement in follower imitation. For example, with \( L_N/L_S = 1, a_N^L/a_S^L = 3, a_N^F/a_S^F = 4/3 \), the market measures are distributed as 25% North dominated, 65% North–South split and 10% South dominated.
4.2. Inefficient Northern Followers. Under inefficient Northern followers, leaders have a large enough advantage over followers in innovation that only leaders undertake innovation. Production of the high quality level of any product is swapped between the North and South: Northern production until the South imitates, then Southern production until the North innovates. Low-type consumers always purchase from Southern firms. Southern market penetration is so pervasive that the North does not dominate any product markets.

The market structure conditions become \( u^e_T n_T = u^e_S n_S \) and \( n_T + n_S = 1 \). The system reduces to four equations. The first two equations are still the resource constraints. The remaining two equations are the R& D conditions for Northern leaders (3.1) and Southern followers (3.4), as the R& D conditions for Northern followers and Southern leaders become inequalities (see Appendix).

The system determines aggregate spending, the relative wage, the rate of innovation and the measure of South-dominated markets, based on the labor supply in the North, the labor supply in the South, the labor requirement in leader innovation and the labor requirement in follower imitation. For example, with \( L_N/L_S = 2/7, a_N^e/a_S^e = 4/3 \), the market measures are uniformly distributed as 50\% North–South split and 50\% South dominated.

4.3. Inefficient Southern Followers. Under inefficient Southern followers, leaders have a large enough advantage over followers in imitation that only leaders undertake imitation. Production of the low quality level of any product is swapped between the North and the South: Northern production until the South imitates, then Southern production until the North innovates. High-type consumers always purchase from Northern firms. Southern market penetration is so slight that the South does not completely penetrate any product markets (never catches up to the Northern technology frontier).

The market structure conditions become \( u^e_N n_N = u^e_S n_N \) and \( n_N + n_T = 1 \). The system also reduces to four equations, although these are a different combination of the original six. The first two equations are again the resource constraints. The remaining two equations are the R& D conditions for Northern followers (3.2) and Southern leaders (3.3), as the R& D conditions for Southern followers and Northern leaders become inequalities.

The system determines aggregate spending, the relative wage, the rate of innovation and the measure of North–South split markets, based on the labor supply in the North, the labor supply in the South, the labor requirement in follower innovation and the labor requirement in leader imitation. For example, with \( L_N/L_S = 30/13, a_N^e/a_S^e = 3 \), the market measures are uniformly distributed as 50\% North dominated and 50\% North–South split.

5. Market Penetration

High-technology markets have experienced increased Southern market penetration. This section searches for the underlying forces behind this behavior. Potential explanations considered include changes in resource availability, intellectual property rights (IPR) protection or R&D subsidies. I find support for a relative
expansion of Southern resources and relative weakening of Southern IPR protection (but not R&D subsidies) as potential underlying forces behind the increased Southern market penetration of East Asian NICs and similar countries. Proofs for the inefficient Northern followers equilibrium appear in the Appendix. Examples for the other two forms of equilibria confirm that the results for the inefficient Northern followers equilibrium are generally valid.

5.1. Relative Resources. The view that increased resources in the South and decreased resources in the North could cause increased Southern market penetration is supported by my model. I define $L_R = L_N/L_S$ as the Northern resource base relative to the Southern resource base.

**Proposition 1.** In the inefficient Northern followers equilibrium, increased resources in the South relative to resources in the North (a decrease in $L_R$) leads to a decreased relative wage, a decreased rate of innovation, and an increased extent of complete Southern market penetration.

With inefficient Northern followers, a relative increase in Southern resources raises the measure of South-dominated markets at the expense of North–South split markets. With all followers efficient, a relative increase in Southern resources raises the measure of South dominated (and North–South split) markets at the expense of North-dominated markets, as in Figure 1. With inefficient Southern followers, a relative increase in Southern resources raises the measure of North–South split markets at the expense of North-dominated markets. Thus increased resources in the South relative to resources in the North can explain increased Southern market penetration.

