Online Appendix for “Military Mobilization and Commitment Problems”*

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1 Appendix

As is common in bargaining models, I assume throughout that $D$ accepts an offer if he is indifferent between accepting it and rejecting it.

1.1 The Ultimatum-Offer Mobilization Model, Incomplete Information Results Without Initial Attack Option

In the main text, I first discuss the case where the two types of $D$ have the same “cost coefficient” $k_c$, and then allow the two types to have different cost coefficients $k_{c_l}$ and $k_{c_h}$, with $k_{c_l} < k_{c_h}$. Rather than do two separate analyses, in this appendix I simply assume that $k_{c_l} \leq k_{c_h}$, which will cover both cases.

The following two lemmas are useful. The first establishes that in any PBE, the belief-threshold for making the big offer (which even the high-resolve type accepts) is $s^* = \frac{c_{D_h} - c_{D_l}}{c_{D_h} + c_S}$ (the same as in the model without mobilization) for any mobilization level $m \in [0, m]$. The second establishes that in any PBE, the high-resolve type $(c_{D_l})$ chooses the same mobilization level that it would under complete information (i.e., it never “bluffs”).

**Lemma 1** In any PBE, upon observing mobilization level $m \in [0, m]$, if $S$’s updated (or off-the-equilibrium-path) probability/belief that she assigns to $D$ being the low-cost type $(c_{D_l})$ is $s'$, then she (i) offers $p(m) - c_{D_l}$ if $s' > s^* = \frac{c_{D_h} - c_{D_l}}{c_{D_h} + c_S} \in (0, 1)$, (ii) offers $p(m) - c_{D_h}$ if $s' < s^*$, and (iii) is indifferent between offering $p(m) - c_{D_l}$ and $p(m) - c_{D_h}$ if $s' = s^*$, and hence can be offering either, or mixing between these two offers, but never chooses any other offer with positive probability.

**PROOF:** Suppose $S$ observes some mobilization level $m \in [0, m]$. At that mobilization level, $c_{D_l}$ accepts any offer $(y, 1 - y)$ such that $y \geq p(m) - c_{D_l}$ and $c_{D_h}$ accepts any $y \geq p(m) - c_{D_h}$, and they go to war for any lower offer (because of the assumption that both types are dissatisfied at all mobilization levels, i.e., $q < p - c_{D_h}$). Therefore, making the small offer of $p(m) - c_{D_h}$ (which only $c_{D_h}$ accepts) is strictly preferred over making the large offer of $p(m) - c_{D_l}$ (which both types accept) if and only if $s'[1 - p(m) - c_S] + (1 - s')[1 - p(m) + c_{D_h}] > 1 - p(m) + c_{D_l}$, which simplifies to $s' < \frac{c_{D_h} - c_{D_l}}{c_{D_h} + c_S}$. Alternatively, if $s' > \frac{c_{D_h} - c_{D_l}}{c_{D_h} + c_S}$, then $p(m) - c_{D_l}$
Lemma 2 In any PBE, type $c_{D_1}$ (i) chooses $m = 0$ with probability 1 if $k_{c_1} > k_p$, and (ii) chooses $m = \overline{m}$ with probability 1 if $k_{c_1} < k_p$.

PROOF: First consider case (i), i.e., $k_{c_1} > k_p$. Suppose there is a PBE in which $c_{D_1}$ chooses some $m' > 0$ with positive probability. By Lemma 1, we know that $S$ (depending on her belief upon observing $m'$) either offers $p(m') - c_{D_1}$ or offers $p(m') - c_{D_h}$, or mixes between them. In any case, $c_{D_1}$’s payoff is $p - c_{D_1} + m'(k_p - k_{c_1})$ (either because $S$ makes the big offer and $c_{D_1}$ accepts it, or $S$ makes the small offer and $c_{D_h}$ goes to war). But $c_{D_1}$ can profitably deviate (from choosing $m'$) by choosing $m = 0$ and going to war, thereby getting a higher (because $k_{c_1} > k_p$) payoff of $p - c_{D_1}$. Hence, such a PBE cannot exist.

Now consider case (ii), i.e., $k_{c_1} < k_p$. Suppose there is a PBE in which $c_{D_1}$ chooses some $m' < \overline{m}$ with positive probability. By Lemma 1, we know that $S$ (depending on her belief upon observing $m'$) either offers $p(m') - c_{D_1}$ or offers $p(m') - c_{D_h}$, or mixes between them. In any case, $c_{D_1}$’s payoff is $p - c_{D_1} + m'(k_p - k_{c_1})$ (either because $S$ makes the big offer and $c_{D_1}$ accepts it, or $S$ makes the small offer and $c_{D_h}$ goes to war). But $c_{D_1}$ can profitably deviate (from choosing $m'$) by choosing $m = \overline{m}$ and going to war, thereby getting a higher (because $k_{c_1} < k_p$) payoff of $p - c_{D_1} + \overline{m}(k_p - k_{c_1})$. Hence, such a PBE cannot exist. Q.E.D.
The following four propositions describe the generically unique PBE (i.e., unique up to the point that there are multiple off-the-equilibrium-path beliefs that are acceptable; the strategies and outcomes are identical) depending on the values of $k_c$, $k_{ch}$, and $k_{cl}$ relative to $k_p$, where $k_{c_l} \leq k_{ch}$.

**Proposition 1** If $k_p < k_{c_l} \leq k_{ch}$, then there is a generically unique PBE in which both types choose $m = 0$ with probability 1.

**PROOF:** By Lemma 2, $c_{D_l}$ chooses $m = 0$ with probability 1 in any PBE. I claim that $c_{D_h}$ also does. Suppose otherwise, i.e., suppose there is a PBE in which $c_{D_h}$ chooses some $m' > 0$ with positive probability. Upon observing $m'$, $S$ knows that it faces $c_{D_h}$ (because $c_{D_l}$ chooses $m = 0$ with probability 1 in any such hypothetical PBE) and hence offers $p(m') - c_{D_h}$ (in particular, it never offers the big amount $p(m') - c_{D_l}$), giving $c_{D_h}$ a payoff of $p - c_{D_h} + m'(k_p - k_{ch})$. But $c_{D_h}$ can profitably deviate (from choosing $m'$) by choosing $m = 0$ and going to war, thereby getting a higher (because $k_{ch} > k_p$) payoff of $p - c_{D_h}$. This establishes that in any PBE, both types must be choosing $m = 0$ with probability 1. Such PBE can indeed be constructed; all we need to do is stipulate that if $S$ observes some $m' > 0$, the off-the-equilibrium-path probability that she assigns to $D$ being the low-cost type, $s'$, satisfies $s' < s^*$, and hence she makes the small offer of $p(m') - c_{D_h}$ rather than the large offer of $p(m') - c_{D_l}$, and hence $c_{D_h}$ never has an incentive to deviate to $m'$. That there are multiple off-the-equilibrium-path beliefs that support this PBE is the only non-uniqueness. Q.E.D.

**Proposition 2** If $k_{c_l} \leq k_{ch} < k_p$, then there is a generically unique PBE in which both types choose $m = m$ with probability 1.

**PROOF:** By Lemma 2, $c_{D_l}$ chooses $m = m$ with probability 1 in any PBE. I claim that $c_{D_h}$ also does. Suppose otherwise, i.e., suppose there is a PBE in which $c_{D_h}$ chooses some $m' < m$ with positive probability. Upon observing $m'$, $S$ knows that it faces $c_{D_h}$ (because $c_{D_l}$ chooses $m = m$ with probability 1 in any such hypothetical PBE) and hence offers $p(m') - c_{D_h}$ (in particular, it never offers the big amount $p(m') - c_{D_l}$), giving $c_{D_h}$ a payoff of $p - c_{D_h} + m'(k_p - k_{ch})$. But $c_{D_h}$ can profitably deviate (from choosing $m'$) by choosing $m = m$.
and going to war, thereby getting a higher (because $k_{ch} < k_p$) payoff of $p - c_{Dh} + \overline{m}(k_p - k_{ch})$. This establishes that in any PBE, both types must be choosing $m = \overline{m}$ with probability 1. Such PBE can indeed be constructed; all we need to do is stipulate that if $S$ observes some $m' < \overline{m}$, the off-the-equilibrium-path probability that she assigns to $D$ being the low-cost type, $s'$, satisfies $s' < s^*$, and hence she makes the small offer of $p(m') - c_{Dh}$ rather than the large offer of $p(m') - c_{Dl}$, and hence $c_{Dh}$ never has an incentive to deviate to $m'$. That there are multiple off-the-equilibrium-path beliefs that support this PBE is the only non-uniqueness. Q.E.D.

**Proposition 3** If $k_a < k_p < k_{ch} < k_p + \frac{c_{Dh} - c_{Dl}}{\overline{m}}$, then there is a generically unique PBE in which $c_{Dl}$ chooses $m = \overline{m}$ with probability 1, and $c_{Dh}$ chooses $m = \overline{m}$ with probability $a = \frac{s(c_S + c_D)}{(1-s)(c_{Dh} - c_{Dl})} \in (0,1)$ and $m = 0$ with probability $1 - a$. Upon observing $m = \overline{m}$, $S$’s updated probability/belief (given $a$) that $D$ is the low-cost type ($c_{Dl}$) satisfies exactly $s'' = s^*$, and she offers $p(\overline{m}) - c_{Dl}$ with probability $b = \frac{\overline{m}(k_{ch} - k_p)}{c_{Dh} - c_{Dl}} \in (0,1)$ and $p(\overline{m}) - c_{Dh}$ with probability $1 - b$, which makes $c_{Dh}$ indifferent between choosing $m = 0$ and $m = \overline{m}$.

