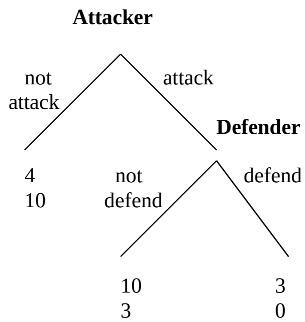
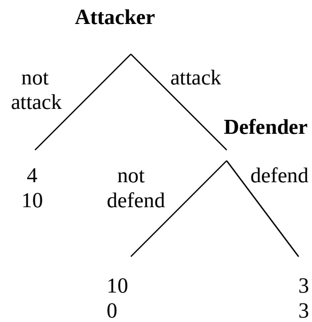
POLS 308: Game Theoretic Methods in Political Science (Spring 2017)
Professor Ahmer Tarar NAME:____________________________
Homework Assignment #5 (due at beginning of class on **Wednesday, April 26**)

(1) Consider the following two deterrence games. A potential attacker is thinking about attacking an ally of a "defender" nation. If the potential attacker attacks, the defender has to decide whether or not to defend its ally.



Game 1



Game 2

(a) (2 pts) In which of the above 2 games can the defender be called the "tough" type, and in which game is the defender the "weak" type?

(b) (8 pts) Solve games 1 and 2 by backwards induction (show each player's action at **every point** in the game by drawing arrows, and circle the outcome).

(c) (6 pts) Now suppose that the attacker doesn't know whether the defender is the tough type or the weak type, i.e. the attacker doesn't know which of the above two games it is playing. Suppose the attacker believes that the defender is tough with probability p (where $1 > p > 0$) and weak with probability $1 - p$. What is the attacker's **expected utility** for attacking?

(d) (4 pts) For what values of p will the attacker find it rational to attack?

(2) Exercise U3 on p.374 (Chapter 9) of the textbook (4^{th} edition of the textbook). (20 points)

(3) (20 points) Consider the game of Chicken, in which each player can choose Straight or Swerve. Suppose that a "normal" type of player has the usual preferences: if I am player 1, I prefer (Straight, Swerve) over (Swerve, Swerve) over (Swerve, Straight) over (Straight, Straight). Suppose that a "crazy" type switches the last two preferences, that is, it prefers (Straight, Straight) over (Swerve, Straight), but otherwise has the same preferences as a "normal" type.

Suppose that player 1 is the "normal" type, and player 2 is the "normal" type with probability $0 < p < 1$, and is the "crazy" type with probability $1 – p$. Player 1 does not know player 2's type. Draw the payoff matrices and find all of the **Bayesian Nash equilibria (BNE)** in pure strategies. **For each player, be sure to use the payoffs of 10 for the most preferred outcome, 9 for the second-best, 6 for the third-best, and 0 for the least-preferred outcome. In drawing the payoff matrices, be sure to list the actions by listing Straight first, and Swerve second.**

(4) One day, long ago, two women and a baby were brought before King Salmon. Each woman claimed to be the true mother of the baby. King Salmon pondered dividing the baby in half and giving an equal share to each mother, but decided that in the interest of the baby he would use game theory to resolve the dispute instead.

Here is what the king knew. One of the women is the true mother, but he does not know which one. The two women know who the true mother is. The true mother places a value of $v_T > 0$ on having the baby and 0 on not having it. The woman who is not the true mother places a value of $v_F > 0$ on having the baby and 0 on not having it. Assume that $v_T > v_F$, so that the true mother values the baby more.

The king proposed the following game to determine who would get to keep the baby. One woman is chosen to go first. She is player 1, and the other woman is player 2. Play 1 announces MINE if she wants to claim to be the real mother, and HERS is she wants to claim that the other woman (player 2) is the true mother. If she says HERS, the baby goes to player 2 and the game ends. If she says MINE, player 2 gets to move. Player 2 announces MINE or HERS. If player 2 says MINE, she gets the baby and pays King Salmon a fixed amount v, where $v_F < v < v_T$. Meanwhile, player 1 also has to pay King Salmon a fixed amount $\delta > 0$. If player 2 says HERS, then player 1 gets the baby and no payments are made to King Salmon.

(a) (10 points) First suppose that player 1 is the true mother. Draw the game-tree and find the subgame-perfect equilibria (SPE).

(b) (10 points) Now suppose that player 2 is the true mother. Draw the game-tree and find the SPE.

(c) (5 points) Based on your answers, discuss the wisdom of the king, and the feasibility of the king being able to implement this mechanism.

(5) Consider the following version of Axelrod's tournaments: There are three individuals using three different strategies: (i) TFT ("tit-for-tat": cooperate in the first round, and then in subsequent rounds do what the opponent did in the previous round), (ii) ALTERNATE-C (alternate between cooperating and defecting, beginning with cooperating), and (iii) TF2T ("tit-for-two-tats": cooperate in the first 2 rounds, and then in each subsequent round defect only if the opponent defected in each of the previous *two* rounds; otherwise cooperate).

Each individual interacts with the other two individuals and plays the following Prisoner's Dilemma game 6 times (not 200 times, like in Axelrod's tournaments).

		2	
		C	D
1	C	3,3	0,5
	D	5,0	1,1

(a) (4 pts) When TFT plays TF2T,
   (i) What is TFT's score (just add up the scores for the 6 rounds)?
   (ii) What is TF2T's score?

(b) (4 pts) When TFT plays ALTERNATE-C,
   (i) What is TFT's score?
   (ii) What is ALTERNATE-C's score?

(c) (4 pts) When TF2T plays ALTERNATE-C,

(i) What is TF2T's score?

(ii) What is ALTERNATE-C's score?

(d) (4 pts) Using the above answers, calculate each strategy's average score.

(e) (2 pts) Why did the strategy with the lowest average score do so bad?

(f) (2 pts) Why did the strategy with the highest average score do so well?