![Figure 1](image1.png)
penetration. The prevailing equilibrium dictates the level where increased Southern market penetration occurs.

Increased relative resources enable the South to allocate relatively more resources to both imitation and production. The increased resources allocated to imitation lead to more imitative activity; the increased resources allocated to production lead to a larger share of world production, which translates to increased Southern market penetration. Because Northern firms maintain a quality advantage in a smaller percentage of all markets, the relative wage falls.

5.2. Relative Intellectual Property Rights Protection. The view that decreased IPR protection in the South and increased IPR protection in the North could cause increased Southern market penetration is supported by my model. Decreasing IPR protection in the South is modeled as decreasing the costs of imitation: the labor requirements in innovation are now \( \kappa_N a_N^L \) for leaders and \( \kappa_N a_N^L \) for followers and the labor requirements in imitation are now \( \kappa_S a_S^L \) for leaders and \( \kappa_S a_S^L \) for followers, with \( 0% < \kappa_i \leq 100% \) (100% meaning fully enforced). I define \( \kappa_R = \kappa_N / \kappa_S \) as the Northern strength of IPR protection relative to the Southern strength of IPR protection.

**Proposition 2.** In the inefficient Northern followers equilibrium, a weakening of IPR protection in the South relative to IPR protection in the North (an increase in \( \kappa_R \)) leads to a decreased relative wage, a decreased rate of innovation, and an increased extent of complete Southern market penetration.

Relative IPR protection has the opposite effects of relative resources. With inefficient Northern followers, a relative decrease in Southern IPR protection raises the measure of South-dominated markets at the expense of North–South split markets. With all followers efficient, a relative decrease in Southern IPR protection raises the measure of South-dominated (and North–South split) markets at the expense of North-dominated markets, as in Figure 2. With inefficient Southern followers, a relative decrease in Southern IPR protection raises the measure of North–South split markets at the expense of North-dominated markets. As with relative resources, the prevailing equilibrium dictates the level where increased Southern market penetration occurs.

Weakened relative IPR protection enables the South to achieve a larger imitation intensity from any given allocation of resources to imitation. By shifting some resources out of R&D, the South can expand both imitation and production and thus increase Southern market penetration. Similarly to expanded relative resources in the South, weakened relative IPR protection provides a potential explanation for increased Southern market penetration. Weak IPR protection makes resources more efficient in R&D, which increases the resources available measured in efficiency units. Whether expanded relative resources in the South or weakened relative IPR protection plays the leading role in generating increased Southern market penetration does not matter from the perspective of the effect on the relative wage and the rate of innovation. From either source, increased Southern market penetration is associated with a fall in the relative wage and a concurrent fall in the rate of innovation.
Instead of applying equally to all R&D activity within a country, IPR protection could single out R&D by followers for more stringent enforcement. If IPR protection raises the R&D costs of followers more than the R&D costs of leaders, then a relative weakening of IPR protection in the South can cause the equilibrium to switch from inefficient Southern followers (where Southern market penetration is at most partial) to all followers efficient, to inefficient Northern followers (where Southern market penetration is at least partial). Thus sizable relative weakening of IPR protection in the South can lead to increased Southern market penetration not only within each form of equilibrium but between them as well.

5.3. Research and Development Subsidies. Increased imitation subsidies in the South and decreased innovation subsidies in the North are suspected of causing increased Southern market penetration, but my model finds no support for this explanation. R&D subsidies have perverse effects on Southern market penetration. Subsidies are introduced by including the coefficient \((1 - s_s)\) on the imitation costs in the imitation conditions (3.3, 3.4) and \((1 - s_N)\) on the innovation costs in the innovation conditions (3.1, 3.2), indicating that each government pays the share \(s_i\) of R&D expenses. Unlike resources and IPR protection, Northern and Southern R&D subsidies have distinct (rather than relative) effects.

**Proposition 3.** In the inefficient Northern followers equilibrium, an increased innovation subsidy in the North \((an\ increase\ in\ s_N)\) leads to an increased relative wage, and an increased rate of innovation, but also an increased extent of complete Southern market penetration.

