Chapter 12

A Game-Theoretic Approach to Strategic Behavior
Chapter Outline

• An Introduction to the Theory of Games
  – Prisoner’s Dilemma
  – Nash Equilibrium
  – Maximin Strategy
  – Strategies for Repeated Play
• Sequential Games
• Commitment Problems
Game Theory

• *Game theory* is a means of analyzing markets where strategies matter.

• A *game* describes the players, timing, possible strategies and outcomes.

• A *strategy* is a statement of what move a player will choose.
Game Theory

• A key distinction in game structure is whether players move simultaneously (prior to knowing other players move) or take turns.
  – In a normal form game, players choose their actions at the same time.
  – In an extensive form (or sequential) game, players take turns choosing their actions.
Dominant Strategy

- **Dominant strategy**: the strategy in a game that produces better results irrespective of the strategy chosen by one’s opponent.
Prisoner’s Dilemma

• Two prisoners are held in separate cells for a serious crime that they did commit.
• The prosecutor has only enough hard evidence to convict them of a minor offense, for which the penalty is 1 year in jail.
• Each prisoner is told that if one confesses while the other remains silent, the confessor will go free while the other will spend 20 years in prison. If both confess, they will get an intermediate sentence of 5 years.
• The two prisoners are not allowed to communicate with one another.
Prisoner’s Dilemma

• A game is a prisoner’s dilemma if:
  1. Each player has a dominant strategy
  2. When both players choose their dominant strategy, each gets a smaller payoff than if each had chosen their dominated strategy
## Prisoner’s Dilemma

### Table 12.1 The Prisoner’s Dilemma

<table>
<thead>
<tr>
<th>Prisoner X</th>
<th>Prisoner Y</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Confess</strong></td>
<td><strong>Confess</strong></td>
</tr>
<tr>
<td>5 years for each</td>
<td>0 years for X, 20 years for Y</td>
</tr>
<tr>
<td><strong>Remain silent</strong></td>
<td><strong>Remain silent</strong></td>
</tr>
<tr>
<td>20 years for X, 0 years for Y</td>
<td>1 year for each</td>
</tr>
</tbody>
</table>
Prisoner’s Dilemma

• The dominant strategy is for each prisoner to confess.
• Yet when each confesses, each does worse (5 years) than if each had remained silent (1 year).
• Situation where rational pursuit of individual self-interest does not yield the best outcome for the players as a group.
Prisoner’s Dilemma

• Oligopolists trying to collude face a similar challenge.
• Suppose two firms agree to each produce half the monopoly level of output to split the market.
• Agreement will not be stable – each will want to undercut the other’s price and capture the entire market.
Prisoner’s Dilemma

• Firms may overspend on advertising for similar reason.
• If neither of two firms advertise, they split the market and have high profits of 500 each.
• If both firms advertise, their efforts cancel each other out and they still split the market but with lower profits of 250 each due to advertising costs.
• If one firm advertises and the other does not, the firm that advertises earns 750 and the other 0.
## Prisoner’s Dilemma

### Table 12.3 Advertising as a Prisoner’s Dilemma

<table>
<thead>
<tr>
<th>Firm 2</th>
<th>Firm 1</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Don’t advertise</td>
</tr>
<tr>
<td>Don’t advertise</td>
<td>$500 for each</td>
</tr>
<tr>
<td>Advertise</td>
<td>$0 for Firm 1, $750 for Firm 2</td>
</tr>
</tbody>
</table>

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Prisoner’s Dilemma

• Each firm has a dominant strategy of Advertise:
  – If Firm 1 advertises, Firm 2 earns more by advertising $250 > $0
  – If Firm 1 does not advertise, Firm 2 earns more by advertising $750 > $500
  – Ditto for Firm 1 wanting to advertise regardless of whether Firm 2 advertises.
Prisoner’s Dilemma

• Both firms pick their dominant strategy of advertising, and each earns $250.
• If they could somehow have committed to not advertise, they could have each earned $500.
• Instead, they wasted money advertising to fight over market share, and the advertising canceled each other out.
Nash Equilibrium

• What is the equilibrium if one (or both) players do not have a dominant strategy?
• *Nash equilibrium*: the combination of strategies in a game such that neither player has any incentive to change strategies given the strategy of his opponent.
  – A Nash equilibrium does not require both players to have a dominant strategy.
## Nash Equilibrium