**PROOF:** By Lemma 2, $c_{Dl}$ chooses $m = \overline{m}$ with probability 1 in any PBE. I now establish that there cannot exist a PBE in which $c_{Dh}$ chooses some $0 < m' < \overline{m}$ with positive probability. Suppose there exists such a PBE. Upon observing $m'$, $S$ knows that it faces $c_{Dh}$ (because $c_{Dl}$ chooses $m = \overline{m}$ with probability 1 in any such hypothetical PBE) and hence offers $p(m') - c_{Dh}$ (in particular, it never offers the big amount $p(m') - c_{Dl}$), giving $c_{Dh}$ a payoff of $p - c_{Dh} + m'(k_p - k_{ch})$. But $c_{Dh}$ can profitably deviate (from choosing $m'$) by choosing $m = 0$ and going to war, thereby getting a higher (because $k_{ch} > k_p$) payoff of $p - c_{Dh}$. This establishes that in any PBE, $c_{Dh}$ cannot be choosing some $0 < m' < \overline{m}$ with positive probability.

Three possibilities for PBE remain: (i) $c_{Dh}$ chooses $m = \overline{m}$ with probability 1, (ii) $c_{Dh}$ chooses $m = 0$ with probability 1, and (iii) $c_{Dh}$ strictly mixes between $m = \overline{m}$ and $m = 0$. I will show that cases (i) and (ii) are impossible, and then construct PBE for case (iii). First consider case (i). Because both types are choosing $m = \overline{m}$ with probability 1, upon observing $m = \overline{m}$ $S$’s posterior belief remains at her prior belief $s$ ($< s^*$) and hence she makes the small offer of $p(\overline{m}) - c_{Dh}$, giving $c_{Dh}$ a payoff of $p - c_{Dh} + \overline{m}(k_p - k_{ch})$. But $c_{Dh}$
can profitably deviate (from choosing \( m = \overline{m} \)) by choosing \( m = 0 \) and going to war, thereby getting a higher (because \( k_{ch} > k_p \)) payoff of \( \overline{p} - c_{D_h} \). Hence, such a PBE cannot exist.

Now consider case (ii). Upon observing \( m = 0 \), \( S \) (knowing that she faces \( c_{D_h} \)) makes the small offer of \( \overline{p} - c_{D_h} \), and upon observing \( m = \overline{m} \), \( S \) (knowing that she faces \( c_{D_l} \)) makes the big offer of \( p(\overline{m}) - c_{D_l} \). If \( c_{D_h} \) were to deviate (from choosing \( m = 0 \)) by choosing \( m = \overline{m} \) and accepting \( S \)'s offer, \( c_{D_h} \)'s utility would be \( \overline{p} - c_{D_l} + \overline{m}(k_p - k_{ch}) \). Thus, such a deviation is strictly profitable if and only if \( \overline{p} - c_{D_h} < \overline{p} - c_{D_l} + \overline{m}(k_p - k_{ch}) \), which simplifies to \( k_{ch} < k_p + \frac{c_{D_h} - c_{D_l}}{\overline{m}} \), which we have assumed to hold in this proposition. Hence, such a PBE is not possible.

The only remaining possibility for a PBE is case (iii), in which \( c_{D_h} \) strictly mixes between \( m = 0 \) and \( m = \overline{m} \). Let \( c_{D_h} \) choose \( m = \overline{m} \) with probability \( a \in (0, 1) \) and \( m = 0 \) with probability \( 1 - a \). For \( c_{D_h} \) to be rationally mixing, he has to be indifferent. But if \( a \) is high enough that \( S \) makes the small offer of \( p(\overline{m}) - c_{D_h} \) upon observing \( m = \overline{m} \) (i.e., if \( a \) is high enough that \( S \)'s updated probability/belief that \( D \) is the low-cost type satisfies \( s'' < s^* \)), then \( c_{D_h} \) strictly prefers \( m = 0 \) to \( m = \overline{m} \), because \( k_{ch} > k_p \) (i.e., if he is going to get the small offer in either case, he would rather not mobilize, because it is too costly). Hence, he cannot be rationally mixing. On the other hand, if \( a \) is low enough that \( S \) makes the large offer of \( p(\overline{m}) - c_{D_l} \) upon observing \( m = \overline{m} \) (i.e., if \( a \) is low enough that \( S \)'s updated probability/belief that \( D \) is the low-cost type satisfies \( s'' > s^* \)), then \( c_{D_h} \) strictly prefers \( m = \overline{m} \) to \( m = 0 \), because of our assumption in this proposition that \( k_{ch} < k_p + \frac{c_{D_h} - c_{D_l}}{\overline{m}} \) (see the previous paragraph). Hence, he cannot be rationally mixing. Thus, the only possibility for a PBE in case (iii) is where \( a \) is exactly such that upon observing \( m = \overline{m} \), \( S \)'s posterior belief satisfies \( s'' = s^* \), and hence she strictly randomizes between offering \( p(\overline{m}) - c_{D_l} \) and offering \( p(\overline{m}) - c_{D_h} \), with the exact probability that makes \( c_{D_h} \) indifferent between choosing \( m = 0 \) and \( m = \overline{m} \). That is, \( a \) has to be such that \( s'' = \frac{s}{s+(1-s)\alpha} = s^* \), which simplifies to \( a = \frac{s(c_{S}+c_{D_l})}{(1-s)(c_{D_h}-c_{D_l})} \in (0, 1) \). (Note that as \( s \) increases from 0 to \( s^* \), \( a \) increases from 0 to 1.) Let \( b \in (0, 1) \) be the probability with which \( S \) offers \( p(\overline{m}) - c_{D_l} \) upon observing \( m = \overline{m} \), and \( 1 - b \) be the probability with which she offers \( p(\overline{m}) - c_{D_h} \). Then, \( b \) has to be such that \( \overline{p} - c_{D_h} = b[p(\overline{m}) - c_{D_l} - k_{ch} \cdot \overline{m}] + (1 - b)[p(\overline{m}) - c_{D_h} - k_{ch} \cdot \overline{m}] \), which simplifies to
\[ b = \frac{\bar{m}(k_c - k_p)}{c_{D_h} - c_{D_l}} \in (0, 1) \]. Note that as \( k_c \) increases from \( k_p \) to \( k_p + \frac{c_{D_h} - c_{D_l}}{\bar{m}} \), \( b \) (the probability with which \( S \) makes the big offer upon observing \( m = \bar{m} \)) increases linearly from 0 to 1.

Hence, as seen in Figure 4, the probability of war decreases linearly from \( s \) to 0.

Such PBE can indeed be constructed; all we need to do is stipulate that if \( S \) observes some \( 0 < m' < \bar{m} \), the off-the-equilibrium-path probability that she assigns to \( D \) being the low-cost type, \( s' \), satisfies \( s' < s^* \), and hence she makes the small offer of \( p(m') - c_{D_h} \) rather than the large offer of \( p(m') - c_{D_l} \), and hence \( c_{D_h} \) never has an incentive to deviate to \( m' \) (because \( k_c > k_p \), if \( S \) is going to make the small offer of \( p(m') - c_{D_h} \) upon observing some \( 0 < m' < \bar{m} \), \( c_{D_h} \) strictly prefers choosing \( m = 0 \) over choosing \( m' \)). That there are multiple off-the-equilibrium-path beliefs that support this PBE is the only non-uniqueness. Q.E.D.

**Proposition 4** If \( k_c < k_p < k_p + \frac{c_{D_h} - c_{D_l}}{\bar{m}} < k_{ch} \), then there is a generically unique PBE in which \( c_{D_l} \) chooses \( m = \bar{m} \) with probability 1 and \( c_{D_h} \) chooses \( m = 0 \) with probability 1.

**PROOF:** By Lemma 2, \( c_{D_l} \) chooses \( m = \bar{m} \) with probability 1 in any PBE. I claim that \( c_{D_h} \) chooses \( m = 0 \) with probability 1 in any PBE. I establish this in two steps. In the next paragraph, I show that there cannot exist a PBE in which \( c_{D_h} \) chooses some \( 0 < m' < \bar{m} \) with positive probability. In the paragraph after that, I show that there cannot exist a PBE in which \( c_{D_h} \) chooses \( m = \bar{m} \) with positive probability. This will establish the claim, and then I will complete the proof.

First suppose there is a PBE in which \( c_{D_h} \) chooses some \( 0 < m' < \bar{m} \) with positive probability. Upon observing \( m' \), \( S \) knows that it faces \( c_{D_l} \) (because \( c_{D_l} \) chooses \( m = \bar{m} \) with probability 1 in any such hypothetical PBE) and hence offers \( p(m') - c_{D_h} \) (in particular, it never offers the big amount \( p(m') - c_{D_l} \)), giving \( c_{D_h} \) a payoff of \( p - c_{D_h} + m'(k_p - k_c) \). But \( c_{D_h} \) can profitably deviate (from choosing \( m' \)) by choosing \( m = 0 \) and going to war, thereby getting a higher (because \( k_c > k_p \)) payoff of \( p - c_{D_h} \). This establishes that in any PBE, \( c_{D_h} \) cannot be choosing some \( 0 < m' < \bar{m} \) with positive probability.