An increase in the Northern innovation subsidy has a perverse effect on Southern market penetration. With inefficient Northern followers, an increase in the North-
ern innovation subsidy raises the measure of South-dominated markets at the expense of North–South split markets. With all followers efficient, an increase in the Northern innovation subsidy raises the measure of South-dominated (and North–South split) markets at the expense of North-dominated markets, as in Figure 3. With inefficient Southern followers, an increase in the Northern innovation subsidy raises the measure of North–South split markets at the expense of North-dominated markets. Each of these results is perverse: Northern subsidies for innovation increase Southern market penetration.

A Northern subsidy to innovation perversely increases Southern market penetration; a Southern subsidy to imitation perversely decreases Southern market penetration or has no effect. Southern imitation subsidies are at best ineffective in furthering Southern market penetration.

**Proposition 4.** *In the inefficient Northern followers equilibrium, an increased imitation subsidy in the South (an increase in \( s_q \)) leads to a decreased relative wage, with no effect on the rate of innovation or the extent of complete Southern market penetration.*

With inefficient Northern followers, an increase in the Southern imitation subsidy has no effect on Southern market penetration or the rate of innovation, but reduces the relative wage. With all followers efficient, an increase in the Southern imitation subsidy slightly raises the measure of North-dominated markets at the expense of South-dominated markets, as in Figure 4. With inefficient Southern followers, an increase in the Southern imitation subsidy slightly raises the measure of North-dominated markets at the expense of North–South split markets.
Why do the effects of R&D subsidies differ from the effects of weakened IPR protection when each lowers the costs of R&D for firms? Weakening IPR protection differs from subsidizing R&D because the lessened labor requirements in R&D show up in the resource constraints, not just the R&D conditions. Weak IPR protection differs from R&D subsidies in that while both lower the cost of R&D, weak IPR protection conserves scarce resources, whereas R&D subsidies do not. The amount of labor needed for a given R&D intensity is independent of any R&D subsidy. The perverse effect of R&D subsidies occurs because labor is pulled out of production into R&D.

5.4. Impact. The extent of Southern market penetration mirrors the gap between the Northern and Southern technology frontiers: Southern market penetration is the greatest when the technology gap is the smallest. In South-dominated markets, the South has caught up to the North and so the gap is zero $G_S = 0$. In North–South split markets, the South is one quality increment behind the North and so the gap is the quality increment $G_T = \lambda$. In North-dominated markets, the South is two quality increments behind the North and so the gap is twice the quality increment $G_N = 2\lambda$. Over all markets, the average technology gap is

\[ G = n_N G_N + n_T G_T + n_S G_S = \lambda(1 - n_S + n_N) \]

Increases in $n_S$ or decreases in $n_N$ shrink the technology gap by the quality increment $\lambda$. In the inefficient Northern followers equilibrium, $G \in (0, \lambda)$; in the inefficient Southern followers equilibrium, $G \in (\lambda, 2\lambda)$; in the all followers efficient equilibrium, $G \in (0, 2\lambda)$. 
The wage in the North relative to the South is positively related to the technology gap and thus inversely related to the extent of Southern market penetration. Only Northern innovation subsidies increase both Southern market penetration and the relative wage; however, if the Northern government must (lump-sum) tax its citizens to finance the subsidy, wages net of the tax must fall. Northern resources earn more than Southern resources due to the exclusive ability of Northern firms to innovate, making the average technology gap positive $G > 0$. Since Southern firms cannot innovate, Northern firms maintain a quality advantage in at least some products. The Northern wage must exceed the Southern wage for innovation and imitation to earn the same return. Increased Southern market penetration decreases the average lead of Northern firms over Southern firms and thus decreases the relative wage.

