Chapter 5

Applications of Rational Choice and Demand Theories

Chapter Outline

• Using The Rational Choice Model To Answer Policy Questions
• Consumer Surplus
• Overall Welfare Comparisons
• Using Price Elasticity Of Demand
• Intertemporal Choice Model
Application: A Gasoline Tax And Rebate Policy

- Policy proposal made during the administration of President Jimmy Carter
- Goal: use gasoline taxes to help limit the quantity demanded of gasoline.
  - Tax revenue would then be used to reduce the payroll tax (tax rebate).
- Would consumers buy the same amount of gasoline as before if the tax is rebated?

A Gasoline Tax And Rebate Policy

- Income M is $150/week, price of gas G is $1/gallon, price of composite good Y is $1.
- Budget constraint \( P_G G + P_Y Y = M \) so \( G + Y = 150 \) or \( Y = 150 - G \).
- Endpoints are maximum \( M/P_Y = 150 \) and maximum \( M/P_G = 150 \).
- Slope (negative relative price of gasoline) - \( P_G/P_Y = -1 \).
- Consume \( G = 58 \) gallons/week.
A Gasoline Tax And Rebate Policy

• Add 50% tax on gas so price of gas including tax is now $1.50.
• Budget constraint $1.5 \ G + Y = 150$ or $Y = 150 – 1.5 \ G$.
• Endpoints $Y = 150$ and $G = 100$.
• Slope now -1.5.
• Consume $G = 30$ gallons/week.

A Gasoline Tax And Rebate Policy

• Now refund the tax to consumers (in form of lower payroll taxes).
• Budget constraint retains the new steeper slope due to the higher price of gasoline (including the tax) but shifts out by the average tax raised per consumer – here $18$. $1.5 \ G + Y = 150 + 18 = 168$.
• Endpoints $Y = 168$ and $G = 112$.
• Consume $G = 36$ (more than 30 without refund but less than 58 before the tax).
Application: A Gasoline Tax And Rebate Policy

- Despite the rebate, the consumer substantially curtails gasoline consumption.
  - If gasoline is a normal good, the effect of the rebate is to offset partially the income effect of the price increase. It does nothing to alter the substitution effect.
Application: School Vouchers

- Policy Proposal: each family be given a **voucher** that could be used toward the tuition at any school of the family’s choosing.
- Current system: families who choose to go to private schools do not receive a refund on their school taxes.
- Question: what is the effect of vouchers on the level of resources devoted to education.

Application: School Vouchers

- Households have income $Y$. Required to pay tax $P_e$ for education regardless of whether send child to public school.
- Must consume at least 1 unit of educational quality. Provided for free at a public school.
- In current system, if send child to private school, must pay $P_e$ again. No credit for tax paid.
- Budget constraint horizontal at $Y - P_e$ from 0 to 1, then drops to $Y - 2P_e$, then has slope reflecting cost of additional units of educational quality.
Application: School Vouchers

- With a voucher system, the budget constraint would still be horizontal at $Y - P_e$ from 0 to 1, but would then reflect the slope of the cost of additional units of educational quality, without the drop down to $Y - 2P_e$.
- Households apt to chose at the kink in the budget constraint under the current system but many would opt to buy a bit more educational quality if with a voucher system as avoid double paying $P_e$. 
Application: School Vouchers

- Result from Consumer Choice Analysis: switching to a voucher system will increase the level of spending on education.
  - Parents no longer have to forfeit their school taxes when they switch from public to private schools
Consumer Surplus

• **Consumer surplus**: a dollar measure of the extent to which a consumer benefits from participating in a transaction.
  – In a graph → area between demand curve and price.

---

For Figure 5.4, suppose the demand for shelter is $P = 15 - Q$. What is consumer surplus at $P = $3/sq yd?

• Find quantity demanded is 12
  \[
  3 = 15 - Q \implies Q = 15 - 3 = 12
  \]

• Consumer surplus is triangle with base $Q = 12$ and height $15 - 3 = 12$

\[
CS = (15 - 3) \frac{12}{2} = 12(6) = 72
\]
Consumer Surplus Lost When Price Rises

- For Figure 5.5, find decline in consumer surplus when price increases from $2 to $3/gal for demand $P = 10 - Q$. $Q = 8$ at $P = 2$, $Q = 7$ at $P = 3$.
- Lost consumer surplus is a trapezoid with a height of $3 - 2 = 1$ and an average base of $(7 + 8)/2 = 7.5$

$$
\Delta CS = -(3 - 2) \frac{7 + 8}{2} = -7.5
$$

- Can also calculate the two consumer surpluses and subtract one from the other.
Two-part Pricing

• For Figure 5.6, demand \( P = 50 - Q/4 \). \( P = $25/hr. \) What is the largest fixed fee would be willing to pay? Willing to pay amount of consumer surplus.
  
