5 PRODUCT DIFFERENTIATION

Introduction

We have seen earlier how pure external IRS can lead to intra-industry trade. Now we see how product differentiation can provide a basis for trade due to consumers valuing variety. When trade occurs due to product differentiation, even identical countries will trade by exchanging different varieties of the same good. The value consumers place on variety generates another source of gains from trade. IRS due to a fixed cost of producing each variety limits the number of varieties produced by a country. Two key versions of modeling preferences for the differentiated products are the *love of variety* approach and the *ideal variety* (bliss point or spatial) approach. Both provide a subutility function that increases in the number of varieties available, but the love of variety approach is easier to employ.

Product Differentiation (Helpman and Krugman 1985)

The CES utility function has proved very useful in models of product differentiation. The typical form of modeling preferences is to assume an upper-tiered utility function

$$u(x_0, V) = U\left(x_0, V(x_1, ..., x_n)\right) \tag{5.1}$$

where x_0 is consumption of some homogeneous numeraire good, $x_1, ..., x_n$ are consumptions of n differentiated goods, and V is a sub-utility function for a set of differentiated products. Utility is separable between the set of differentiated goods and the numeraire. Apply a two stage budgeting procedure to allocate spending across differentiated products and then between the set of differentiated products and the numeraire.

We assume that preferences are homothetic between the numeraire good x_0 and the set of differentiated goods $x_1 \ldots x_n$ so *consumers spend a fixed share of their income on the two categories of goods.* Suppose the upper tier utility function is Cobb-Douglas in the numeraire good and the set of differentiated goods

$$u(x_0, V) = x_0^{\alpha} V^{1-\alpha} \tag{5.2}$$

so the elasticity of substitution between the differentiated goods and the numeraire good equals one. Normalizing the price of the numeraire good to one $p_0 = 1$, the consumer's budget constraint sets expenditure equal to income

$$x_0 + \sum_{i=1}^{n} p_i x_i = I \tag{5.3}$$

where p_i is the price of good i and I is income in terms of the numeraire good. When preferences are homothetic, the consumer spends a fixed proportion of income I on the two set of goods: $x_0 = \alpha I$ on the numeraire good and $\sum_{i=1}^{n} p_i x_i = (1-\alpha)I$ on all the differentiated goods. Let

$$E \equiv I - x_0 = (1-\alpha)I \tag{5.4}$$

be expenditure on the set of differentiated products. Thus, the budget constraint for spending on differentiated goods, given E, is

$$\sum_{i=1}^{n} p_i x_i = E \tag{5.5}$$

Love of Variety

Suppose the sub-utility function is a symmetrical CES function

$$V = \left(\sum_{i=1}^{n} x_i^{\rho} \right)^{\frac{1}{\rho}}, \ \rho < 1 \tag{5.6}$$

This subutility function has several nice properties:

- Every pair of varieties is equally substitutable:

$$\sigma = \frac{1}{1-\rho} > 1 \longleftrightarrow \rho = 1 - \frac{1}{\sigma} \tag{5.7}$$

- Degree of substitution does not depend upon the level of consumption of the goods.

- Variety has value. Suppose n varieties are available at the same price p. Then the consumer buys equal amounts of all goods. Subutility can be written as

$$V = \left(\sum_i x_i^\rho\right)^{\frac{1}{\rho}} = (nx^\rho)^{\frac{1}{\rho}} \tag{5.8}$$

$$= \left[n\left(\frac{E}{np}\right)^\rho\right]^{\frac{1}{\rho}} = n^{\frac{1}{\rho}}\frac{E}{np} = n^{\frac{1}{\rho}-1}\frac{E}{p}$$

and increases as the number of varieties n increases.

$$\frac{\partial V}{\partial n} = \left(\frac{1}{\rho} - 1\right) nV > 0 \tag{5.9}$$

If the number of differentiated goods is large, the set of differentiated goods may be represented by a continuum, so the sum is then replaced by an integral in the subutility function,

$$V = \left[\int_0^n x_i^\rho di\right]^{\frac{1}{\rho}} \tag{5.10}$$

which can be more convenient. The goal is to maximize subutility V subject to the budget constraint

$$\int_0^n p_i x_i di = E \tag{5.11}$$

Given an expenditure E on the differentiated goods (from the first stage), the consumer's problem becomes

$$\max \quad \left[\int_0^n x_i^\rho di\right]^{\frac{1}{\rho}} + \lambda\left[E - \int_0^n p_i x_i di\right] \tag{5.12}$$

The first order condition for good i is

$$\frac{1}{\rho}\left[\int_0^n x_i^\rho di\right]^{\frac{1}{\rho}-1} \rho x_i^{\rho-1} - \lambda p_i = 0 \tag{5.13}$$

and similarly for another good j

$$\frac{1}{\rho}\left[\int_0^n x_i^\rho di\right]^{\frac{1}{\rho}-1} \rho x_j^{\rho-1} - \lambda p_j = 0 \tag{5.14}$$