The relative wage decline associated with increased Southern market penetration provides some justification for why Southern governments seek to increase Southern market penetration while Northern governments seek to decrease it. Whether increased Southern market penetration occurs due to increased Southern relative resources or weakened Southern relative IPR protection, an increase in $n_s$ is associated with a decline in the relative wage and the rate of innovation. Focusing on the inefficient Northern followers equilibrium, low-type consumers pay $w$ regardless of the market structure, but in North–South split markets consume a lower quality level. Thus, low-type consumers enjoy a lower quality-adjusted price when a market shifts from North–South split to South dominated. High-type consumers purchase the highest available quality level regardless of the market structure, but in North–South split markets pay a premium $\lambda^B$ over the price in South-dominated markets. Thus high-type consumers also enjoy a lower quality-adjusted price when a market shifts from North–South split to South dominated. From the perspective of quality adjusted prices (for a fixed rate of innovation), increased Southern market penetration benefits both low- and high-type consumers. However, the rate of innovation concurrently declines.

How do the lower quality-adjusted prices (a level effect) weigh against the lower rate of innovation (a growth effect)? By the law of large numbers, the expected number of innovations arriving in time period $t$ is $\bar{m} = \bar{w}$. Instantaneous utility (2.2) is

$$u^{\omega}(t) = \log E_i^{\omega} - \log \bar{p}^{\omega} + \bar{m}^{\omega} \log \lambda^{\omega}$$

where the average price paid by low-type consumers is

$$\bar{p}^A = n_s w + n_T w = w,$$

the average price paid by high-type consumers is

$$\bar{p}^B = n_s w + n_T \lambda^B w = w [n_s + (1 - n_s) \lambda^B],$$

the average quality level consumed by low-type consumers is

$$\bar{m}^A = n_s \bar{m} + n_T (\bar{m} - 1) = \bar{m} - (1 - n_s),$$
and the average quality level consumed by high-type consumers is

\begin{equation}
\overline{m}^B = \overline{m}
\end{equation}

Increased Southern market penetration has no (direct) effect on the average price paid by low-type consumers \( \partial \overline{p}^A / \partial n_S = 0 \), lowers the average price paid by high-type consumers \( \partial \overline{p}^B / \partial n_S < 0 \), raises the average quality level consumed by low-type consumers (for a fixed rate of innovation) \( \partial \overline{m}^A / \partial n_S > 0 \), and has no effect on the average quality level consumed by high-type consumers (for a fixed rate of innovation) \( \partial \overline{m}^B / \partial n_S = 0 \).

Lifetime utility (2.1) for low-type consumers is

\begin{equation}
U^A_i = \frac{\log E_i^A - \log w + \frac{t}{\rho} \log \lambda^A - (1 - n_S) \log \lambda^A}{\rho}
\end{equation}

and lifetime utility for high-type consumers is

\begin{equation}
U^B_i = \frac{\log E_i^B - \log w + \frac{t}{\rho} \log \lambda^B - \log \left[ \lambda^B - n_S(\lambda^B - 1) \right]}{\rho}
\end{equation}

For both types of consumers, the third term causes utility to fall with increased Southern market penetration due to delayed innovation as \( \partial n / \partial n_S < 0 \), while the second and fourth terms cause utility to rise due to lower prices and lessened quality distortion. The first term falls for Northern consumers due to the income effects of increased Southern market penetration (the lower relative wage).

The lower prices swamp the delay in innovation initially leading to higher instantaneous utility, but the delay in innovation swamps the lower prices, eventually leading to lower instantaneous utility. Weighing the short-term benefits against the long-term costs of increased Southern market penetration depends on the discount rate. Due to the decline in the Northern relative wage, Southern consumers fare better than Northern consumers when Southern market penetration increases. Thus, Southern governments seeking to maximize the welfare of their citizens prefer a higher degree of Southern market penetration than Northern governments. For some range of Southern market penetration, Northern governments desire less Southern market penetration while Southern governments prefer more.

6. CONCLUSION

The model developed in this paper determines the extent of Southern penetration of high-technology product markets, where Southern firms imitate and Northern firms innovate in response to profit incentives. Through imitating existing quality levels, Southern firms penetrate product markets; through inventing higher quality levels, Northern firms repel Southern market penetration. For high-technology products, the product cycle involves movements in the location of production of
different quality levels of existing products. Firms in developed countries advance the quality frontier, while firms in newly industrialized countries eventually take over production.

