Quality Ladders and Product Cycles

Gene M. Grossman and Elhanan Helpman


Product Cycle & Quality Improvement

- Product differentiation can take two forms: horizontal (variety) and vertical (quality).
- GH had product cycle model where innovations introduced new varieties.
- Also had model of endogenous quality upgrading.
- Put together to get product cycle model where innovations introduce higher quality levels of existing products.
- Following innovation, Northern firm earns profits until a Southern firm imitates, who earns profits until another innovation occurs.
Product Quality

- Each product can be improved repeatedly.
- Increments to quality are common across all products and exogenously given by the parameter $\lambda > 1$.
- Initial lowest quality level of a good is normalized to $q_0(j) = 1$ [j is $\omega$ in article].
- After $m$ [j in article] improvements in product $j$, its highest available quality is $q_m(j) = \lambda^m$.

Consumers Problem

- Maximize additively separable intertemporal utility function:
  $$U_i = \int_0^\infty e^{-\rho t} \log u_i(t) dt$$
- Instantaneous utility
  $$\log u_i(t) = \int_0^t \log \sum m \lambda^m x_{im}(j, t) dj$$
- Subject to intertemporal budget constraint
  $$\int_0^\infty e^{-R(t)} E_i(t) dt \leq A_i(0) \int_0^\infty e^{-R(t)} Y_i(t) dt$$
Consumers Solution

- Choose the highest quality level available of each product at each point in time as offers lowest quality-adjusted price.
  \[ \frac{p_m(j,t)}{p_{m-1}(j,t)} = \lambda > 1 \]
- Evenly spread expenditure across continuum of products and across time
  \[ E_i(j,t) = E_i(t) = E_i \]
- Aggregate expenditure \( E = E_N + E_S \)
  \[ E_i = \int_0^1 \sum_m p_m(j,t)x_m(j,t)\,dj \]

Innovation

- Follows a Poisson process. At each time interval \( dt \), the probability of success is \( t_N \, dt \). The intensity of innovation \( t_N \) indicates a firm’s effort.
- The cost of innovation is \( w_N a_{Nk} \, t_N \, dt \). When innovation occurs with finite positive intensity, its value \( v_{Nk} \) has to equal to \( w_N a_{Nk} \).
Leaders and Followers

- Distinguish between innovation undertaken by firms who successfully developed the current generation of the product (called leaders), and all other firms (called followers).
- Innovation is easier for leaders than followers $a_{NL} < a_{NF}$ due to product-specific knowledge acquired.
  - Outsiders observe only the product and not the process that led to its discovery.

Imitation

- Follows a Poisson process. At each time interval $dt$, the probability of success is $\mu_s dt$. The intensity of imitation $\mu_s$ indicates a firm’s effort.
- The cost of imitation is $w_s a_s \mu_s dt$. When imitation occurs with finite positive intensity, its value $v_s$ has to equal to $w_s a_s$. 
Three Types of Market Structures

- NN: Northern firm who innovated over another Northern firm and so has a Northern firm one quality level down as closest rival.
- NS: Northern firm who innovated over a Southern firm and so has a Southern firm one quality level down as closest rival.
- S: Southern firm who imitated the product of a Northern firm producing same quality level.

Production Costs

- Normalize unit labor requirement in production to one in each country.
- Normalize prices by the Southern wage (set $w_S = 1$).
- Define $w = w_N / w_S = w_N$ as the Northern relative wage, the wage in the North relative to the wage in the South.
- Producing one unit of output in the North costs $w > 1$ and in the South costs one.
Limit Pricing

- NN: Set price equal to quality increment times Northern wage (Northern wage is cost of closest rival).
  \[ p_{NN} = \lambda w \]

- NS: Set price equal to quality increment (closest rival has cost of one).
  \[ p_{NS} = \lambda \]

- S: Set price equal to Northern wage (cost of closest rival). Same quality level so no quality premium.
  \[ p_S = w \]

Profits

\[ \pi_{NN} = (p_{NN} - w)x_{NN} = (\lambda w - w)E / \lambda w = (1 - \lambda / \lambda)E \]
\[ \pi_{NS} = (p_{NS} - w)x_{NS} = (\lambda - w)E / \lambda = (1 - w / \lambda)E \]
\[ \pi_S = (p_S - 1)x_S = (w - 1)E / w = (1 - 1 / w)E \]
Valuations

- Northern firms of both types face risk of innovation and imitation.
  \[ v_{NN} = \frac{\pi_{NN}}{\rho + t_{NN} + \mu_S}, \quad v_{NS} = \frac{\pi_{NS}}{\rho + t_{NN} + \mu_S} \]
- Southern firms face risk of only innovation.
  \[ v_S = \frac{\pi_S}{\rho + t_{NS}} \]

Valuation Conditions

- Northern followers (compete against Northern rival)
  \[ E(1 - \delta) = wa_{NF} (\rho + t_{NN} + \mu_S) \]
- Northern leaders (compete against Southern rival)
  \[ E(1 - w\delta) = wa_{NL} (\rho + t_{NN} + \mu_S) \]
- Southern firms
  \[ E(1 - 1/ w) = a_S (\rho + t_{NS}) \]
Aggregation

- Measures of production sum to one:
  \[ n_{NN} + n_{NS} + n_S = 1 \]
- Aggregate rate of innovation equals:
  \[ \iota = \iota_{NS} n_S + \iota_{NN} (n_{NN} + n_{NS}) \]
- Aggregate rate of imitation equals:
  \[ \eta = \mu_S (n_{NN} + n_{NS}) \]
- Flows in must equal flows out of each type of market structure.

Resource Constraints

- Northern labor constraint requires labor demand for innovation and production to not exceed Northern labor supply:
  \[ a_{NN} t_{NN} n_N + a_{NS} t_{NS} n_S + \left( \frac{n_{NN}}{W} + n_{NS} \right) E \delta = L_N \]
- Southern labor constraint requires labor demand for production to not exceed Southern labor supply:
  \[ a_S \mu_S n_N + n_S \frac{E}{W} = L_S \]
Comparative Statics

- The main equations are the innovation and imitation valuation conditions and the Northern and Southern labor constraints.
- Consider separately case with efficient followers (innovation targets all products) and with inefficient followers (innovation targets only products where latest quality level has been imitated).

Results

- For inefficient followers, aggregate rate of innovation equals that of imitation. Both increase with increases in either labor supply or increases in subsidies to either innovation or imitation.
- For efficient followers, aggregate rate of innovation increases with increases in the Northern labor supply or innovation subsidy, decreases with imitation subsidy, and unclear result for Southern labor supply. Aggregate rate of imitation increases with increases in either labor supply or imitation subsidy and decreases with increases in innovation subsidy.
## Conclusion

- Effects of parameters on rates of innovation and imitation depend on whether:
  - Innovation enhances variety or quality
  - Innovation by leaders or followers
  - Imitation endogenous or exogenous