The above two first order conditions imply

$$\frac{x_i}{x_j} = \left[\frac{p_j}{p_i}\right]^{\frac{1}{1-\rho}} = \left[\frac{p_j}{p_i}\right]^\sigma \tag{5.15}$$

If goods are equally priced, then they will be equally demanded

$$p_i = p_j \to \frac{p_j}{p_i} = 1 \to \frac{x_i}{x_j} = 1 \to x_i = x_j \tag{5.16}$$

For a CES utility function, the elasticity of substitution between two different varieties is $\sigma = \frac{1}{1-\rho}$. Using demand functions,

$$E = \int_0^n x_j p_i^{1-\sigma} p_j^{\sigma} di = x_j p_j^{\sigma} \int_0^n p_i^{1-\sigma} di \tag{5.17}$$

implies

$$x_j = \frac{E p_j^{-\sigma}}{\int_0^n p_i^{1-\sigma} di} = \frac{E p_j^{\frac{1}{\rho-1}}}{\int_0^n p_i^{-\frac{\rho}{1-\rho}} di} \tag{5.18}$$

An individual firm views $\int_0^n p_i^{1-\sigma} di$ as fixed and thus faces a constant elasticity demand curve

$$x_j = k p_j^{-\sigma} \tag{5.19}$$

where

$$k \equiv \frac{E}{\int_0^n p_i^{1-\sigma} di} \tag{5.20}$$

with demand elasticity equal to the elasticity of substitution σ between a pair of the differentiated goods.

Each firm chooses the price of its variety to maximize its profits, taking as given the price charged by other firms. Assume that every variety is produced with the same production function. Focus on a representative firm (producing a unique variety), whose problem is to pick its price to maximize its profits

$$\pi = px - C(x) \tag{5.21}$$

Suppose the cost function takes the forms of a fixed cost plus a constant marginal cost

$$C(x) = b + cx \tag{5.22}$$

Then profits are

$$\pi = (p-c)x - b = (p-c)kp^{-\sigma} - b \tag{5.23}$$

The first order condition for profit maximization is

$$p\left(1 - \frac{1}{\sigma}\right) = c \tag{5.24}$$

which implies that all varieties are priced equally at

$$p = \frac{c}{1 - \frac{1}{\sigma}} \tag{5.25}$$

and in the limit as elasticity becomes infinite, price equals cost and $\lim_{\sigma\to\infty} p = c$. With all varieties priced equally, the budget constraint $nxp = E$ implies that consumers evenly spread consumption over all available varieties

$$x = \frac{E}{np} \tag{5.26}$$

A zero profit condition $\pi = 0$ then pins down the measure of varieties available.

$$n = \frac{E}{b\sigma} \tag{5.27}$$

The measure of varieties available decreases in the elasticity σ and the fixed cost of variety b

$$\frac{\partial n}{\partial \sigma} = -\frac{E}{b\sigma^2}, \ \frac{\partial n}{\partial b} = -\frac{E}{b^2\sigma} \tag{5.28}$$

as intuitively should happen based on cost and benefit.

Krugman *AER* 1980

Krugman's model gives an explanation for the trade between countries with similar (even identical) factor endowments (same technology and tastes too) that illustrates how having a large domestic market operates as a source of comparative advantage (export goods for which have a larger domestic market than other goods relative to ROW). The model is derived from the famous Dixit and Stiglitz (1977) model of monopolistic competition (horizontal product differentiation).

Consumers

A large number of potential goods enter symmetrically into utility according to the utility function

$$U = \sum_{i=1}^{n} c_i^{\theta}, \ 0 < \theta < 1 \tag{5.29}$$

where c_i is consumption of good i. Utility exhibits love of variety. Preferences exhibit a constant elasticity of substitution between any two goods. Consumers choose their consumptions

c_i to maximize their utility (5.29) subject to the budget constraint

$$\sum_{i=1}^{n} p_i c_i = I$$

where p_i is the price of good i and I is income. Consumers solve

$$\max_{c_i} \sum_{i=1}^{n} c_i^{\theta} + \lambda \left(I - \sum_{i=1}^{n} p_i c_i \right)$$

where λ is the shadow price on the budget constraint (the marginal utility of income). The first order conditions (FOC) are

$$\theta c_i^{\theta-1} - \lambda p_i = 0, \; i = 1, \ldots, n \tag{5.30}$$

which can be rearranged to give individual consumer's demand

$$p_i = \frac{\theta}{\lambda} c_i^{\theta-1} \tag{5.31}$$

Since everyone is identical it must be that consumption of each good equals production of each good per consumer

$$c_i = \frac{x_i}{L} \tag{5.32}$$

where L is the labor supply (measure of consumers as each consumer has one unit of labor supply). Substituting for c_i, we get the market demand function for good i

$$p_i = \frac{\theta}{\lambda} \left(\frac{x_i}{L} \right)^{\theta-1} \tag{5.33}$$

Firms face a CES demand function with elasticity $\frac{1}{1-\theta}$. No strategic interdependence among firms as the number of available varieties is assumed to be large. Each good i will be produced by only one firm: all firms differentiate their product from all products offered by other firms.