The extent of Southern market penetration differs between the forms of steady-state equilibria involving positive rates of innovation and imitation. Which equilibrium prevails depends on the labor requirements in R&D of followers relative to leaders. Large disparities in innovation efficiency lead to extensive Southern market penetration, while large disparities in imitation efficiency lead to minimal Southern market penetration.

The model provides potential explanations for the increased penetration of East Asian countries into markets for high-technology products through increased resources or weakened IPR protection relative to developed countries. Increased resources enable a country to allocate more resources to both production and R&D. Weakened IPR protection resembles increased resources by reducing the resource requirements in R&D. Developing countries (including NICs) may be using weak IPR protection as a substitute for increased resources in furthering their penetration of high-technology product markets. On the other hand, R&D subsidies do not conserve on scarce resources, and thus subsidizing imitation does not further Southern market penetration and subsidizing innovation does not repel it.

Increased Southern market penetration, whether due to increased Southern relative resources or weakened Southern IPR protection, causes the rate of innovation and the Northern relative wage to fall. Increased Southern market penetration has a positive level effect on welfare due to lower prices, but a negative growth effect due to delayed innovation. The decline in the Northern relative wage stemming from increased Southern market penetration causes Southern consumers (and their representative governments) to prefer a further extent of Southern market penetration than Northern consumers (and their representative governments). Conflicts over IPR protection such as the current conflict between the U.S. and China may arise due to this asymmetry.

This model provides a basis for examining other international trade issues related to quality levels. Glass and Saggi (1997) use the inefficient Southern followers equilibrium as the basis for a model where Northern firms undertake foreign direct investment (FDI) in the South only for appropriate technologies, with quality levels one step above the Southern technological frontier for each product. Because FDI at high-quality levels is limited by the Southern technology frontier, the Southern government can encourage FDI at high-quality levels not only by taxing FDI at low-quality levels but also by subsidizing imitation by Southern firms.

APPENDIX

A.1. Condition for Separation. Were a top firm in a North–South split market to select pooling, the firm would charge $p^p = \lambda^A$ and make sales $x^p = E/\lambda^A$, yielding instantaneous profits.

$$\pi^p = E \left(1 - \frac{w}{\lambda^A}\right)$$
Separation occurs in North–South split markets if the value of the top firm that results from pooling

\[(A.2) \quad v_T^P = \frac{\pi_T^P}{\rho + \pi_T^L + \pi_T^S}\]

does not exceed \(v_T^1\), which is assured by \(\pi_T^P < \pi_T^1\), that occurs when enough income is in the hands of high-type consumers.

\[(A.3) \quad f(B) > f(B)^* = \frac{1 - \frac{w}{\lambda^A}}{1 - \frac{1}{\lambda^B}}\]

Thus, pooling occurs when \(f(B) < f(B)^*\) and separation when \(f(B) \geq f(B)^*\). By similar arguments, separation occurs in North-dominated markets if it occurs in North–South split markets. Separation occurs in North-dominated markets if the value of the top firm that results from pooling

\[(A.4) \quad v_N^P = \frac{\pi_N^P}{\rho + \pi_N^L + \pi_N^S}\]

does not exceed \(v_N^1\), which is assured by \(\pi_N^P < \pi_N^1\). As \(w < \lambda^A\), profits for a top firm when separation occurs are higher in North-dominated markets than in North–South split markets, \(\pi_N^1 > \pi_T^1\), and thus whenever \(\pi^P < \pi_T^1\), then \(\pi^P < \pi_N^1\) by transitivity.