Now suppose there is a PBE in which \( c_{D_h} \) chooses \( m = \bar{m} \) with positive probability \( a \in (0, 1] \). There are three cases to consider, depending on the value of \( a \). (i) If \( a \) is high enough that \( S \)'s updated belief/probability \( s'' \) that she assigns to \( D \) being type \( c_{D_l} \) (upon
observing \( m = \overline{m} \) satisfies \( s'' < s^* \) (from the previous proof, we know that the critical threshold is \( a > a^* = \frac{s(c_s + c_{D_l})}{(1-s)(c_{D_h} - c_{D_l})} \in (0, 1) \)), then \( S \) makes the small offer of \( p(\overline{m}) - c_{D_h} \), giving \( c_{D_h} \) a payoff of \( p - c_{D_h} + \overline{m}(k_p - k_{c_h}) \). But because \( k_{c_h} > k_p \), \( c_{D_h} \) strictly benefits (from choosing \( m = \overline{m} \)) by instead choosing \( m = 0 \) and going to war, thereby getting a higher payoff of \( p - c_{D_h} \), and hence such a PBE cannot exist. (ii) If \( a \) is low enough that \( S \)'s updated belief/probability \( s'' \) that she assigns to \( D \) being type \( c_{D_l} \) (upon observing \( m = \overline{m} \)) satisfies \( s'' > s^* \) (i.e., if \( a < a^* \)), then \( S \) makes the big offer of \( p(\overline{m}) - c_{D_l} \), giving \( c_{D_h} \) a payoff of \( p - c_{D_h} + \overline{m}(k_p - k_{c_h}) \). But if \( c_{D_h} \) deviates (from choosing \( m = \overline{m} \)) by choosing \( m = 0 \) and going to war, then his payoff is \( p - c_{D_h} \), which is a strictly profitable deviation if and only if \( p - c_{D_h} > p - c_{D_l} + \overline{m}(k_p - k_{c_h}) \), which simplifies to \( k_{c_h} > k_p + \frac{c_{D_h} - c_{D_l}}{\overline{m}} \), which we have assumed to hold in this proposition; hence, such a PBE cannot exist. (iii) If \( a \) is exactly such that \( S \)'s updated belief/probability \( s'' \) that she assigns to \( D \) being type \( c_{D_l} \) (upon observing \( m = \overline{m} \)) satisfies \( s'' = s^* \) (i.e., if \( a = a^* \)), then \( S \) is indifferent between making the small offer of \( p(\overline{m}) - c_{D_h} \) and the big offer of \( p(\overline{m}) - c_{D_l} \), and hence she might be offering either, or mixing between them. The strictly best scenario for \( c_{D_h} \) is if she makes the big offer with probability 1. But even in this scenario, because \( k_{c_h} > k_p + \frac{c_{D_h} - c_{D_l}}{\overline{m}} \), \( c_{D_h} \) strictly benefits (from choosing \( m = \overline{m} \)) by instead choosing \( m = 0 \) and going to war (as in case ii), and hence such a PBE cannot exist.

The previous two paragraphs establish that type \( c_{D_h} \) chooses \( m = 0 \) with probability 1 in any PBE, and by Lemma 2 we know that type \( c_{D_l} \) chooses \( m = \overline{m} \) with probability 1 in any PBE. Such PBE can indeed be constructed; all we need to do is stipulate that if \( S \) observes some \( 0 < m' < \overline{m} \), the off-the-equilibrium-path probability that she assigns to \( D \) being the low-cost type, \( s' \), satisfies \( s' < s^* \), and hence she makes the small offer of \( p(m') - c_{D_h} \) rather than the large offer of \( p(m') - c_{D_l} \), and hence \( c_{D_h} \) never has an incentive to deviate to \( m' \). Our assumption in this proposition that \( k_{c_h} > k_p + \frac{c_{D_h} - c_{D_l}}{\overline{m}} \) ensures that \( c_{D_h} \) does not have an incentive to deviate to \( m = \overline{m} \), even though \( S \) makes the big offer of \( p(\overline{m}) - c_{D_l} \) upon observing \( m = \overline{m} \). That there are multiple off-the-equilibrium-path beliefs that support this PBE is the only non-uniqueness. Q.E.D.
1.2 The Ultimatum-Offer Mobilization Model, Incomplete Information Results With Initial Attack Option

From the previous section, we know that, depending on the values of $k_c$, and $k_{ch}$, there is a generically unique PBE of the subgame that begins when $S$ chooses not to engage in an initial attack. Therefore, we can break the analysis into four cases (corresponding to Propositions 1, 2, 3, and 4, respectively): when the parameters are such that if $S$ chooses not to engage in an initial attack, (i) the two types pool on $m = 0$, (ii) the two types pool on $m = \overline{m}$, (iii) the semi-separating equilibrium exists, and (iv) the fully separating equilibrium exists. In the main text, I analyzed case (iv), and here I analyze the other three cases.

In case (i), $S$ does not engage in an initial attack. Doing so would result in a payoff of $1 - p - c_s$, whereas by not attacking, with probability $1 - s$ $D$ accepts her low offer of $y = p - c_{Dh}$, leaving her with strictly more than her war payoff (which is the payoff she gets with probability $s$). Because neither type of $D$ engages in any extra mobilizing, there is no commitment problem.

In case (ii), if $S$ chooses not to initially attack, both types of $D$ choose $m = \overline{m}$, and $S$ then offers $y = p(\overline{m}) - c_{Dh}$, which only $c_{Dh}$ accepts. Hence, $S$’s expected payoff for not initially attacking is $EU_S(\text{not attack}) = (s)[1 - p(\overline{m}) - c_s] + (1 - s)[1 - p(\overline{m}) + c_{Dh}]$. Setting this strictly less than $EU_S(\text{attack}) = 1 - p - c_s$ and solving for $s$, we obtain that $S$ strictly prefers to initially attack if and only if $s > s_{crit} \equiv \frac{c_s + c_{Dh} - kp \overline{m}}{c_s + c_{Dh}}$ ($< 1$), i.e., $S$ is sufficiently confident that she faces the high-resolve type. Note that $s_{crit} \leq 0$ simplifies to $p(\overline{m}) - p \geq c_s + c_{Dh}$, meaning that when the latter condition holds, $S$ initially attacks for any prior belief $s \in (0, 1)$. This condition is simply Powell’s general inefficiency condition with the low-resolve type $c_{Dh}$. Note that if $S$ is sufficiently confident that she faces the high-resolve type, then she certainly wants to attack, because war is going to occur anyway with the high-resolve type (recall that no signaling will occur if $S$ allows $D$ to mobilize), and hence she would rather fight before he has mobilized because it will be a war on better terms (and note that whether Powell’s general inefficiency condition holds with the high-resolve type therefore plays no role here, as agreement will not be reached with the high-resolve type after mobilization, and hence it is not of interest to $S$ whether the offer she would
have to make to $c_D$, upon mobilization lies outside of the pre-mobilization preferred-to-war bargaining range). Hence, whether $S$ has any incentive to not initially attack depends entirely on the situation with the low-resolve type. If Powell’s general inefficiency condition holds with the low-resolve type, then $S$ certainly chooses to initially attack (i.e., for any prior $s$), because the power-shift exceeds the bargaining surplus with the low-resolve type,\footnote{Note that the bargaining surplus with the low-resolve type, $c_{D_l} + c_S$, is strictly greater than the bargaining surplus with the high-resolve type, $c_{D_h} + c_S$.} and hence $S$ prefers to initially fight even if she is very confident that she faces the low-resolve type. On the other hand, if Powell’s general inefficiency condition does not hold with the low-resolve type (i.e., $p(m) < c_{D_h}c_{D_l} - c_{D_h}c_{D_l} + k_p m$), then $S$ only initially attacks if she is sufficiently confident that she faces the high-resolve type. If $s > s_{crit}$ and $S$ initially attacks, then if $D$ ends up being the low-resolve type, then $S$ will regret that she initially attacked. On the other hand, if $s \leq s_{crit}$ and $S$ does not initially attack, then if $D$ ends up being the high-resolve type, then $S$ will regret that she did not initially attack.

In case (iii), if $S$ chooses not to initially attack, type $c_{D_l}$ chooses $m = \overline{m}$ with probability 1, whereas type $c_{D_h}$ chooses $m = \overline{m}$ with probability $a$ (as given in Proposition 3) and $m = 0$ with probability $1 - a$. Upon observing $m = 0$, $S$ offers $y = p - c_{D_h}$ (which is accepted for sure), and upon observing $m = \overline{m}$, $S$ offers $y = p(\overline{m}) - c_{D_l}$ (which is accepted for sure) with probability $b$ (as given in Proposition 3) and offers $y = p(\overline{m}) - c_{D_h}$ (which only $c_{D_h}$ accepts) with probability $1 - b$. Moreover, $S$’s updated belief that she faces $c_{D_l}$ upon observing $m = \overline{m}$ is such that she is indifferent between making these two offers, i.e., her expected payoff upon observing $m = \overline{m}$ is simply $1 - p(\overline{m}) + c_{D_l}$ (i.e., as if she makes the big offer with certainty). Hence, $S$’s expected payoff for not initially attacking is $EU_S(not \text{ attack}) = (1 - s)(1 - a)[1 - p + c_{D_h}] + [s + (1 - s)a][1 - p(\overline{m}) + c_{D_l}]$. Setting this strictly less than $EU_S(attack) = 1 - p - c_S$ and solving for $s$, we obtain that $S$ strictly prefers to initially attack if and only if $s > s_{crit} = \frac{c_{D_h} - c_{D_l}}{c_{D_h} - c_{D_l} + k_p \overline{m}} \in (0, 1)$, i.e., $S$ is sufficiently confident that she faces the high-resolve type.
1.2.1 Proof That It Is Possible For $s_{\text{crit}} < s^*$ In Case (iv)

In the main text, I claimed that there exist parameter ranges for which $s_{\text{crit}} < s^*$ in case (iv). This is important to establish because the range of priors where $s_{\text{crit}} < s < s^*$ is perhaps the most interesting one in case (iv). Note that $s_{\text{crit}} < s^*$ can be rearranged to obtain $k_p' > \frac{(c_{Dh} + c_S)^2 - (c_{Dh} - c_{Dl})^2}{c_{Dh} - c_{Dl}} (> 0)$. It is easy to see that there exist parameter values that satisfy this inequality.

1.3 The Infinite-Horizon Model

Note that, with $n = 3$ (i.e., $D$ can mobilize a maximum of 3 times), there are four possible “states of the world” (henceforth just “states”) that the players can be in — a state where the probability that $D$ wins a war is $p$ (no extra mobilization has occurred), where it is $p + m$, $p + 2m$, and $p + 3m$ (fully mobilized). I will call these states $m0, m1, m2$, and $m3$, respectively. The game begins in state $m0$, where it remains until $D$ chooses to mobilize. The state variable can only increase in increments of 1, and can never decrease (because $D$ is not allowed to de-mobilize). Once state $m3$ is reached, the state can never change.