<table>
<thead>
<tr>
<th>Firm 2</th>
<th>Firm 1</th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Don’t advertise</td>
<td>$500 for each</td>
<td>$750 for Firm 1, $0 for Firm 2</td>
</tr>
<tr>
<td>Don’t advertise</td>
<td>$0 for Firm 1, $450 for Firm 2</td>
<td></td>
<td>$250 for each</td>
</tr>
<tr>
<td>Advertise</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Nash Equilibrium

• Firm 1 still has a dominant strategy to Advertise but now Firm 2 does not:
  – If Firm 1 advertises, Firm 2 earns more by advertising $250 > $0
  – If Firm 1 does not advertise, Firm 2 earns more by not advertising $500 > $450

• The Nash Equilibrium is for both firms to advertise.
The Maximin Strategy

• **Maximin strategy**: choosing the option that makes the lowest payoff one can receive as large as possible.

• Cautious behavior when do not have a dominant strategy and not sure what other player is apt to do.
Tit-for-Tat

• If a prisoner’s dilemma game is repeated over and over (with the same two players), can the players somehow learn to cooperate?
• Or will they keep defecting over and over?
• Tit-for-tat is a way to support cooperation in a repeated play prisoner’s dilemma game.
Tit-for-Tat

- **Tit-for-tat strategy:** The first time you interact with someone, you cooperate. In each subsequent interaction you do what that person did in the previous interaction.

- Thus, if your partner defected on your first interaction, you would then defect on your next interaction with her. If she then cooperates, your move next time will be to cooperate as well.
  - Requirements: there not be a known, fixed number of future interactions (will unravel). Care about future payoffs. Can remember last strategy your partner played.
**Sequential Games**

- **Sequential game**: one player moves first, and the other is then able to choose his strategy with full knowledge of the first player’s choice.
  - Example - United States and the former Soviet Union (USSR) during much of the cold war.
Sequential Games

• If USSR launches missiles (node B), US best choice is to not retaliate (node E).
• If USSR does not launch missiles (node C), US best choice is to not launch either (node G).
• USSR chooses between 100 if launches (with no US launch) and 0 if does not launch (with no US launch), and decides to launch.
• US can threaten to retaliate against any USSR launch, but not optimal to do if payoffs are as in this decision tree.
Figure 12.1: Nuclear Deterrence as a Sequential Game
Sequential Games

• *Strategic entry deterrence* – change potential rivals’ expectations about how the firm will respond when its market position is threatened.

• Sears has an option to add a platform on top of building that would make adding to height cheaper in the future.
Sequential Game

• Without the platform, a new entrant knows that Sears will not decide to add on to the tower’s height, and entrant decides to enter and earns 60 (while Sears earns 40).

• With the platform installed, a new entrant knows that Sears will respond to its entry by adding height, and the entrant would earn -50 so entrant stays out and earns 0 instead.

Sears earns 90 by keeping the entrant out.
Figure 12.2: The Decision to Build the Tallest Building
Figure 12.3: Strategic Entry Deterrence

Company X

- Enter
  - Sears
    - Build higher
      - D: 40 for Sears, -50 for X
    - Don’t build higher
      - E: 30 for Sears, 60 for X

- Don’t enter
  - C: 90 for Sears, 0 for X

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Commitment Problems

• *Commitment problems*: games where the common feature is that people can do better if they can commit themselves to behave in a way that will later be inconsistent with their own material interests.

• *Commitment device*: a device that commits a person to behave in a certain way in the future, even though he may wish to behave otherwise when the time comes.
## Problem 1

<table>
<thead>
<tr>
<th>Tariff Wars</th>
<th></th>
<th>The United States</th>
</tr>
</thead>
<tbody>
<tr>
<td>China</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Free trade</td>
<td>$100 for each</td>
<td>$200 for the United States, $500 for China</td>
</tr>
<tr>
<td>Tariff</td>
<td>-$500 for the United States, $200 for China</td>
<td>-$300 for each</td>
</tr>
</tbody>
</table>

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Problem 1

1. If the United States adopts a tariff, would China choose a tariff or free trade? If the United States picks free trade, would China choose a tariff or free trade? Does China have a dominant strategy? If yes, what? Does the United States have a dominant strategy? If yes, what? What is the equilibrium outcome in this game? Is this game a prisoner’s dilemma?
Solution 1