A.2. Producing Firms Do Not Innovate. Leaders still producing do not undertake R&D provided followers are not too disadvantaged in R&D. If a leader who is still the top firm in a North-dominated market were to successfully conduct further R&D, the firm would charge \(p_{N_2} = (\lambda^B)^2\lambda^A\), yielding instantaneous profits.

\[(A.5) \quad \pi_{N_2}^S = E^B \left(1 - \frac{w}{(\lambda^B)^2 \lambda^A} \right)\]

The value of the top firm that would result

\[(A.6) \quad v_{N_2}^S = \frac{\pi_{N_2}^S}{\rho + \pi_N^L + \pi_S^L} = \frac{\pi_{N_2}^S}{\pi_N^1} v_N^1\]

implies an incremental gain in value.

\[(A.7) \quad v_{N_2}^S - v_N^1 = \frac{w(\lambda^A - 1)}{\lambda^B(\lambda^B\lambda^A - w)} v_N^1\]
For such R&D to be undertaken, costs need to not exceed expected benefits, \( u_{N_2} - v_N^e \geq w a_N^L \), which requires

\[
\frac{a_N^e}{a_N^L} \geq \frac{\lambda^B (\lambda^B a^A - w)}{w (\lambda^A - 1)}
\]  

(A.8)

Since \( w < \lambda^A, a_N^e/a_N^L < \lambda^B \) is sufficient for such R&D to never occur. If a leader who is still the top firm in a North-South split market were to conduct further R&D, the firm would charge \( p_{T_2} = (\lambda^B)^2 w \), yielding instantaneous profits.

\[
\pi_{T_2}^S = E^B \left( 1 - \frac{1}{(\lambda^B)^2} \right)
\]  

(A.9)

The value of the top firm that would result

\[
v_{T_2}^S = \frac{\pi_{T_2}^S}{\rho + \iota_N^e + \nu_N^e} = \frac{\pi_{T_2}^S}{\pi_T^S} v_T^1
\]  

(A.10)

implies an incremental gain in value.

\[
v_{T_2}^S - v_T^1 = \frac{1 - \lambda^B}{(\lambda^B)^2} v_T^1
\]  

(A.11)

For such R&D to be undertaken, costs need to not exceed expected benefits, \( u_{T_2} - v_T^1 \geq w a_N^L \), which requires \( 1 \leq (1 - \lambda^B)/(\lambda^B)^2 \). Since \( \lambda^B > 1 \), such R&D never occurs. Similar reasoning applies to Southern firms.

A.3. Inefficient Northern Followers. Construct the system for inefficient Northern followers as follows. The rate of innovation becomes \( \iota = \iota_N n_S \). By constant market measures \( \iota_N^2 n_T = \iota_N^2 n_S \) and now \( n_T = 1 - n_S \). Define \( a_R = a_N^L/a_N^S, S_N = 1 - s_N, S_S = 1 - s_S, K_R = \kappa_R a_R, K_S = \kappa_S a^S, e = E/w, f = f(B) = 1 - f(A), \) and \( \delta = 1/\lambda^B \). Making these substitutions, the system of four equations becomes

\[
\iota K_R K_S + (1 - n_S) \delta e f = L_R L_S
\]  

(A.12)

\[
\iota K_S + [1 - (1 - n_S) f] e = L_S
\]  

(A.13)

\[
(1 - n_S)(1 - \delta) e f = [\iota + \rho (1 - n_S)] K_R K_S s_N
\]  

(A.14)

\[
(w - 1) \left[ n_S \left( \rho + \frac{\iota}{1 - n_S} \right) + \iota (1 - f) \right] e
\]

\[
= (n_S \rho + \iota) \left( \rho + \frac{\iota}{1 - n_S} \right) K_S S_S
\]  

(A.15)
The relative wage is determined exclusively by the last equation

\[
(A.16) \quad w = 1 + \frac{K_S S_S}{e} \left( \rho n_S + \frac{\rho + \frac{\nu}{1-n_S}}{\rho n_S + \frac{\nu}{1-n_S}} \right) > 1
\]

