REVIEW 4.1 SOLUTIONS (U.S. PRODUCTION)

In either country, producing one yard of cloth uses 2 units of labor and 1 acre of land, while producing one pound of food uses 2 units of labor and 3 acres of land. The United States has 900 units of labor and 600 acres of land.

a. What is the United States labor constraint?
\[ a_{LC} Q_C + a_{LF} Q_F = L \rightarrow 2 Q_C + 2 Q_F = 900 \]
\[ \rightarrow Q_F = 450 - Q_C \]

b. What is the most cloth the United States can produce, if constrained by only labor?
\[ Q_C^L = \frac{L}{a_{LC}} = \frac{900}{2} = 450 \]

c. What is the most food the United States can produce, if constrained by only labor?
\[ Q_F^L = \frac{L}{a_{LF}} = \frac{900}{2} = 450 \]

d. What is the United States land constraint?
\[ a_{TC} Q_C + a_{TF} Q_F = T \rightarrow Q_C + 3 Q_F = 600 \]
\[ \rightarrow Q_F = 200 - \frac{1}{3} Q_C \]
e. What is the most cloth United States can produce, if constrained by only land?

\[
\overline{Q}_C^T = \frac{T}{a_{TC}} = \frac{600}{1} = 600
\]

f. What is the most food United States can produce, if constrained by only land?

\[
\overline{Q}_F^T = \frac{T}{a_{TF}} = \frac{600}{3} = 200
\]

g. What United States production bundle fully employs both factors?

*The United States produces 75 pounds of food and 375 yards of cloth.*

\[
450 - Q_C = 200 - \frac{1}{3}Q_C
\]

\[
\left(1 - \frac{1}{3}\right)Q_C = 450 - 200
\]

\[
\frac{2}{3}Q_C = 250 \rightarrow Q_C = 375
\]

\[
Q_F = 450 - Q_C = 450 - 375 = 75
\]
REVIEW 4.2 SOLUTIONS (RUSSIAN PRODUCTION)
Russia has 900 units of labor and 900 acres of land.

a. What is the Russian land constraint?
   \[ a_{TC} Q_c^* + a_{TF} Q_f^* = T^* \rightarrow Q_c^* + 3 Q_f^* = 900 \]
   \[ \rightarrow Q_f^* = 300 - \frac{1}{3} Q_c^* \]

b. What is the most cloth Russia can produce, if constrained by only land?
   \[ \bar{Q}_c^{T^*} = \frac{T^*}{a_{TC}} = \frac{900}{1} = 900 \]

c. What is the most food Russia can produce, if constrained by only land?
   \[ \bar{Q}_f^{T^*} = \frac{T^*}{a_{TF}} = \frac{900}{3} = 300 \]

d. What Russian production bundle fully employs both factors?
   Russia produces 225 pounds of food and 225 yards of cloth.
   \[ 450 - Q_c^* = 300 - \frac{1}{3} Q_c^* \]
   \[ \rightarrow \left( 1 - \frac{1}{3} \right) Q_c^* = 450 - 300 \]
   \[ \rightarrow \frac{2}{3} Q_c^* = 150 \rightarrow Q_c = 225 \]
\[ Q_{F}^* = 450 - Q_{C}^* = 450 - 225 = 225 \]

e. Compare the two countries’ supplies of cloth relative to food. The United States produces more cloth relative to food than Russia.

\[
\frac{Q_{C}}{Q_{F}} = 5 \cdot \frac{375}{75} > \frac{225}{225} = 1 = \frac{Q_{C}^*}{Q_{F}^*}
\]
REVIEW 4.3 SOLUTIONS (U.S. AUTARKY PRICING)
The price of food is always $240/pound. In the United States, the price of cloth is $120/yard in autarky.