In any given period in state $m0, m1, m2$, and $m3$, I denote the status-quo by $q_0, q_1, q_2$, and $q_3$, respectively. Within a given state, the status-quo can be different in different periods (because any agreement reached becomes the new status-quo in the next period), but because it is not necessary in the analysis to use the more complicated notation of $q_{01}, q_{02}, q_{03}, \ldots$ for different status-quo’s in different periods in state $m0$ (for example), I use the simpler notation with the understanding that the status-quo can be different for different periods within the same state (as well as in different states, of course).

The model is analyzed by a process akin to backwards induction. The analysis is broken down into 4 different cases, when the mobilization increment $m$ is high, higher-medium, lower-medium, and low. In each case, I derive the SPE and then summarize the SPE outcome at the end.

**CASE 1**: $c_S + c_D < m$ (high)

Suppose state $m3$ is reached. Then agreement would be reached immediately on $(p + 3m) - c_D$, and this agreement will never be renegotiated since $D$ cannot mobilize further.
(Note that in any SPE, going into state \( m3 \) it will always be the case that \( q_3 < (p+3m) - c_D \), so that \( D \) is dissatisfied and hence will immediately attack if \( S \) makes too small an offer.)

Suppose state \( m2 \) is reached. Then \( D \)'s payoff for rejecting and mobilizing is \( q_2 + \frac{\delta}{1-\delta}[(p+3m) - c_D] \). (\( D \)'s average per-period payoff for rejecting and mobilizing is \( (1-\delta)q_2 + \delta[(p+3m)-c_D] \), which approaches \( (p+3m)-c_D \) as \( \delta \) approaches \( 1 \).) For \( \delta \) close to \( 1 \), this is certainly better than attacking. It is also better than rejecting without mobilizing, because the payoff in the current period is the same either way, and by mobilizing, \( D \) gets \( (p+3m)-c_D \) beginning in the next period, which is the best payoff that \( D \) can possibly get in any period. So if \( S \) makes too low an offer, \( D \) will reject and mobilize. What then is \( D \)'s minimal demand in the current period? Because acceptance can lead to new agreements being reached in the future, deriving this is not as straightforward as when acceptance locks in that agreement forever (in which case we can derive that \( D \) accepts any offer \( y \) such that \( y \geq q_2 + \frac{\delta}{1-\delta}[(p+3m) - c_D] \), or \( y \geq (1-\delta)q_2 + \delta[(p+3m) - c_D] \)).

But we now know that because \( D \) rejects and mobilizes if he gets too small an offer, in any given period in state \( m2 \), if \( S \) wants to make an acceptable offer, she has no need to actually offer \( (p+3m) - c_D \), because by rejecting and mobilizing, \( D \) gets the lower payoff of \( q_2 \) in the current period (i.e., in any given period in state \( m2 \), \( D \)'s acceptance threshold will always be less than \( (p+3m) - c_D \)). Therefore, a possible best response for \( S \) in state \( m2 \) is (i) to make an infinite sequence of minimally acceptable offers that move closer to \( (p+3m) - c_D \) but never actually gets there. Because \( D \)'s average per-period payoff for rejecting and mobilizing gets arbitrarily close to \( (p+3m) - c_D \) as \( \delta \) approaches \( 1 \), \( D \)'s average per-period payoff for (i) must be arbitrarily close to \( (p+3m) - c_D \) (for \( D \) to opt for this infinite sequence of offers rather than rejecting and mobilizing), meaning that \( S \)'s average per-period payoff for (i) is arbitrarily close to \( 1 - (p+3m) + c_D \). Her other possible best responses in state \( m2 \) are to (ii) attack right away, (iii) make minimally acceptable offers for a time and then attack, or (iv) make an unacceptable offer, either right away or after making some minimally acceptable offers.

Note that for \( \delta \) close to \( 1 \), \( S \) strictly prefers (i) to (iv) - her only incentive for the latter strategy is keeping a more favorable status quo for one period but then reaching agreement.
on \((p+3m) - c_D\) in the next period. Thus, (iv) is never a best response for \(S\). Now consider a comparison between (i) and (ii). By attacking right away, \(S\) gets a payoff of \(1 - (p+2m) - c_S\) in every period. Thus, \(S\) prefers (i) to (ii) if \(1 - (p+2m) - c_S < 1 - (p+3m) + c_D\), which simplifies to \(m < c_S + c_D\). Note that if this is the case, then \(S\) also prefers (i) to (iii), because these two strategies are the same up to the point where \(S\) attacks, and beginning from that point the comparison is exactly the same. Thus, if \(m < c_S + c_D\), then \(S\)’s best response in state \(m2\) is (i), namely to make an infinite sequence of minimally acceptable offers, which as \(\delta\) approaches 1 gives \(D\) an average per-period payoff that approaches \((p+3m) - c_D\) and \(S\) an average per-period payoff that approaches \(1 - (p+3m) + c_D\).

If \(m > c_S + c_D\), on the other hand, then \(S\) prefers (ii) to (i), and also prefers (iii) to (i). Note that as \(\delta\) approaches 1, \(S\) prefers (ii) to (iii), i.e., she prefers to attack right away rather than make some minimally acceptable offers and then attack. This is because as \(\delta\) approaches 1, the very first minimally acceptable agreement will be arbitrarily close to \((p+3m) - c_D\).

To summarize, when \(\delta\) is close to 1, \(S\) will attack right away in state \(m2\) if \(m > c_S + c_D\), and will make an infinite sequence of minimally acceptable (and increasing) offers in state \(m2\) if \(m < c_S + c_D\) (which, as \(\delta\) approaches 1, has an average per-period payoff for \(D\) that is arbitrarily close to \((p+3m) - c_D\) and for \(S\) that is arbitrarily close to \(1 - (p+3m) + c_D\)). Thus what will happen in states \(m3\) and \(m2\) have been fully analyzed by a process akin to backwards induction, and below we need to consider what will happen in the earlier states when \(m > c_S + c_D\) and when \(m < c_S + c_D\) (so that different behaviors will occur in state \(m2\)).

Suppose that \(m > c_S + c_D\), so that \(S\) would immediately attack if state \(m2\) is reached. Then, in state \(m1\), \(D\)’s payoff for rejecting and mobilizing is \(q_1 + \frac{\delta}{1-\delta}[(p+2m) - c_D]\). An argument very similar to the above one implies that since \(m > c_S + c_D\), \(S\) will attack right away if state \(m1\) is reached. Below (in brackets) are the details of the argument, which need to be slightly modified.

[ ]

Suppose state \(m1\) is reached. Then \(D\)’s payoff for rejecting and mobilizing is \(q_1 + \frac{\delta}{1-\delta}[(p+
\[ (D's \text{ average per-period payoff for rejecting and mobilizing is } (1 - \delta) q_1 + \delta [(p + 2m) - c_D], \text{ which approaches } (p + 2m) - c_D \text{ as } \delta \text{ approaches 1.}) \] For \( \delta \) close to 1, this is certainly better than attacking. It is also better than rejecting without mobilizing, because the payoff in the current period is the same either way, and by mobilizing, \( D \) gets \( (p + 2m) - c_D \) beginning in the next period, which is the best payoff that \( D \) can possibly get in the future given that \( S \) will immediately attack if state \( m_2 \) is reached. So if \( S \) makes too low an offer, \( D \) will reject and mobilize. What then is \( D \)'s minimal demand in the current period? Because acceptance can lead to new agreements being reached in the future, deriving this is not as straightforward as when acceptance locks in that agreement forever.

But we now know that because \( D \) rejects and mobilizes if he gets too small an offer, in any given period in state \( m_1 \), if \( S \) wants to make an acceptable offer, she does not need to actually offer \( (p + 2m) - c_D \), because by rejecting and mobilizing, \( D \) gets the lower payoff of \( q_1 \) in the current period (i.e., in any given period in state \( m_1 \), \( D \)'s acceptance threshold will always be less than \( (p + 2m) - c_D \)). Therefore, a possible best response for \( S \) in state \( m_1 \) is (i) to make an infinite sequence of minimally acceptable offers that move closer to \( (p + 2m) - c_D \) but never actually gets there. Because \( D \)'s average per-period payoff for rejecting and mobilizing gets arbitrarily close to \( (p + 2m) - c_D \) as \( \delta \) approaches 1, \( D \)'s average per-period payoff for (i) must be arbitrarily close to \( (p + 2m) - c_D \) (for \( D \) to opt for this infinite sequence of offers rather than rejecting and mobilizing), meaning that \( S \)'s average per-period payoff for (i) is arbitrarily close to \( 1 - (p + 2m) + c_D \). Her other possible best responses in state \( m_1 \) are to (ii) attack right away, (iii) make minimally acceptable offers for a time and then attack, or (iv) make an unacceptable offer, either right away or after making some minimally acceptable offers.

Note that for \( \delta \) close to 1, \( S \) strictly prefers (i) to (iv), as (i) gives an average per-period payoff arbitrarily close to \( 1 - (p + 2m) + c_D \), whereas (iv) gives an average per-period payoff arbitrarily close to \( 1 - (p + 2m) - c_S \) (her only incentive for (iv) over (i) is keeping a more favorable status quo for one period, and this incentive disappears as \( \delta \) approaches 1). Thus, strategy (iv) is never \( S \)'s best response. Now consider a comparison between (i) and (ii). By attacking right away, \( S \) gets a payoff of \( 1 - (p + m) - c_S \) in every period, and hence \( S \) prefers
(ii) to (i) if \(1 - (p + m) - c_S > 1 - (p + 2m) + c_D\), which simplifies to \(m > c_S + c_D\), which we have stipulated to be true. Thus, our stipulation that \(m > c_S + c_D\) implies that \(S\) prefers (ii) to (i), and also prefers (iii) to (i) (since the comparison is exactly the same). Also note that as \(\delta\) approaches 1, \(S\) prefers (ii) to (iii), i.e., she prefers to attack right away rather than make some minimally acceptable offers and then attack, because as \(\delta\) approaches 1, the very first minimally acceptable offer will be arbitrarily close to \((p + 2m) - c_D\).