An increase in the Southern imitation subsidy causes a decrease in the relative wage \( \partial w / \partial S_S < 0 \). An increase in the Southern imitation subsidy fails to affect the other variables: \( \partial n_S / \partial S_S = 0 \), \( \partial v / \partial S_S = 0 \), and \( \partial e / \partial S_S = 0 \). Solve

\[
(A.17) \quad B \begin{bmatrix} \partial e \\ \partial \nu \\ \partial n_S \end{bmatrix} = \begin{bmatrix} L_S \partial L_R - K_S a_R \partial \kappa_R \\ 0 \\ [\nu + \rho(1-n_S)] K_N a_R \partial \kappa_R - K_R \partial S_N \end{bmatrix}
\]

where

\[
(A.18) \quad B = \begin{bmatrix} (1-n_S) \delta f & K_R K_S & -\delta ef \\ 1 - (1-n_S)f & K_S & ef \\ (1-n_S)(1-\delta) f & -K_R K_S S_N & -(1-\delta) ef + \rho K_R K_S S_N \end{bmatrix}
\]

noting

\[
(A.19) \quad |B| = K_R K_S^2 S_N \left[ \frac{K_R S_N \delta}{1-\delta} \left( \rho + \frac{\nu}{1-n_S} \right) + \frac{K_R \nu}{1-n_S} + \rho(1-n_S)(K_R + \delta) f \right] > 0
\]

to generate the effect of relative resources, relative IPR protection and Northern innovation subsidy on the measure of South dominance

\[
(A.20) \quad \frac{\partial n_S}{\partial L_R} = -L_S K_S \frac{K_R S_N \left[ 1 - (1-n_S)f \right] + (1-n_S)(1-\delta) f}{|B|} < 0
\]

\[
(A.21) \quad \frac{\partial n_S}{\partial \kappa_R} = a_R K_S^2 (1-n_S) S_N \left[ [\nu + \rho(1-n_S)] \delta f - \rho K_R [1 - (1-n_S)f] \right] \frac{\nu(1-\delta) f}{|B|} > 0
\]

\[
(A.22) \quad \frac{\partial n_S}{\partial S_N} = K_R K_S^2 \frac{[\nu + \rho(1-n_S)] [K_R - (1-n_S)(K_R + \delta) f]}{|B|} > 0
\]
the rate of innovation

\[
\frac{\partial \nu}{\partial L_R} = L_S K_R K_S S_N \left[ \frac{\nu}{1-n_S} + \rho(1-n_S)f \right] \frac{|B|}{|B|} > 0
\] 

(A.23)

\[
\frac{\partial \nu}{\partial K_R} = -a_R K_R K_S^2 S_N \left[ \frac{(1-\delta)(1-n_S)}{(1-\delta)(1-n_S)} + \frac{\nu}{1-n_S} + \rho(1-n_S)f \right] \frac{|B|}{|B|} < 0
\]

(A.24)

\[
\frac{\partial \nu}{\partial S_N} = K_R K_S \left[ \nu + \rho(1-n_S) \right] \delta ef \frac{|B|}{|B|} > 0
\]

(A.25)

and real expenditure

\[
\frac{\partial e}{\partial L_R} = L_S K_R K_S^2 S_N \left[ \frac{K_R S_N}{1-\delta} - 1 \right] \frac{\nu}{1-n_S} + \frac{\rho K_R S_N}{1-\delta} \frac{|B|}{|B|} > 0
\] 

(A.26)

\[
\frac{\partial e}{\partial K_R} = a_R K_R K_S^3 S_N \left( \frac{\nu}{1-n_S} + \rho \right) \frac{K_R(1-n_S) + \delta \left[ \nu + \rho(1-n_S) \right]}{1-\delta} + \frac{\nu^2}{1-n_S} \frac{|B|}{|B|} > 0
\]

(A.27)

\[
\frac{\partial e}{\partial S_N} = -K_R K_S^2 \left( K_R + \delta \right) \left[ \nu + \rho(1-n_S) \right] ef \frac{|B|}{|B|} < 0
\]

(A.28)

Recover the effect of the other parameters on the relative wage by employing the chain rule, noting \( \partial w/\partial e < 0 \), \( \partial w/\partial \nu > 0 \), and \( \partial w/\partial n_S < 0 \). Consequently, \( \partial w/\partial L_R > 0 \), \( \partial w/\partial K_R < 0 \), and \( \partial w/\partial S_N > 0 \).

REFERENCES