  • At \( P = $25/hr, \) \( Q = 100 \) so consumer surplus is
    
    \[ CS = (50 - 25) \frac{100}{2} = 25(50) = $1,250 \]

  • If price decreases to \( P' = $20, \) \( Q' = 120 \) so
    
    \[ CS' = (50 - 20) \frac{120}{2} = 30(60) = $1,800 \]
Comparing Budget Constraints

• For Figure 5.7, first year prices $P_X = 10$, $P_Y = 20$, consume $X = 50$ and $Y = 25$.
• Second year prices $P_X = 10$, $P_Y = 10$ and income $M = 750$. In which year is the individual better off?
• New budget constraint $10X + 10Y = 750$ includes the original consumption $X = 50$, $Y = 25$ so no worse off.
• Relative price of $X$ increased, so will shift to consuming less $X$ and be better off.
Application: Welfare Effects of Changes in Housing Prices

Two scenarios:

1. You have just purchased a house for $200,000. The very next day, the prices of all houses, including the one you just bought, double.
2. You have just purchased a house for $200,000. The very next day, the prices of all houses, including the one you just bought, fall by half.

- In each case, how does the price change affect your welfare? (Are you better off before the price change or after?)
Figure 5.8: Rising Housing Prices and Welfare of Homeowners

Figure 5.9: Falling Housing Prices and Welfare of Homeowners
Application: A Bias in the Consumer Price Index

- **Consumer price index (CPI):** measures changes in the “cost of living,” the amount a consumer must spend to maintain a given standard of living.
  - Fails to take substitution into account hence overestimating the cost of living.
  - The bias will be larger when there are greater differences in the rates of increase of different prices.

CPI Bias

- For Figure 5.10, suppose rice and wheat are each priced at $1/lb. Rice and wheat are perfect substitutes at a one-to-one ratio.
- Initially, 20 lbs of each are consumed.
- Then the price of rice rises to $2/lb and the price of wheat rises to $3/lb.
- Initial expenditure $20 on rice plus $20 on wheat so $M = $40. $R + W = 40
CPI Bias

- The new budget constraint is $2R + 3W = M$.
- If consume $W = R = 20$, $2(20) + 3(20) = $100 compared to $40$ originally, a 150% increase.
- But cheaper to switch to 40 lbs of rice and no wheat, at a cost of $2(40) = $80, which leads to same level of satisfaction as initial bundle. Compared to $40$, $80$ is a 100% increase but less than the 150% increase for a fixed bundle.

Figure 5.10: The Bias Inherent in the Consumer Price Index
Application: The Marta Fare Increase

• In 1987 the Metropolitan Atlanta Rapid Transit Authority (MARTA) raised its basic fare from 60 to 75 cents/ride.
• In the 2 months following the fare increase, total system revenues rose 18.3 percent in comparison with the same period a year earlier.
• What do these figures tell us about the original price elasticity of demand for rides on the MARTA system? Demand inelastic.

Intertemporal Choice Model

• How would rational consumers distribute their consumption over time?
• Two time periods: current and future.
• Two goods: current consumption ($C_1$) versus future consumption ($C_2$).
Intertemporal Budget Constraint

- Current income $M_1$, future income $M_2$, real interest rate $r$
- Present value form
  \[ C_1 + \frac{C_2}{1 + r} = M_1 + \frac{M_2}{1 + r} \]
- Future value form (multiply above by $1+r$)
  \[(1 + r)C_1 + C_2 = (1 + r)M_1 + M_2\]
**Intertemporal Budget Constraint**

- **Maximums (endpoints):** Can consume
  - \( M_1 + M_2/(1+r) \), current income plus borrow against future income (present value of income), if consume everything today, or
  - \( (1+r) M_1 + M_2 \), current income with interest plus future income (future value of income), if wait and consume everything in the future.
- **Slope is** \(- (1 + r)\): every dollar you do not consume today permits you to consume \(1+r\), principal plus interest, in the future period.