Producers

Suppose there is only one factor, labor. Let the cost function for each good be given by

$$l_i = \alpha + \beta x_i, \; \alpha > 0, \; \beta > 0 \tag{5.34}$$

where l_i is the labor needed to produce x_i units of good i. α gives the fixed cost and β the marginal cost (in terms of labor). Hence have IRS internal to the firm: average cost declines with output (at a decreasing rate). Each producer chooses its output x_i to maximize its profits

$$\pi_i = p_i x_i - w l_i \tag{5.35}$$

where w is the wage paid to labor. Substituting the demand function (5.33) for p_i and the cost function (5.34) for l_i, the firm picks its output x_i to maximize its profits

$$\pi_i = \frac{\theta x_i^\theta}{\lambda}\left(\frac{1}{L}\right)^{\theta-1} - w(\alpha + \beta x_i) \tag{5.36}$$

The first order conditions are

$$\frac{\partial \pi_i}{\partial x_i} = \left(\frac{\theta^2}{\lambda}\right) x_i^{\theta-1}\left(\frac{1}{L}\right)^{\theta-1} - \beta w = 0,\ i = 1, \ldots, n \tag{5.37}$$

which can be rearranged to read

$$\frac{\theta}{\lambda}\left(\frac{x_i}{L}\right)^{\theta-1} = \frac{\beta w}{\theta} \tag{5.38}$$

The left-hand side is recognized as p_i from the demand function, so the FOC implies

$$p = p_i = \frac{\beta w}{\theta} \tag{5.39}$$

where $\frac{1}{\theta} > 1$ is the markup over marginal cost βw: price is a fixed mark up over marginal cost reflecting the substitutability of varieties. Clearly, the price increases with the marginal cost parameter β and decreases with θ

$$\frac{\partial p}{\partial \beta} = \frac{w}{\theta} > 0,\ \frac{\partial p}{\partial \theta} = -\frac{\beta w}{\theta^2} < 0 \tag{5.40}$$

Profits can be expressed as

$$\pi_i = (p - \beta w)\, x_i - w\alpha = \left[\left(\frac{1}{\theta} - 1\right)\beta x_i - \alpha\right] w \tag{5.41}$$

Free entry drives equilibrium profits to zero. Solving $\pi_i = 0$ for x_i gives

$$x = x_i = \frac{\alpha}{\frac{p}{w} - \beta} = \frac{\alpha\theta}{\beta(1-\theta)} \tag{5.42}$$

Equilibrium output increases in the fixed cost of variety α and θ but decreases in the marginal cost parameter β

$$\frac{\partial x}{\partial \alpha} = \frac{\theta}{\beta(1-\theta)} > 0,\ \frac{\partial x}{\partial \theta} = \frac{\alpha}{\beta(1-\theta)^2} > 0,\ \frac{\partial x}{\partial \beta} = -\frac{\alpha\theta}{\beta^2(1-\theta)} < 0 \tag{5.43}$$

Equilibrium output is constant across goods and independent of the resources in the economy. An increase in resources L increases only the number of varieties n that are produced (see below), not the output of each variety x.

Full Employment

Full employment requires that labor demand nl_i equal labor supply L

$$L = n(\alpha + \beta x) = \frac{n\alpha}{1-\theta} \tag{5.44}$$

where n is the number of varieties actually produced. Solving the full employment condition for n gives

$$n = \frac{L}{\alpha + \beta x} = \frac{L(1-\theta)}{\alpha} \tag{5.45}$$

The above equations provide a complete description of the autarkic equilibrium in an economy. As resources L increase, the number of available varieties n increases.

$$\frac{dn}{dL} = \frac{1-\theta}{\alpha} > 0 \tag{5.46}$$

An increase in the substitutability of goods θ decreases n.

$$\frac{dn}{d\theta} = -\frac{L}{\alpha} < 0 \tag{5.47}$$

An increase in the fixed cost of variety α decreases n.

$$\frac{dn}{d\alpha} = -\frac{L\,(1-\theta)}{\alpha^2} < 0 \tag{5.48}$$

The gains from trade are related to expanded resources L and imperfect substitutability of goods $\theta < 1$.