Thus, if \(m > c_S + c_D\) and \(\delta\) is sufficiently close to 1, \(S\) chooses to attack immediately if state \(m1\) is reached.

Then, in state \(m0\), \(D\)'s payoff for rejecting and mobilizing is \(q_0 + \frac{\delta}{1 - \delta}[(p + m) - c_D]\). A completely analogous argument as above (the one in brackets) implies that if \(m > c_S + c_D\), then \(S\) will attack right away in state \(m0\).

**SPE:** Thus, when \(\delta\) is close to 1 and \(m > c_S + c_D\), there is a SPE in which \(S\) immediately attacks in the first period of the game.

**CASE 2:** \(\frac{c_S + c_D}{2} < m < c_S + c_D\) (higher-medium)

Now suppose that \(m < c_S + c_D\), so that if state \(m2\) is reached, \(S\) would make an infinite sequence of minimally acceptable offers which when \(\delta\) is close to 1 gives \(D\) an average per-period payoff that is arbitrarily close to \((p + 3m) - c_D\) and that for \(S\) is arbitrarily close to \(1 - (p + 3m) + c_D\).

Suppose state \(m1\) is reached. Then \(D\)'s payoff for rejecting and mobilizing is arbitrarily close to \(q_1 + \frac{\delta}{1 - \delta}[(p + 3m) - c_D]\). \((D\)'s average per-period payoff for rejecting and mobilizing is arbitrarily close to \((1 - \delta)q_1 + \delta[(p + 3m) - c_D]\), which approaches \((p + 3m) - c_D\) as \(\delta\) approaches 1.) For \(\delta\) close to 1, this is certainly better than attacking. It is also better than rejecting without mobilizing, because the payoff in the current period is the same either way, and by mobilizing, \(D\) gets an average per-period payoff that is arbitrarily close to \((p + 3m) - c_D\) beginning in the next period, which is the best payoff that \(D\) can possibly get in any period. So if \(S\) makes too low an offer, \(D\) will reject and mobilize. What then is \(D\)'s minimal demand in the current period? Because acceptance can lead to new agreements being reached in future, deriving this is not as straightforward as when acceptance locks in that agreement...
forever.

But we now know that because $D$ rejects and mobilizes if he gets too small an offer, in any given period in state $m_1$, if $S$ wants to make an acceptable offer, she has no need to actually offer $(p + 3m) - c_D$, because by rejecting and mobilizing, $D$ gets the lower payoff of $q_1$ in the current period (i.e., in any given period in state $m_1$, $D$’s acceptance threshold will always be less than $(p + 3m) - c_D$). Therefore, a possible best response for $S$ is (i) to make an infinite sequence of minimally acceptable offers that move closer to $(p + 3m) - c_D$ but never actually gets there. Because $D$’s average per-period payoff for rejecting and mobilizing is arbitrarily close to $(p + 3m) - c_D$ as $\delta$ approaches $1$, $D$’s average per-period payoff for (i) must be arbitrarily close to $(p + 3m) - c_D$ (for $D$ to opt for this infinite sequence of offers rather than reject and mobilize), meaning that $S$’s average per-period payoff for (i) is arbitrarily close to $1 - (p + 3m) + c_D$. Her other possible best responses are to (ii) attack right away, (iii) make minimally acceptable offers for a time and then attack, or (iv) make an unacceptable offer, either right away or after making some minimally acceptable offers.

Note that (i) and (iv) both ultimately result in $S$ making an infinite sequence of increasing offers that approach $(p + 3m) - c_D$, and that for $\delta$ close to $1$, (i) and (iv) both give $S$ an average per-period payoff that is arbitrarily close to $1 - (p + 3m) + c_D$. Now consider a comparison between (i)/(iv) and (ii). By attacking right away, $S$ gets a payoff of $1 - (p + m) - c_S$ in every period. Thus, $S$ prefers (i)/(iv) to (ii) if $1 - (p + m) - c_S < 1 - (p + 3m) + c_D$, which simplifies to $2m < c_S + c_D$. Note that if this is the case, then $S$ also prefers (i)/(iv) to (iii), because (i) and (iii) are identical up to the point where $S$ attacks, and beginning from that point the comparison is exactly the same. Thus, if $2m < c_S + c_D$, then $S$’s best response is either (i) or (iv) in state $m_1$, namely to ultimately make an infinite sequence of minimally acceptable offers, which as $\delta$ approaches $1$ gives $D$ an average per-period payoff of $(p + 3m) - c_D$ and $S$ an average per-period payoff of $1 - (p + 3m) + c_D$.

If $2m > c_S + c_D$, on the other hand, then $S$ prefers (iii) to (i)/(iv), and also prefers (iii) to (i)/(iv). As $\delta$ approaches $1$, $S$ prefers (ii) to (iii), i.e., she prefers to attack right away rather than make some minimally acceptable offers and then attack, because as $\delta$ approaches $1$, the very first minimally acceptable agreement will be arbitrarily close to $(p + 3m) - c_D$. 

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Suppose that $2m > c_S + c_D$, so that $S$ will attack right away in state $m1$. Then the exact same argument given above in Case 1 implies that since $m < c_S + c_D$, in state $m0$ $S$ will make an infinite sequence of minimally acceptable offers that moves closer to $(p + m) - c_D$.

**SPE:** Thus, when $\delta$ is close to 1 and $\frac{c_S + c_D}{2} < m < c_S + c_D$, there is a SPE in which $S$ makes an infinite sequence of minimally acceptable offers in state $m0$ that moves closer to $(p + m) - c_D$. State $m1$ is never reached.

**CASE 3:** $\frac{c_S + c_D}{3} < m < \frac{c_S + c_D}{2}$ (lower-medium)

Now suppose that $2m < c_S + c_D$, so that if state $m1$ were reached, $S$ would ultimately make an infinite sequence of minimally acceptable offers that, when $\delta$ is close to 1, gives $D$ an average per-period payoff that is arbitrarily close to $(p + 3m) - c_D$ and that for $S$ is arbitrarily close to $1 - (p + 3m) + c_D$.

Then, in state $m0$, $D$’s payoff for rejecting and mobilizing is arbitrarily close to $q_0 + \frac{\delta}{1-\delta}([p + 3m] - c_D)$. ($D$’s average per-period payoff for rejecting and mobilizing is arbitrarily close to $(1 - \delta)q_0 + \delta([p + 3m] - c_D)$, which approaches $(p + 3m) - c_D$ as $\delta$ approaches 1.) For $\delta$ close to 1, this is certainly better than attacking. It is also better than rejecting without mobilizing, because the payoff in the current period is the same either way, and by mobilizing, $D$ gets an average per-period payoff that is arbitrarily close to $(p + 3m) - c_D$ beginning in the next period, which is the best payoff that $D$ can possibly get in any period. So if $S$ makes too low an offer, $D$ will reject and mobilize. What then is $D$’s minimal demand in the current period? Because acceptance can lead to new agreements being reached in the future, deriving this is not as straightforward as when acceptance locks in that agreement forever.

But we now know that because $D$ rejects and mobilizes if he gets too small an offer, in any given period in state $m0$, if $S$ wants to make an acceptable offer, she has no need to actually offer $(p + 3m) - c_D$, because by rejecting and mobilizing, $D$ gets the lower payoff of $q_0$ in the current period (i.e., in any given period in state $m0$, $D$’s acceptance threshold will always be less than $(p + 3m) - c_D$). Therefore, a possible best response for $S$ is (i) to make an infinite sequence of minimally acceptable offers that move closer to $(p + 3m) - c_D$ but never actually gets there. Because $D$’s average per-period payoff for rejecting and mobilizing gets
arbitrarily close to \((p + 3m) - c_D\) as \(\delta\) approaches 1, \(D\)'s average per-period payoff for (i) must be arbitrarily close to \((p + 3m) - c_D\) (for \(D\) to opt for such an infinite sequence rather than reject and mobilize), meaning that \(S\)'s average per-period payoff for (i) is arbitrarily close to \(1 - (p + 3m) + c_D\). Her other possible best responses are to (ii) attack right away, (iii) make minimally acceptable offers for a time and then attack, or (iv) make an unacceptable offer, either right away or after making some minimally acceptable offers.

Note that (i) and (iv) both ultimately result in \(S\) making an infinite sequence of increasing offers that approach \((p + 3m) - c_D\), and that for \(\delta\) close to 1, (i) and (iv) both give \(S\) an average per-period payoff that is arbitrarily close to \(1 - (p + 3m) + c_D\). Now consider a comparison between (i)/(iv) and (ii). By attacking right away, \(S\) gets a payoff of \(1 - p - c_S\) in every period. Thus, \(S\) prefers (i)/(iv) to (ii) if \(1 - p - c_S < 1 - (p + 3m) + c_D\), which simplifies to \(3m < c_S + c_D\). Note that if this is the case, then \(S\) also prefers (i)/(iv) to (iii), because (i) and (iii) are identical up to the point where \(S\) attacks, and beginning from that point the comparison is exactly the same. Thus, if \(3m < c_S + c_D\), then \(S\)'s best response is either (i) or (iv) in state \(m_0\), namely to ultimately make an infinite sequence of minimally acceptable offers, which as \(\delta\) approaches 1 gives \(D\) an average per-period payoff of \((p + 3m) - c_D\) and \(S\) an average per-period payoff of \(1 - (p + 3m) + c_D\). This is Case 4 below.