---

**Intertemporal Budget Constraint**

- For Figure 5.13, \( M1 = 50,000 \), \( M2 = 60,000 \) and \( r = 20\% = 0.2 \). So \( 1 + r = 1.2 \).
  \[
  C_1 + \frac{C_2}{1 + r} = M_1 + \frac{M_2}{1 + r}
  \]
  \[
  C_1 + \frac{C_2}{1.2} = 50,000 + \frac{60,000}{1.2} = 100,000
  \]
- **Maximum now** 100,000.
- **Maximum later** \( 1.2(50,000) + 60,000 = 120,000 \).
- **Slope** \(- (1 + r) = -1.2 \). \( C_2 = 120,000 − 1.2C_1 \).
Figure 5.13: The Intertemporal Budget Constraint

Figure 5.14: Intertemporal Budget Constraint with Income in Both Periods, and Browsing or Lending at the Rate $r$
The Intertemporal Choice Model

- **Marginal rate of time preference**: the number of units of consumption in the future a consumer would exchange for 1 unit of consumption in the present.
  - It declines as one moves downward along an indifference curve.

![Figure 5.15: An Intertemporal Indifference Map](image)
Figure 5.16: The Optimal Intertemporal Allocation

Figure 5.17: Patience and Impatience
Rise in Interest Rate

- For Figure 5.18, $M_1 = 100,000$, $M_2 = 154,000$, $r = 10\%$, $r' = 40\%$.
- Initial budget constraint
  \[(1 + r)C_1 + C_2 = (1 + r)M_1 + M_2\]
  \[1.1C_1 + C_2 = 1.1(100,000) + 154,000 = 264,000 \rightarrow C_2 = 264,000 - 1.1C_1\]
- New budget constraint
  \[1.4C_1 + C_2 = 1.4(100,000) + 154,000 = 294,000 \rightarrow C_2 = 294,000 - 1.4C_1\]
Application: The Permanent Income And Life-Cycle Hypotheses

• *Permanent income hypothesis*: says that the primary determinant of current consumption is not current income but what he called *permanent income*.
  – *Permanent income*: the present value of lifetime income.

Figure 5.19: Permanent Income, not Current Income, is the Primary Determinant of Current Consumption
Problem 1

1. Larry demands strawberries according to the schedule \( P = 4 - \frac{Q}{2} \), where \( P \) is the price of strawberries ($/pint) and \( Q \) is the quantity (pints/wk). Assuming that the income effect is negligible, how much will he be hurt if the price of strawberries goes from $1/pint to $2/pint?

Solution 1

- \( P = 4 - \frac{Q}{2} \) or \( Q = 8 - 2P \) so \( Q = 6 \) at \( P = 1 \); \( Q = 4 \) at \( P = 2 \).
- Lost consumer surplus is a trapezoid with a height of \( 2 - 1 = 1 \) and an average base of \( \frac{4 + 6}{2} = 5 \)

\[
\Delta CS = -(2 - 1) \frac{4 + 6}{2} = -5
\]
- Can also calculate the two consumer surpluses and subtract one from the other: \( 4 - 9 = -5 \).
Problem 2

2. The only video rental club available to you charges $4 per movie per day. If your demand curve for movie rentals is given by \( P = 20 - 2Q \), where \( P \) is the rental price ($/day) and \( Q \) is the quantity demanded (movies per year), what is the annual maximum membership fee you would be willing to pay to join this club?
Solution 2

• Would be willing to pay a membership fee of up to the amount of consumer surplus.
• $P = 20 - 2Q$ so $Q = 10 - P/2$.
• At $P = $4, $Q = 8$ so consumer surplus is triangle with height $20 - 4 = 16$ and base $Q = 8$.

$$CS = (20 - 4) \frac{8}{2} = 16(4) = $64$$
Problem 3

3. Smith lives in a world with two time periods, this period and the next period. His income in each period, which he receives at the beginning of each period, is $210. If the interest rate is 0.05 per time period, what is the present value of his lifetime income? Draw his intertemporal budget constraint. On the same axes, draw Smith’s intertemporal budget constraint when $r = 0.20$.

Solution 3

- $M_1 = M_2 = 210$, $r = 5\%$, $r' = 20\%$. Present value of lifetime income $M_1 + M_2/(1+r) = 210 + 210/1.05 = 410$ for $r=5\%$ and $210 + 210/1.2 = 385$ for $r' = 20\%$.
- Initial budget constraint
  \[(1 + r)C_1 + C_2 = (1 + r)M_1 + M_2\]
  \[1.05C_1 + C_2 = 1.05(210) + 210\]
  \[= 430.5 \rightarrow C_2 = 430.5 - 1.05C_1\]
- New budget constraint
  \[1.2C_1 + C_2 = 1.2(210) + 210\]
  \[= 462 \rightarrow C_2 = 462 - 1.2C_1\]
4. Smith receives $100 of income in this period and $100 next period. At an interest rate of 10 percent, he consumes all his current income in each period. He has a diminishing marginal rate of time preference between consumption next period and consumption this period. True or false: If the interest rate rises to 20 percent, Smith will save some of his income this period.
Solution 4

• True. See Figure 5.18 (different numbers but same result).