International Trade

Assume two countries (alike in every way except size) engage in free trade (with zero transportation costs). What does trade do? Fixed costs in the production of each good create an incentive to concentrate production within a firm. *Increasing returns to scale imply that concentration is efficient so that two identical countries will specialize in the production of different sets of goods and intraindustry trade will occur in similar but differentiated products.* Gains from trade stem from the increased diversity of goods available under free trade due to market expansion. Full employment determines the varieties produced in each country under free trade

$$n = \frac{L}{\alpha + \beta x} = \frac{L(1-\theta)}{\alpha} \tag{5.49}$$

$$n^* = \frac{L^*}{\alpha + \beta x} = \frac{L^*(1-\theta)}{\alpha}$$

Each country produces the same number of varieties under free trade as under autarky. However, under free trade, consumers can consume all $n + n^*$ varieties. Since consumers love variety, they gain from trade. The simplifying assumptions of this model imply that free trade does not alter the prices charged by firms or their output levels. Under free trade, consumers solve the same problem but now distribute their expenditures over $n + n^*$ goods so that they enjoy greater variety.

Results

1. *Both countries gain from trade because of increased diversity of goods.* To see the gains from trade, compare (domestic) utility under autarky

$$U_A = \frac{w}{np} = n\left[\frac{w}{np}\right]^{\theta} = n^{1-\theta}\left(\frac{\theta}{\beta}\right)(1-\theta) \tag{5.50}$$

to utility under free trade,

$$U_T = (n + n^*)^{1-\theta}\left(\frac{\theta}{\beta}\right)(1-\theta) \tag{5.51}$$

Utility under free trade is higher

$$U_R \equiv \frac{U_T}{U_A} = \left(\frac{n+n^*}{n}\right)^{1-\theta} = \left(1 + \frac{n^*}{n}\right)^{1-\theta} > 1 \tag{5.52}$$

due to the greater availability of variety $n + n^* > n$ as $n^* > 0$ due to $L^* > 0$ and $\theta < 1$. Furthermore, *the gains from trade decrease with θ as the goods become closer substitutes so variety is less important*

$$\frac{dU_R}{d\theta} = -\left(\frac{n+n^*}{n}\right)^{1-\theta}\ln\left(\frac{n+n^*}{n}\right) < 0 \tag{5.53}$$

Considering the two extremes is illustrative of the vital role of the goods being differentiated

$$\lim_{\theta\to 1} U_R = 1, \lim_{\theta\to 0} U_R = 1 + \frac{n^*}{n} \tag{5.54}$$

Gains from trade vanish as goods become perfect substitutes (lose love of variety).

2. *Direction of trade is indeterminate*, but nothing important depends on which country produces (and thus exports) which good as the goods are symmetric. Of the $n + n^*$ goods consumed, n^* are imported by the home country. The value of home country imports measured in wage units equals value of foreign country imports, so trade is balanced. Thus, *the volume of trade is determinate.*

3. One peculiarity of the model is that the *output of each good remains the same even under trade.* Krugman (*JIE* 1979) develops a more general (but more tedious) model where the scale of production of each variety does increase with trade.

4. *When there are transportation costs, bigger country has the higher wage.* Iceberg type transportation costs are assumed: when send one unit of any good abroad, only $g \leq 1$ arrives as the rest melts on the way. The higher wage in the bigger country gives a production cost advantage in the smaller country to offset the higher total transportation costs paid to reach consumers.

5. *Transportation costs lead to the home market effect* – countries export those products for which they have bigger markets at home.

Krugman *JIE* 1979 (optional)

The added complication in this paper comes solely from a more general utility function. Generality eliminates the no scale effects aspect of Krugman (*AER* 1980) – recall output and price as well as real wage did not respond to trade in that model. In fact, trade is nothing but an expansion in the market size in that model.

Consumers

Assume a general utility function

$$u = \sum_i v(c_i)$$

where $v' > 0, v'' < 0$. Define

$$\varepsilon_i = -\frac{v'}{v'' c_i} \text{ and assume that } \frac{\partial \varepsilon_i}{\partial c_i} < 0$$

Later we will see that ε_i turns out to be the elasticity of demand facing a producer. To do this note, that the FOC for consumer optimization is

$$v'(c_i) = \lambda p_i \tag{5.55}$$

Totally differentiating the above equation we have

$$v''(c_i) dc_i = \lambda dp_i$$

This implies

$$v''(c_i) \frac{dc_i}{c_i} \frac{1}{p_i} = \lambda \frac{dp_i}{p_i} \frac{1}{c_i}$$

which gives

$$\varepsilon \equiv -\frac{dc_i / c_i}{dp_i / p_i} = -\frac{\lambda p_i}{v''(c_i) c_i} = -\frac{v'(c_i)}{v''(c_i) c_i}$$

Since all consumers are identical, $cL = x_i$. This along with (5.55) gives

$$p_i = \frac{1}{\lambda} v'(\frac{x_i}{L})$$

Producers

Cost function for a producer

$$l_i = \alpha + \beta x_i$$

A producer solves

$$\max \; p_i x_i - w l_i = \frac{1}{\lambda} v'(\frac{x_i}{L}) x_i - w(\alpha + \beta x_i)$$