If \(3m > c_S + c_D\), on the other hand, then \(S\) prefers (ii) to (i)/(iv), and also prefers (iii) to (i)/(iv). As \(\delta\) approaches 1, \(S\) prefers (ii) to (iii), i.e., she prefers to attack right away rather than make some minimally acceptable offers and then attack, because as \(\delta\) approaches 1, the very first minimally acceptable agreement will be arbitrarily close to \((p + 3m) - c_D\).

\textbf{SPE:} Thus, when \(\delta\) is close to 1 and \(c_S + c_D \frac{m}{3} < m < c_S + c_D \frac{2}{3}\), there is a SPE in which \(S\) immediately attacks in state \(m_0\).

\textbf{CASE 4:} \(m < c_S + c_D \frac{m}{3}\) (low)

\textbf{SPE:} The argument above in Case 3 establishes that when \(\delta\) is close to 1 and \(m < c_S + c_D \frac{m}{3}\), there is a SPE in which \(S\) ultimately makes an infinite sequence of minimally acceptable offers that moves closer to \((p + 3m) - c_D\).
1.4 An Alternative Infinite-Horizon Model

Because it may be of interest to some readers, below I present the analysis of an alternative infinite-horizon model that differs from the previous one only in that if an agreement is reached in any period, the game immediately ends with that agreement being locked in forever, and $D$ not having the opportunity to threaten further mobilization. This model is considerably simpler to analyze than the previous one, and fully closed-formed solutions are presented. As the discount factor $\delta$ approaches 1, the results of the two infinite-horizon models are essentially identical.

In the following proofs, I use the “one-stage-deviation principle,” henceforth OSDP, for infinite horizon games with discounting of future payoffs (Fudenberg and Tirole 1991, 108-110). This principle states that, to verify that a profile of strategies comprises a SPE, one just has to verify that, given the other players’ strategies, no player can improve her payoff at any history at which it is her turn to move by deviating from her equilibrium strategy at that history and then reverting to her equilibrium strategy afterwards.

Note that, with $n = 3$ (i.e., $D$ can mobilize a maximum of 3 times), there are four possible “states of the world” (henceforth just “states”) that the players can be in — a state where the probability that $D$ wins a war is $p$ (no extra mobilization has occurred), where it is $p + m$, $p + 2m$, and $p + 3m$ (fully mobilized). I will call these states $m0, m1, m2$, and $m3$, respectively. The game begins in state $m0$, where it remains until $D$ chooses to mobilize. The state variable can only increase in increments of 1, and can never decrease (because $D$ is not allowed to de-mobilize). Once state $m3$ is reached, the state can never change.

The following four propositions describe SPE of the model, when $\delta$ is sufficiently large and when the mobilization increment $m$ is low, lower-medium, higher-medium, or high, respectively. These SPE are Markov-perfect (Fudenberg and Tirole 1991, 501), i.e., within a given state, a player uses a stationary strategy, which might change when the state changes.

**Proposition 5** If the following three lower bounds on $\delta$ hold (note that all three are satisfied as $\delta \to 1$): (i) $\delta^3 \geq \frac{(p-c_D)-q}{(p+3m)-c_D} \in (0, 1)$, (ii) $\delta^2 \geq \frac{(p+m-c_D)-q}{(p+3m)-c_D} \in (0, 1)$, and (iii) $\delta \geq \frac{(p+2m-c_D)-q}{(p+3m)-c_D} \in (0, 1)$, and the following three upper bounds on $m$ hold (note that as $\delta \to 1$, the binding condition becomes $m \leq \frac{c_D+c_S}{3}$): (i) $m \leq \frac{(1-\delta^3)(p-q)+\delta c_D+c_S}{3\delta^3} \geq (0)$, (ii) $m \leq \frac{(1-\delta^2)(p-q)+\delta c_D+c_S}{3\delta^2} \geq (0)$, (iii) $m \leq \frac{(1-\delta)(p-q)+\delta c_D+c_S}{3\delta} \geq (0)$.
\[
\frac{(1-\delta^2)(p-q)+\delta^2 c_D+c_S}{\delta^2-1} \quad \text{when } \delta^2 > \frac{1}{3} \quad \text{(when } \delta^2 \leq \frac{1}{3}, \text{ this restriction on } m \text{ is not needed), and (iii) }
\]
\[
m \leq \frac{(1-\delta)(p-q)+\delta c_D+c_S}{\delta^2-2} \quad \text{when } \delta > \frac{2}{3} \quad \text{(when } \delta \leq \frac{2}{3}, \text{ this restriction on } m \text{ is not needed), then}
\]
the following is a SPE in which S allows D to fully mobilize and then agreement is reached in the fourth period:

In state \(m_0\):

(a) S always proposes some \(y < y' = q(1-\delta^3)+(p+3m) - c_D\) (this RHS is the minimal amount that D is demanding in this state).

(b) D always accepts any offer such that \(y \geq q(1-\delta^3)+(p+3m) - c_D\), and always peacefully rejects and mobilizes for any lower offer.

In state \(m_1\):

(a) S always proposes some \(y < y'' = q(1-\delta^2)+(p+3m) - c_D\) (this RHS is the minimal amount that D is demanding in this state).

(b) D always accepts any offer such that \(y \geq q(1-\delta^2)+(p+3m) - c_D\), and always peacefully rejects and mobilizes for any lower offer.

In state \(m_2\):

(a) S always proposes some \(y < y''' = q(1-\delta)+(p+3m) - c_D\) (this RHS is the minimal amount that D is demanding in this state).

(b) D always accepts any offer such that \(y \geq q(1-\delta)+(p+3m) - c_D\), and always peacefully rejects and mobilizes for any lower offer.

In state \(m_3\):

(a) S always proposes \(y''' = (p+3m) - c_D\).

(b) D always accepts any offer such that \(y \geq (p+3m) - c_D\), and always fights for any lower offer.

PROOF:

State \(m_0\):

(a) First consider S’s proposal. Does S making an unacceptable proposal satisfy the one-stage-deviation principle (OSDP)? By sticking to her equilibrium strategy, agreement is reached on \(y = (p+3m) - c_D\) in period \(t+3\) (where \(t\) is the current period), giving S a utility
of \((1-q) + \delta(1-q) + \delta^2(1-q) + \frac{\delta^3}{1-\delta}[1-(p+3m) + c_D] = \frac{1-q(1-\delta^3) - \delta^3[(p+3m) - c_D]}{1-\delta}\). If S chooses to make a one-shot deviation from her equilibrium strategy and make a different unacceptable proposal, her payoff remains the same. If she chooses to make a one-shot deviation and make an acceptable proposal in the current period, then the best possible (for herself) proposal that she can make, given D’s acceptance rule, is \(y = q(1-\delta^3) + \delta^3[(p+3m) - c_D]\), giving S a utility of \(\frac{1-q(1-\delta^3) - \delta^3[(p+3m) - c_D]}{1-\delta}\), which is the same. Finally, if she chooses to make a one-shot deviation and attack, her payoff is \(\frac{1-p-c_S}{1-\delta}\), and \(\frac{1-q(1-\delta^3) - \delta^3[(p+3m) - c_D]}{1-\delta} \geq \frac{1-p-c_S}{1-\delta}\) simplifies to \(m \leq \frac{(1-\delta)(p-q)+\delta^3c_D+c_S}{3\delta^3} (>0)\), which we have stipulated to hold in this proposition. (Note that as \(\delta \to 1\), this inequality approaches \(m \leq \frac{c_D+c_S}{3}\).) Therefore, S’s proposal satisfies the OSDP.

(b) Now we need to verify that D’s acceptance rule satisfies the OSDP. If D fights upon getting an unacceptable offer (this would be a one-shot deviation from his equilibrium strategy), then his payoff is \(EU_D(F) = \frac{E_p-c_D}{1-\delta}\). If he peacefully rejects and mobilizes (which is his equilibrium strategy), then by sticking to his equilibrium strategy in the future, agreement is reached in period \(t+3\) and his payoff is \(EU_D(M) = q + \delta q + \delta^2 q + \frac{\delta^3[(p+3m) - c_D]}{1-\delta}\). If he peacefully rejects without mobilizing (which would be a one-shot deviation from his equilibrium strategy), then by sticking to his equilibrium strategy in the future, agreement is reached in period \(t+4\) and his payoff is \(EU_D(NM) = q + \delta q + \delta^2 q + \delta^3 q + \frac{\delta^4[(p+3m) - c_D]}{1-\delta}\), which is strictly less than \(EU_D(M)\). Thus, mobilizing gives him his best continuation value as long as \(EU_D(M) \geq EU_D(F)\), which simplifies to \(\delta^3 \geq \frac{(p-c_D)-q}{(p+3m)-c_D-q} \in (0,1)\), which we have stipulated to hold in this proposition. Thus, D’s acceptance rule must be to accept any offer such that \(\frac{y}{1-\delta} \geq EU_D(M)\), which simplifies to \(y \geq q(1-\delta^3) + \delta^3[(p+3m) - c_D]\).