a. What is the pricing equation for food?
\[ a_{LF}w + a_{TF}r = P_F \rightarrow 2w + 3r = 240 \]
\[ r = 80 - \frac{2}{3}w \]

b. What is the maximum wage, if constrained by only the price of food?
\[ \bar{w}_F = \frac{P_F}{a_{LF}} = \frac{240}{2} = 120 \]

c. What is the maximum rent, if constrained by only the price of food?
\[ \bar{r}_F = \frac{P_F}{a_{TF}} = \frac{240}{3} = 80 \]

d. What is the U.S. pricing equation for cloth in autarky?
\[ a_{LC}w^A + a_{TC}r^A = P_C^A \rightarrow 2w^A + r^A = 120 \]
\[ r^A = 120 - 2w^A \]
e. What is the maximum U.S. autarky wage if constrained by only the price of cloth?

\[ \bar{w}_C^A = \frac{P_C^A}{a_{LC}} = \frac{120}{2} = 60 \]

g. What U.S. autarkic factor prices allow both goods to be priced at cost?

*United States autarkic factor prices are wage 30 and rent 60.*

\[ 120 - 2w^A = 80 - \frac{2}{3}w^A \]

\[ \rightarrow \left( 2 - \frac{2}{3} \right)w^A = 120 - 80 \]

\[ \rightarrow \left( \frac{4}{3} \right)w^A = 40 \rightarrow w^A = 30 \]

\[ r^A = 120 - 2w^A = 120 - 2(30) = 120 - 60 = 60 \]
REVIEW 4.4 SOLUTIONS (FREE TRADE PRICING)
In both countries, the price of cloth is $144/yard under free trade.

a. What is the pricing equation for cloth under free trade?
\[ a_{LC} w + a_{TC} r = P_C \rightarrow 2w + r = 144 \]
\[ \rightarrow r = 144 - 2w \]

b. What is the maximum wage if constrained by only the price of cloth under free trade?
\[ \bar{w}_C = \frac{P_C}{a_{LC}} = \frac{144}{2} = 72 \]

c. What is the maximum rent, if constrained by only the price of cloth under free trade?
\[ \bar{r}_C = \frac{P_C}{a_{TC}} = \frac{144}{1} = 144 \]

d. What factor prices under free trade allow both goods to be produced at cost?
*Factor prices are wage 48 and rent 48 under free trade.*

\[ 144 - 2w = 80 - \frac{2}{3}w \]
\[ \rightarrow \left(2 - \frac{2}{3}\right)w = 144 - 80 \]
\[ \rightarrow \frac{4}{3}w = 64 \rightarrow w = 48 \]
\[ r = 144 - 2w = 144 - 2(48) = 144 - 96 = 48 \]
e. How should factor prices in the United States and Russia compare under free trade.

*The factor prices are the same across countries due to the same technology and the same prices of goods.*

f. Compare U.S. factor prices under free trade to autarky.

*The wage to rent ratio rises in the United States in the move from autarky to free trade because labor is relatively abundant in the United States.*

\[
\frac{w}{r} = 1 = \frac{48}{48} > \frac{30}{60} = \frac{1}{2} = \frac{w^A}{r^A}
\]

g. How should Russian factor prices under free trade compare to autarky?

*In Russia, the wage to rent ratio falls in the move from autarky to free trade because labor is relatively scarce in Russia.*

h. Who would oppose free trade in each country?

*United States landlords*

\[
\hat{r} < \hat{P}_F < \hat{P}_C < \hat{w} \rightarrow -20\% < 0 < 20\% < 60\%
\]

*and Russian workers would oppose a free trade agreement because the purchasing power of their income would fall.*
**REVIEW 4.5 SOLUTIONS (COMPARATIVE ADVANTAGE)**

a. Which country is relatively abundant in labor to land?

*United States has a relative abundance in labor to land.*

\[
\frac{3}{2} = \frac{900}{600} = \frac{L}{T} > \frac{L^*}{T^*} = \frac{900}{900} = 1
\]

b. Which good makes relatively intensive use of labor to land?

*Cloth production is relatively intensive in labor to land.*

\[
\frac{2}{1} = \frac{a_{LC}}{a_{TC}} > \frac{a_{LF}}{a_{TF}} = \frac{2}{3}
\]

c. Determine the pattern of comparative advantage.

*The United States has comparative advantage in cloth and Russia in food.*

d. Determine the pattern of trade.

*The United States exports cloth and Russia exports food.*