We know that, in general, a monopolist prices as follows

$$p \left[1 - \frac{1}{\varepsilon} \right] = \beta w$$

which implies that

$$p = \frac{\varepsilon}{\varepsilon - 1} \beta w$$

Note that ε depends upon c_i so in the above equation price still depends upon an endogenous variable. Therefore, we need to use the free entry zero profit condition in conjunction with the above equation to determine the equilibrium price. The above equation defines the PP curve. This curve is upward sloping because $\varepsilon(\cdot)$ decreases in c and ε decreases faster than $\varepsilon - 1$. From zero profits, we have

$$px = (\alpha + \beta x)\, w$$

This implies

$$\frac{p}{w} = \beta + \frac{\alpha}{x} = \beta + \frac{\alpha}{cL}$$

This defines the ZZ curve. Equilibrium price and quantity are determined by the intersection of the two curves. Finally, the equilibrium number of products is given by

$$n = \frac{L}{\alpha + \beta x} = \frac{L}{\alpha + \beta c L}$$

Results

1. Effects of an increase in L: As L increases, the PP curve is unaffected but the ZZ curve shifts to the left. Thus, both p/w and c fall. This implies n increases. Also, since p/w falls, x also increases. Thus, with an increase in the labor force, price to wage ratio falls (real wage increases), output of each good increases, and the number of goods produced increases. Conceptually, some of the increased labor goes to increased output while some of it goes to the production of more varieties, this is why per capita consumption of each good falls – not all of the increase in L goes toward increasing x.
2. Trade: Very much like the first model. Expansion in the number of varieties for each consumer, also increased output and lower prices. Welfare gains.
3. Factor Mobility: The bigger country will have a higher real wage. Incentive to migrate from the small country to the big.

Krugman *JPE* 1981 (optional)

This third paper builds on the first two papers. Its main contribution is to provide a model in which trade occurs both within an industry as well as cross industries (intra- and inter- industry). In addition, it gives the realistic prediction that the degree of intra-industry trade increases with the similarity of the two countries factor endowments. The model also makes it feasible to assess the distributional effects of the two types of trade.

Model

Two sectors with differentiated goods

$$u = \ln\left(\sum_{i}^{N_1} c_{1i}^{\theta}\right)^{\frac{1}{\theta}} + \ln\left(\sum_{j}^{N_2} c_{2j}^{\theta}\right)^{\frac{1}{\theta}}$$

Properties of U: half of income is spent on each industry's products and if number of products in each industry are large, elasticity of substitution equals $\frac{1}{1-\theta}$.

Two types of labor: each is specific to one industry.

$$l_{1i} = \alpha + \beta x_{1i}$$

$$l_{2j} = \alpha + \beta x_{2j}$$

Total labor force equals 2. When we introduce a second country, we will assume that while the two countries differ in their relative endowment of each kinds of labor, the absolute size of the total labor stock is the same in each country.

$$\sum l_{1i} = L_1 = 2 - z$$

and

$$L_2 = z$$

Equilibrium

As before, we have

$$p_1 = \frac{\beta}{\theta} w_1 \text{ and } p_2 = \frac{\beta}{\theta} w_2$$

Free entry into each sector implies that

$$x_1 = x_2 = \frac{\alpha}{\beta}\frac{\theta}{1-\theta}$$

and

$$n_1 = \frac{2-z}{\alpha + \beta x_1} \text{ and } n_2 = \frac{z}{\alpha + \beta x_2}$$

Since consumer's spend equal amounts on both goods,

$$w_1 L_1 = w_2 L_2$$

This implies

$$\frac{w_1}{w_2} = \frac{z}{2-z}$$

The model has two key parameters: z and θ. As z increases, relative wage in sector 1 increases. θ is an index of substitutability among intra-industry products as well a measure of economies of scale: ratio of marginal to average cost. As θ goes up, the degree of product differentiation as well as real wage increase. Define

$$I = 1 - \frac{\sum |X_k - M_k|}{\sum X_k + M_k}$$

as an index of intra-industry trade (Grubel-Loyd).

Consider two identical countries that are mirror images of each other except that they differ in their endowments of the two types of labor.

$$\begin{aligned} L_1 &= 2 - z; L_2 = z \\ L_1^* &= z; L_2^* = 2 - z \end{aligned}$$

If $z = 1$, resources are identical in the two countries. As z increases, factor proportions diverge: when $z = 0$, each country has only one factor.

Under free trade, prices and wages are equalized (note that $w_1 = w_2$ in each country). The zero profit condition gives the output of each good in terms of parameters. We have

$$x_i = x^* = \frac{\alpha\theta}{\beta(1-\theta)}$$

As always, full employment condition gives the number of equilibrium products in each country. What can we say about the volume of trade? Let Y denote the national income in both countries ($Y = w_i L_i$). Equal factor prices imply equal income.