1. If the United States adopts a tariff, China would choose a tariff over free trade (\(-$300 > -$500\)). If the United States picks free trade, China would choose a tariff over free trade (\($200 > $100\)). China has a dominant strategy of imposing a tariff. The United States also has a dominant strategy of imposing a tariff. Thus, the equilibrium outcome in this game is both countries adopt tariffs.
Solution 1

1. This game is a prisoner’s dilemma. Each player has a dominant strategy and when both players choose their dominant strategy, each gets a smaller payoff than if each had chosen their dominated strategy. The countries would be better off if somehow they could both commit to free trade and each earn $200 instead of both imposing tariffs and each earning -$300.
## Problem 2

<table>
<thead>
<tr>
<th>Water Wars</th>
<th>Crystal Springs</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td><strong>P = $1</strong></td>
</tr>
<tr>
<td><strong>Castle Springs</strong></td>
<td>$500 for each</td>
</tr>
<tr>
<td><strong>P = $1</strong></td>
<td>$0 for Crystal Springs, $990 for Castle Springs</td>
</tr>
<tr>
<td><strong>P = $0.99</strong></td>
<td></td>
</tr>
</tbody>
</table>
Problem 2

2. Two water suppliers can collude by charging price $1 and splitting the market. If one undercuts the price to $0.99, will capture the whole market. If both charge $0.99, the split the market but at a lower price than were both to charge $1. Do the firms have a dominant strategy? If so what? What will happen in equilibrium? What if the firms then decide between $0.99 and $0.98?
Solution 2

2. If Crystal Springs charges $1, Castle Springs will charge $0.99 (as $990 > $500) and capture the whole market. If Crystal Springs charges $0.99, Castle Springs will charge $0.99 (as $495 > $0), preferring to split the market over letting its rival sell everything. Castle Springs has a dominant strategy of charging $0.99, as does Crystal Springs (the game is symmetric). Equilibrium will be each charging $0.99. Next equilibrium would be $0.98 and so on until price equals marginal cost.
## Problem 3

### Car Wars

<table>
<thead>
<tr>
<th>Car</th>
<th>Chevrolet Corvette</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>No hybrid</td>
</tr>
<tr>
<td>Dodge Viper</td>
<td>$50 million for each</td>
</tr>
<tr>
<td>No hybrid</td>
<td>$70 million for Chevy, $80 million for Dodge</td>
</tr>
<tr>
<td>Hybrid</td>
<td></td>
</tr>
</tbody>
</table>
Problem 3

3. Chevrolet (Chevy) and Dodge are deciding whether to offer a hybrid version of the Corvette and Viper. Do the firms have a dominant strategy? If so what? What will happen in equilibrium? Is this game a Prisoner’s Dilemma? Why or why not?
3. If Chevy makes a hybrid Corvette, Dodge will opt to not make a hybrid version of its Viper ($70 > $60). If Chevy does not make a hybrid Corvette, Dodge wants to make a hybrid Viper ($80 > $50). Dodge does not have a dominant strategy as its strategy depends on the strategy chosen by its rival. Similarly for Chevy.
3. There are two Nash equilibrium in the game: 
{Chevy hybrid, Dodge not} and {Chevy not, Dodge hybrid}. If Chevy makes a Corvette hybrid and Dodge does not make a hybrid version of its viper, neither firm would want to change its strategy, given the strategy of its rival. Same for if the roles are reversed. There is no telling which of the two possible equilibria will happen when the game is played simultaneously.
Problem 4

4. Suppose similar situation as in Problem 3, but now Dodge gets to decide first, then Chevy. What would the equilibrium be now? Why does timing matter?
4. If Dodge decides to make a hybrid Viper, Chevy will opt to not offer a hybrid Vette (as $70 > $60). If Dodge decides to not offer a hybrid Viper, then Chevy will decide to offer a hybrid Vette (as $80 > $50). Anticipating Chevy’s response, Dodge will offer a hybrid Viper, preferring a payoff of $80 million from {Dodge offers Viper hybrid, Chevy not} over a payoff of $70 million from {Dodge not, Chevy offers Vette hybrid}. The order of play permits Dodge to pick its most preferred of the two possible outcomes.