State \(m_1\):

(a) First consider S’s proposal. Does S making an unacceptable proposal satisfy the OSDP? By sticking to her equilibrium strategy, agreement is reached on \(y = (p+3m) - c_D\) in period \(t+2\) (where \(t\) is the current period, in state \(m_1\)), giving S a utility of \((1-q) + \delta(1-q) + \frac{\delta^2}{1-\delta}[1-(p+3m) + c_D] = \frac{1-q(1-\delta^2) - \delta^2[(p+3m) - c_D]}{1-\delta}\). If S chooses to make a one-shot deviation from her equilibrium strategy and make a different unacceptable proposal, her payoff remains the same. If she chooses to make a one-shot deviation and make an
acceptable proposal in the current period, then the best possible (for herself) proposal that she can make, given $D$’s acceptance rule, is $y = q(1 - \delta^2) + \delta^2[(p + 3m) - c_D]$, giving $S$ a utility of $\frac{1 - q(1 - \delta^2) - \delta^2[(p + 3m) - c_D]}{1 - \delta}$, which is the same. Finally, if she chooses to make a one-shot deviation and attack, her payoff is $\frac{1 - p - m - cs}{1 - \delta}$, and note that $\frac{1 - q(1 - \delta^2) - \delta^2[(p + 3m) - c_D]}{1 - \delta} \geq \frac{1 - p - m - cs}{1 - \delta}$ is always satisfied (in fact, strictly) when $\delta^2 \leq \frac{1}{3}$, and simplifies to $m \leq \frac{(1 - \delta^2)(p - q) + \delta^2c_D + cs}{3\delta^2 - 1}$ when $\delta^2 > \frac{1}{3}$, which we have stipulated to hold in this proposition.\(^2\) (Note that as $\delta \to 1$, the upper bound on $m$ approaches $m \leq \frac{c_D + cs}{3}$, which is already implied by our limiting (as $\delta \to 1$) upper bound on $m$ in state $m0$ that $m \leq \frac{c_D + cs}{3}$.) Therefore, $S$’s proposal satisfies the OSDP.

(b) Now we need to verify that $D$’s acceptance rule satisfies the OSDP. If $D$ fights upon getting an unacceptable offer (this would be a one-shot deviation from his equilibrium strategy), then his payoff is $EU_D(F) = \frac{p + m - cD}{1 - \delta}$. If he peacefully rejects and mobilizes (which is his equilibrium strategy), then by sticking to his equilibrium strategy in the future, agreement is reached in period $t + 2$ and his payoff is $EU_D(M) = q + \delta q + \frac{\delta^2[(p + 3m) - cD]}{1 - \delta}$. If he peacefully rejects without mobilizing (which would be a one-shot deviation from his equilibrium strategy), then by sticking to his equilibrium strategy in the future, agreement is reached in period $t + 3$ and his payoff is $EU_D(NM) = q + \delta q + \delta^2q + \frac{\delta^2[(p + 3m) - cD]}{1 - \delta}$, which is strictly less than $EU_D(M)$. Thus, mobilizing gives him his best continuation value as long as $EU_D(M) \geq EU_D(F)$, which simplifies to $\delta^2 \geq \frac{[(p + m) - cD] - q}{[(p + 3m) - cD] - q} \in (0, 1)$, which we have stipulated to hold in this proposition. Thus, $D$’s acceptance rule must be to accept any offer such that $\frac{p}{1 - \delta} \geq EU_D(M)$, which simplifies to $y \geq q(1 - \delta^2) + \delta^2[(p + 3m) - cD]$.

State $m2$:

(a) First consider $S$’s proposal. Does $S$ making an unacceptable proposal satisfy the OSDP? By sticking to her equilibrium strategy, agreement is reached on $y = (p + 3m) - c_D$ in period $t + 1$ (where $t$ is the current period, in state $m2$), giving $S$ a utility of $(1 - q) + \frac{\delta}{1 - \delta}[1 - (p + 3m) + c_D] = \frac{1 - q(1 - \delta) - \delta[(p + 3m) - c_D]}{1 - \delta}$. If $S$ chooses to make a one-shot deviation from her equilibrium strategy and make a different unacceptable proposal, her payoff remains the

\(^2\)To see this, note that the original inequality can be rearranged to obtain $(1 - \delta^2)(p - q) + \delta^2c_D + cs \geq m(3\delta^2 - 1)$. LHS > 0 always, and RHS \leq 0 for $\delta^2 \leq \frac{1}{3}$. When $\delta^2 > \frac{1}{3}$, the inequality can be simplified to $m \leq \frac{1 - \delta^2(p - q) + \delta^2c_D + cs}{3\delta^2 - 1} (> 0)$. 
same. If she chooses to make a one-shot deviation and make an acceptable proposal in the current period, then the best possible (for herself) proposal that she can make, given D’s acceptance rule, is

\[ y = q(1 - \delta) + \delta[(p + 3m) - c_D], \]

giving S a utility of \( \frac{1 - q(1 - \delta) - \delta[(p + 3m) - c_D]}{1 - \delta} \), which is the same. Finally, if she chooses to make a one-shot deviation and attack, her payoff is \( \frac{1 - p - 2m - c_S}{1 - \delta} \), and note that \( \frac{1 - q(1 - \delta) - \delta[(p + 3m) - c_D]}{1 - \delta} \geq \frac{1 - p - 2m - c_S}{1 - \delta} \) is always satisfied (in fact, strictly) when \( \delta \leq \frac{2}{3} \), and simplifies to \( m \leq \frac{(1 - \delta)(p - q) + \delta c_D + c_S}{36 - 2} \) when \( \delta > \frac{2}{3} \), which we have stipulated to hold in this proposition.3 (Note that as \( \delta \rightarrow 1 \), the upper bound on \( m \) approaches \( c_D + c_S \), which is already implied by our limiting (as \( \delta \rightarrow 1 \)) upper bound on \( m \) in state \( m0 \) that \( m \leq \frac{c_D + c_S}{3} \).) Therefore, S’s proposal satisfies the OSDP.

(b) Now we need to verify that D’s acceptance rule satisfies the OSDP. If D fights upon getting an unacceptable offer (this would be a one-shot deviation from his equilibrium strategy), then his payoff is \( EU_D(F) = \frac{p + 2m - c_D}{1 - \delta} \). If he peacefully rejects and mobilizes (which is his equilibrium strategy), then by sticking to his equilibrium strategy in the future, agreement is reached in period \( t + 1 \) and his payoff is \( EU_D(M) = q + \frac{\delta[(p + 3m) - c_D]}{1 - \delta} \). If he peacefully rejects without mobilizing (which would be a one-shot deviation from his equilibrium strategy), then by sticking to his equilibrium strategy in the future, agreement is reached in period \( t + 2 \) and his payoff is \( EU_D(NM) = q + \delta q + \frac{\delta^2[(p + 3m) - c_D]}{1 - \delta} \), which is strictly less than \( EU_D(M) \). Thus, mobilizing gives him his best continuation value as long as \( EU_D(M) \geq EU_D(F) \), which simplifies to \( \delta \geq \frac{[\delta(p + 3m) - c_D] - q}{[\delta(p + 3m) - c_D] - q} \in (0, 1) \), which we have stipulated to hold in this proposition. Thus, D’s acceptance rule must be to accept any offer such that \( \frac{q}{1 - \delta} \geq EU_D(M) \), which simplifies to \( y \geq q(1 - \delta) + \delta[(p + 3m) - c_D] \).

State m3:

(a) First consider S’s proposal. Does offering \( y''' = (p + 3m) - c_D \) (which is the lowest offer that D will accept) satisfy the OSDP? By making this offer, S’s payoff is \( \frac{1 - p - 3m + c_D}{1 - \delta} \).

If S makes a higher offer, D will accept it and S’s payoff will be lower, and hence such a deviation is not beneficial. If S makes a lower offer, D will reject it and fight, giving S a payoff of \( \frac{1 - p - 3m - c_S}{1 - \delta} \) (this is also her payoff if she makes a one-shot deviation by attacking

3To see this, note that the original inequality can be rearranged to obtain \( (1 - \delta)(p - q) + \delta c_D + c_S \geq m(3\delta - 2) \). \( LHS \geq 0 \) always, and \( RHS \leq 0 \) for \( \delta \leq \frac{2}{3} \). When \( \delta > \frac{2}{3} \), the inequality can be simplified to \( m \leq \frac{(1 - \delta)(p - q) + \delta c_D + c_S}{36 - 2} \) (\( > 0 \)).

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instead of making an offer), which is strictly lower than her payoff for offering \(y''\). Thus, \(S\)'s proposal satisfies the OSDP.

(b) Now we need to verify that \(D\)'s acceptance rule satisfies the OSDP. If \(D\) fights upon getting an unacceptable offer (which is his equilibrium strategy), then his payoff is \(\frac{p + 3m - c_D}{1 - \delta}\). If he peacefully rejects (which would be a one-shot deviation), then by sticking to his equilibrium strategy in the future, agreement is reached in period \(t + 1\) and his payoff is \(q + \frac{\delta}{1 - \delta}[(p + 3m) - c_D]\), which is strictly lower. Thus, fighting gives a higher continuation value than peacefully rejecting, and hence \(D\)'s acceptance rule must be to accept any offer such that \(\frac{y}{1 - \delta} \geq \frac{p + 3m - c_D}{1 - \delta}\), which simplifies to \(y \geq (p + 3m) - c_D\). Q.E.D.

**Proposition 6** If the following three lower bounds on \(\delta\) hold (note that all three are satisfied as \(\delta \to 1\)): (i) \(\delta^3 \geq \frac{(p-c_D)-q}{(p+3m)-c_D} \in (0, 1)\), (ii) \(\delta^2 \geq \frac{(p+m)-c_D-q}{(p+3m)-c_D-q} \in (0, 1)\), and (iii) \(\delta \geq \frac{(p+2m)-c_D-q}{(p+3m)-c_D-q} \in (0, 1)\), and the following three conditions on \(m\) hold (note that as \(\delta \to 1\), the binding condition becomes \(c_D + c_S \leq m \leq \frac{c_D + c_S}{2}\): (i) \(m \geq \frac{(1-\delta^3)(p-q)+\delta^3 c_D+c_S}{3\delta^3} > 0\), (ii) \(m \leq \frac{(1-\delta^2)(p-q)+\delta^2 c_D+c_S}{3\delta^2-1}\) when \(\delta^2 > \frac{1}{3}\), (when \(\delta^2 \leq \frac{1}{3}\), this restriction on \(m\) is not needed), and (iii) \(m \leq \frac{(1-\delta)(p-q)+\delta c_D+c_S}{3\delta-2}\) when \(\delta > \frac{2}{3}\) (when \(\delta \leq \frac{2}{3}\), this restriction on \(m\) is not needed), then the following is a SPE in which \(S\) attacks in the first period:

In state \(m_0\):

(a) \(S\) always attacks.