Let X_i denote home exports in industry i and let M_i denote home imports in industry i. Then we have

$$X_1 = \frac{Y}{2}\frac{2-z}{z} \text{ and } X_2 = \frac{Y}{2}\frac{z}{2}$$

Similarly,

$$M_2 = \frac{Y(2-z)}{2} \text{ and } M_1 = Y\frac{z}{2}$$

Substitute the above exports and imports into the Grubel-Loyd measure of intra-industry trade and we get

$$I = z$$

This is a striking result! The degree of intra-industry trade exactly equals the degree of similarity between the factor endowment of the two countries. What can we say about the welfare consequences of such trade? The basic message is that distributional consequences are not serious if the countries are similar enough – intra-industry trade is politically more palatable since everyone benefits from it.

Individuals split their income equally into the two sectors and within each sector, consumers buy all goods. Thus we have

$$U = \ln\left[n_1\left(\frac{w}{2n_1p_1}\right)^{\theta}\right]^{\frac{1}{\theta}} + \ln\left[n_2\left(\frac{w}{2n_2p_2}\right)^{\theta}\right]^{\frac{1}{\theta}}$$

which can be rewritten as

$$-2\ln 2 + \ln\frac{w}{p_1} + \ln\frac{w}{p_2} + \frac{1-\theta}{\theta}\ln n_1 + \frac{1-\theta}{\theta}\ln n_2$$

Thus two things determine utility: the *diversity* of products as well as *real wages*. Trade increases variety but also causes FPE. The wage of the abundant factor increases so this factor is better off due to both effects. However, for the scarce factor we have

$$\begin{aligned} U^T - U^A &= \ln\frac{z}{2-z} + \frac{1-\theta}{\theta}\ln\frac{2}{2-z} + \frac{1-\theta}{\theta}\ln\frac{2}{z} \\ &= \frac{2\theta-1}{\theta}\ln z - \frac{1}{\theta}\ln 2 - z + \frac{2-2\theta}{\theta}\ln 2 \end{aligned}$$

From the above we can conclude the following. First if $\theta < 0.5$, scarce factor also gains. When θ is small, diversity matters a lot to the consumer. Second, if $\theta > 0.5$ and if $z > \overline{z}$, both factors still gain – z above a threshold implies sufficient similarity in factor endowments.

Ethier *AER* 1982

A large proportion of world trade involves exchange of similar products between industrialized countries but these products are typically intermediate production goods rather than consumption goods as in Krugman's models. Ethier (1979) argued that if IRS depend upon the size of the world market (due to increased specialization of production into small steps which lead to tradeable intermediate goods), we will derive both an explanation for intraindustry trade and get rid of the multiplicity problem as well as the possibility of losses from trade that may arise IRS are external and national in scope. Ethier (1982) constructs a model in which two kinds of IRS coexist: one is the usual external IRS (as the number of intermediate goods expands, final output increases) and the second is IRS internal to the firm due to the presence of fixed costs involved in the production of each intermediate good. Unlike rest of the literature, Ethier's model is a full general equilibrium model with two goods and two factors of production. Therefore, it allows us to examine the fate of the traditional HOS theory in the presence of international IRS. Surprisingly, Ethier finds that most of the traditional theorems of the HOS model survive quite well and that (like in Krugman's model), intra-industry trade too has a factor endowments basis. However, such trade is shown to be complementary to international factor mobility. Lastly, while IRS are essential to obtain intra-industry trade, the degree of IRS is not of much consequence in determining the magnitude of intra-industry trade.

Model

- Two goods: wheat W and manufactures M
- Two factors: capital K and labor L
- Wheat has CRS technology
- Manufactures has IRS. Define $M = km$ where k is an index of scale economies and m is the size of operation in the M industry (how much of the economy's K and L is in M. You can think of m as the output from a neoclassical production function).

- Let $W = T(m)$ define the transformation curve between W and M.

- M has two sub-sectors: component production and assembly of finished components.

Each components firm produces one type of input x_i using m as an input. Let n be the (endogenous) number of components produced

$$m = \sum_i m_i$$

where m_i is the factor use in component i. Cost function

$$m_i = b + ax_i \Rightarrow m = nb + a \sum_i x_i$$

Component producers are monopolistic competitors and under free entry (so zero profits).

Final Good

All components are assembled through a symmetric CES production function

$$M = n^{\alpha - \frac{1}{\beta}} \left(\sum_i x_i^{\beta} \right)^{\frac{1}{\beta}}, \ \alpha > 1, 0 < \beta < 1$$

- Every pair of varieties is equally substitutable.

- The degree of substitution does not depend upon the level of usage.

- Variety is valued. Suppose n varieties are available at price q. Then final producer buys equal amounts of all components. If x equals the quantity purchased of each component, then

  $$M = n^{\alpha - \frac{1}{\beta}} (\sum_i x^{\beta})^{\frac{1}{\beta}} = n^{\alpha - \frac{1}{\beta}} (nx^{\beta})^{\frac{1}{\beta}} = n^{\alpha} x = n^{\alpha - 1}[nx]$$

  Clearly, output increases as n increases. Since $\alpha > 1$, there are IRS in n. These represent the productivity gains that accrue from an increased division of labor. These IRS are external to the final good producer who cannot control n. M displays CRS with respect to x.

Let q_i be the price of component i in W units.

$$\min \sum_i q_i x_i \text{ such that } n^{\alpha-\frac{1}{\beta}} (\sum_i x^\beta)^{\frac{1}{\beta}} = 1$$

First order conditions

$$n^{\alpha-\frac{1}{\beta}} \frac{1}{\beta} \left[\sum_{i=1}^{n} x_i^\beta \right]^{\frac{1}{\beta}-1} \beta x_i^{\beta-1} - \lambda p_i = 0$$

Take another good j and we have

$$n^{\alpha-\frac{1}{\beta}} \frac{1}{\beta} \left[\sum_{i=1}^{n} x_i^\beta \right]^{\frac{1}{\beta}-1} \beta x_j^{\beta-1} - \lambda p_j = 0$$

The above two conditions imply

$$\frac{x_i}{x_j} = \left[\frac{q_j}{q_i} \right]^{\frac{1}{1-\beta}}$$

Components

We know

$$x_i = x_j \left[\frac{q_j}{q_i} \right]^{\frac{1}{1-\beta}}$$

which leads to a constant elasticity demand curve facing the producer of component i and that the price of component j is a markup over marginal cost, where the markup equals β.

Total cost in terms of numeraire good W is given by

$$C(x_i) = -T^{'}(m)(b + a x_i)$$

The above cost function implies that any given firm is too small to influence $T^{'}(m)$, the opportunity cost of a small amount of M in terms of W. Marginal cost for producer i equals

$$c(x_i) = C'(x_i) = -T^{'}(m)a$$

Profit maximization requires marginal revenue equal marginal cost

$$q = q_j = \frac{-T^{'}(m)a}{\beta}$$

Since in equilibrium, all components will have the same price $q_i = q$,

$$\pi_i = qx + T^{'}(m)(b + ax)$$

Free entry implies

$$x = \frac{-T'(m)b}{q + T'(m)a}$$

Substituting for price of components we have

$$x = \frac{-T'(m)b}{\frac{-T'(m)a}{\beta} + T'(m)a} = \frac{b\beta}{a(1-\beta)}$$

Total resources devoted to component production are given by m. Therefore,

$$m = n(a + bx)$$

This implies

$$n = \frac{m(1-\beta)}{b}$$

Our index of IRS can be rewritten by noting that

$$n^\alpha x = M = km$$

so that

$$k = \left(\frac{(1-\beta)}{b}\right)^{\alpha-1} \frac{\beta}{a} m^{\alpha-1}$$

k has the property that it increases in m and decreases in a and b.

Autarky

Set $P_W = 1$ and let $P = \frac{P_M}{P_W} = P_M$. Zero profits in supply of M requires that

$$P_S M = qnx \longleftrightarrow P_S n^\alpha x = qnx$$

which gives

$$P_S = qn^{1-\alpha}$$

which can be rewritten using values of q and k and n as

$$P_S = \frac{-T'(m)m}{M} = \frac{-T'(m)}{k(m)}$$

Note that due to IRS, the supply function is downward sloping.

On the demand side we assume homothetic preferences

$$U(M, W) = M^\gamma W^{1-\gamma}$$

Constant expenditure shares imply

$$P_D M = \gamma(W + P_D M)$$

where γ equals the fraction of income spent on M and $W + P_D M$ equals total expenditure of consumers. This implies the demand price equals

$$P_D = \frac{\gamma}{1-\gamma}\frac{T(m)}{k(m)m}$$

Intersection of the two curves gives autarkic equilibrium.

International Trade (optional)

Two economies that differ only in terms of factor endowments. Denote foreign country by asterisks. If both countries continue to make both goods, they will specialize in different components. Let n_H and n_F denote their numbers.

$$n = n_H + n_F = \frac{(m+m^*)(1-\beta)}{b}$$

The above implies that the world output of M is given as follows

$$M+M^* = k(m+m^*)(m+m^*) = \overbrace{\frac{\beta}{\alpha}\left(\frac{(1-\beta)}{b}\right)^{\alpha-1}(m+m^*)^{\alpha-1}}^{k(m+m^*)}(m+m^*)$$

Of course, still need to determine the pattern of production in the two countries under free trade. How are m and m^* and the relative price determined? Given Cobb-Douglas preferences in two countries, we have

$$P_D = \frac{\gamma}{1-\gamma}\frac{T(m)+S(m^*)}{M+M^*} = \frac{\gamma}{1-\gamma}\frac{\alpha}{\beta}\left(\frac{b}{(1-\beta)}\right)^{\alpha-1}\frac{T(m)+S(m^*)}{(m+m^*)^{\alpha}}$$

As before, home supply curve is given by

$$P_S^H = \frac{-T'(m)}{k(m+m^*)} = [(1-\beta)(m+m^*)/b]^{1-\alpha}\frac{aT'(m)}{\beta}$$

Equilibrium in the home market requires $P_S^H = P_D$ so that we have

$$\frac{-T'(m)}{k(m+m^*)} = \frac{\gamma}{1-\gamma}\frac{T(m)+S(m^*)}{M+M^*}$$

This equation defines the *Home Allocation Curve* (HAC) drawn in the (m, m^*) space. Each point in the (m, m^*) space represents a particular allocation of resources in the world economy.

The HAC indicates for each m the foreign allocation of resources for which the home economy is in equilibrium in world markets: m and m^* are on the HAC if the demand price that clears the world market for the two goods is equal to home supply price for which the domestic producers are willing to supply $W = T(m)$ and $M = k(m + m^*)m$: their share of the world output that is required to clear the market for both goods, given that the foreigners are supplying the rest ($S(m^*)$ and M^*). Simple calculations show that it is downward sloping. Furthermore, $P_D < P_S^H$ above the curve and vice versa below it. m_o denotes complete specialization points. Similarly, the Foreign Allocation Curve (FAC) is defined by

$$\frac{-S'(m^*)}{k(m+m^*)} = \frac{\gamma}{1-\gamma}\frac{T(m)+S(m^*)}{M+M^*}$$

International equilibrium is given by the intersection of the two curves.

Intraindustry Trade and Complementarity

Intraindustry trade has a factor endowments basis and it is complimentary to international factor mobility —similarity of endowments leads to more intraindustry trade.

First, both countries specialize in different types of components, if they both produce M. Home country's import and export of components are given by

$$M_C = n_F g x \text{ and } X_C = n_H x(1-g)$$

where g equals domestic national income as a fraction of world income, i.e.,

$$g = \frac{PM + W}{P(M+M^*) + W + W^*}$$

The index for intraindustry trade is given by

$$\rho = 1 - \frac{|X_C - M_C|}{X_C + M_C}$$

which gives

$$\rho = \frac{2gn_F}{(1-g)n_H + gn_F} \text{ if } n_H \geq n_F$$

and

$$\rho = \frac{2(1-g)n_H}{(1-g)n_H + gn_F} \text{ if } n_H \leq n_F$$

Higher values of ρ indicate more intraindustry trade. Let h and h^* denote K/L ratios at home and abroad, where $h > h^*$. Consider a small exchange of factors that reduces the gap between h and h^* such that incomes remain unchanged as do prices of goods and commodities (FPE continues to hold). This implies home will make less components and foreign country will make more than before. Since Home is K abundant, its share of world output of components will exceed its share of world income g. This will imply that ρ will equal the first formula in which ρ increases with an increase in n_F and a decrease in n_H. Maximum intra-industry trade when identical factor endowments ($h = h^*$). If there were no differences in factor endowments at all, all trade would be intraindustry where the two countries will exchange only components.

Effect of Technological Parameters

- Intersectoral allocation of resources is not determined by β and a, b.
- How about intraindustry trade? Stare at the expression for ρ, it is unaffected by these parameters since such changes produce offsetting changes on the number of components produced in the two countries. Hence, these parameters play a *knife-edge role*: their existence is absolutely critical for intraindustry trade but their magnitude does not affect the extent of such trade.

Fate of the HOS Theory

Assume that manufactures is relatively capital intensive.

1. *Factor Price Equalization*: Assume diversification. From the equations determining the two allocation curves, it must be that $S^{'}(m^*) = T^{'}(m)$. Therefore prices of the components are equal, regardless of where they are produced so that FPE holds.
2. *Rybczynski*: Say we change the ratio of capital to labor but keep the relative price of components constant (T' constant). Then while the output of each component remains unchanged (because price has not changed), the increased m implies that the number of components increases (like increasing labor supply in Krugman *AER* 1980). Due to external IRS, the output of manufactures increases.

3. *Stolper Samuelson*: The standard result holds with respect to the change in price of components relative to wheat. But what we care about is manufactures since that's what's consumed — the essence of the Stolper-Samuelson theorem is its prediction about changes in real rewards. So what can we say about that? Things are more complicated here. There are two competing effects and results depend upon which of the two dominates. First define the nominal price of a component

$$P_C = qP_W$$

Note that

$$P_M = n^{1-\alpha} P_C$$

which gives

$$\widehat{P}_M = \widehat{P}_C - (\alpha - 1)\widehat{n}$$

Say $\frac{P_C}{P_W}$ increases. Then, cost of producing manufactures increases since components are more expensive. However, a change in relative price alters resource allocation so that more resources will move into manufactures leading to an increase in n. Due to the external IRS, an increase in n essentially lowers the cost of manufactures. This latter *scale effect* (captured by the term $\alpha - 1$ or the degree of external IRS) works against the former *intersectoral effect* and results depend upon which of these dominates. If the intersectoral effect dominates, we get conventional results whereas if the latter dominates we get an anti-Stolper-Samuelson theorem.

4. *Heckscher-Ohlin* – quantity version remains intact. The scale effect can alter the price version of the theorem.