(b) \(D\) always accepts any offer such that \(y \geq q(1 - \delta^3) + \delta^3[(p + 3m) - c_D]\), and always peacefully rejects and mobilizes for any lower offer.

In state \(m_1\):

(a) \(S\) always proposes some \(y < y'' = q(1 - \delta^2) + \delta^2[(p + 3m) - c_D]\) (this RHS is the minimal amount that \(D\) is demanding in this state).

(b) \(D\) always accepts any offer such that \(y \geq q(1 - \delta^2) + \delta^2[(p + 3m) - c_D]\), and always peacefully rejects and mobilizes for any lower offer.

In state \(m_2\):

(a) \(S\) always proposes some \(y < y''' = q(1 - \delta) + \delta[(p + 3m) - c_D]\) (this RHS is the minimal amount that \(D\) is demanding in this state).
(b) $D$ always accepts any offer such that $y \geq q(1 - \delta) + \delta[(p + 3m) - c_D]$, and always peacefully rejects and mobilizes for any lower offer.

In state $m_3$:

(a) $S$ always proposes $y'' = (p + 3m) - c_D$.

(b) $D$ always accepts any offer such that $y \geq (p + 3m) - c_D$, and always fights for any lower offer.

PROOF:

State $m_0$:

(a) First consider $S$’s strategy. Does attacking satisfy the OSDP? By attacking, $S$’s payoff is $1 - p - c_S 1 - q$. If $S$ makes a one-shot deviation by making an unacceptable offer in the current period (and then reverting to her equilibrium strategy), agreement is reached in period $t + 3$ (at state $m_3$) and $S$’s payoff is $(1 - q) + \delta(1 - q) + \delta^2(1 - q) + \frac{\delta^3[1 - (p + 3m) - c_D]}{1 - \delta} = \frac{1 - q(1 - \delta^3) - \delta^3[(p + 3m) - c_D]}{1 - \delta}$. If $S$ makes a one-shot deviation by making an acceptable offer in the current period, then the best possible (for herself) proposal that she can make, given $D$’s acceptance rule, is $y = q(1 - \delta^3) + \delta^3[(p + 3m) - c_D]$, giving $S$ a utility of $\frac{1 - q(1 - \delta^3) - \delta^3[(p + 3m) - c_D]}{1 - \delta}$, which is the same as for making an unacceptable offer. Thus, attacking satisfies the OSDP as long as $\frac{1 - p - c_S}{1 - \delta} \geq \frac{1 - q(1 - \delta^3) - \delta^3[(p + 3m) - c_D]}{1 - \delta}$, which simplifies to $m \geq \frac{(1 - \delta^3)(p - q) + \delta^3 c_D + c_S}{\delta^3} (> 0)$, which we have stipulated to hold in this proposition. (Note that as $\delta \to 1$, this lower bound on $m$ approaches $c_D + c_S$.)

(b) Now we need to verify that $D$’s acceptance rule satisfies the OSDP. If $D$ fights upon getting an unacceptable offer (this would be a one-shot deviation from his equilibrium strategy), then his payoff is $EU_D(F) = \frac{p - c_D}{1 - \delta}$. If he peacefully rejects and mobilizes (which is his equilibrium strategy), then by sticking to his equilibrium strategy in the future, agreement is reached in period $t + 3$ and his payoff is $EU_D(M) = q + \delta q + \delta^2 q + \frac{\delta^3[(p + 3m) - c_D]}{1 - \delta}$. If he peacefully rejects without mobilizing (which would be a one-shot deviation from his equilibrium strategy), then war occurs in the next period and his payoff is $EU_D(NM) = q + \frac{\delta(p - c_D)}{1 - \delta}$, which is strictly less than $EU_D(F)$. Thus, mobilizing gives him his best continuation value as long as $EU_D(M) \geq EU_D(F)$, which simplifies to $\delta^3 \geq \frac{(p - c_D) - q}{(p + 3m) - c_D} - q \in (0, 1)$, which we have stipulated to hold in this proposition. Thus, $D$’s acceptance rule must be to accept
any offer such that \( \frac{y}{1-\delta} \geq EU_D(M) \), which simplifies to \( y \geq q(1 - \delta^3) + \delta^3[(p + 3m) - c_D] \).

State m1:

The proof is exactly the same as in Proposition 5, as the strategies of the two players in states m1, m2, and m3 (the only three states that matter once state m1 is reached) are exactly the same as in Proposition 5. Therefore, the conditions of Proposition 5 (for state m1) are needed, namely that (i) \( m \leq \frac{(1-\delta)(p-q)+\delta^2c_D+c_S}{3\delta^2-1} \) when \( \delta^2 > \frac{1}{3} \), and (ii) \( \delta^2 \geq \frac{|(p+m)-c_D|-q}{|(p+3m)-c_D|-q} \in (0,1) \), both of which we have stipulated to hold in this proposition. (Note that as \( \delta \to 1 \), the first condition approaches \( m \leq \frac{c_D+c_S}{2} \).)

State m2:

The proof is exactly the same as in Proposition 5, as the strategies of the two players in states m2 and m3 (the only two states that matter once state m2 is reached) are exactly the same as in Proposition 5. Therefore, the conditions of Proposition 5 (for state m2) are needed, namely that (i) \( m \leq \frac{(1-\delta)(p-q)+\delta^2c_D+c_S}{3\delta^2-2} \) when \( \delta > \frac{2}{3} \), and (ii) \( \delta^2 \geq \frac{|(p+2m)-c_D|-q}{|(p+3m)-c_D|-q} \in (0,1) \), both of which we have stipulated to hold in this proposition. (Note that as \( \delta \to 1 \), the first condition approaches \( m \leq c_D + c_S \).)

State m3:

The proof is exactly the same as in Proposition 5, as the strategies of the two players in state m3 are exactly the same as in Proposition 5. Q.E.D.

**Proposition 7** If the following four lower bounds on \( \delta \) hold (note that all four are satisfied as \( \delta \to 1 \)): (i) \( \delta \geq \frac{(p-c_D)-q}{(p+m)-c_D-q} \in (0,1) \), (ii) \( \delta^2 > \frac{1}{3} \), (iii) \( \delta^2 \geq \frac{|(p+m)-c_D|-q}{|(p+3m)-c_D|-q} \in (0,1) \), and (iv) \( \delta \geq \frac{|(p+2m)-c_D|-q}{|(p+3m)-c_D|-q} \in (0,1) \), and the following three conditions on \( m \) hold (note that as \( \delta \to 1 \), the binding condition becomes \( \frac{c_D+c_S}{2} \leq m \leq c_D + c_S \)): (i) \( m \leq \frac{(1-\delta)(p-q)+\delta^2c_D+c_S}{3\delta^2-1} \) \( > 0 \) under our above stipulation that \( \delta^2 > \frac{1}{3} \), and (iii) \( m \leq \frac{(1-\delta)(p-q)+\delta^2c_D+c_S}{3\delta^2-2} \) when \( \delta > \frac{2}{3} \) (when \( \delta \leq \frac{2}{3} \), this restriction on \( m \) is not needed), then the following is a SPE in which agreement is reached in the first period:

In state m0:

(a) \( S \) always proposes \( y' = q(1 - \delta) + \delta[(p + m) - c_D] \).

(b) \( D \) always accepts any offer such that \( y \geq q(1 - \delta) + \delta[(p + m) - c_D] \), and always
peacefully rejects and mobilizes for any lower offer.

In state \( m_1 \):

(a) \( S \) always attacks.

(b) \( D \) always accepts any offer such that \( y \geq q(1 - \delta^2) + \delta^2 [(p + 3m) - c_D] \), and always peacefully rejects and mobilizes for any lower offer.

In state \( m_2 \):

(a) \( S \) always proposes some \( y < y''' = q(1 - \delta) + \delta [(p + 3m) - c_D] \) (this RHS is the minimal amount that \( D \) is demanding in this state).

(b) \( D \) always accepts any offer such that \( y \geq q(1 - \delta) + \delta [(p + 3m) - c_D] \), and always peacefully rejects and mobilizes for any lower offer.

In state \( m_3 \):

(a) \( S \) always proposes \( y''' = (p + 3m) - c_D \).

(b) \( D \) always accepts any offer such that \( y \geq (p + 3m) - c_D \), and always fights for any lower offer.

**PROOF:**

State \( m_0 \):

(a) First consider \( S \)'s proposal. Does offering \( y' = q(1 - \delta) + \delta [(p + m) - c_D] \) (which is the lowest offer that \( D \) will accept) satisfy the OSDP? By making this offer, \( S \)'s payoff is \( \frac{1-q(1-\delta)-\delta[p+m]-c_D}{1-\delta} \). If \( S \) makes a one-shot deviation by making a higher offer, \( D \) will accept it and \( S \)'s payoff will be lower, and hence such a deviation is not beneficial. If \( S \) makes a one-shot deviation by making a lower offer, \( D \) will reject it by mobilizing and war will occur in the next period (given the future equilibrium strategies), giving \( S \) a payoff of \( (1 - q) + \frac{\delta(1-p-m-c_S)}{1-\delta} = \frac{1-q(1-\delta)-\delta(p+m+c_S)}{1-\delta} \), which is strictly lower than her payoff for offering \( y' \). Finally, if \( S \) adopts a one-shot deviation by attacking, her payoff is \( \frac{1-p-c_S}{1-\delta} \), and \( \frac{1-q(1-\delta)-\delta[p+m]-c_D}{1-\delta} \geq \frac{1-p-c_S}{1-\delta} \) simplifies to \( m \leq \frac{(1-\delta)(p-q)+\delta c_D+c_S}{\delta} (> 0) \), which we have stipulated to hold in this proposition. (Note that as \( \delta \to 1 \), this inequality approaches \( m \leq c_D + c_S \).) Therefore, \( S \)'s proposal satisfies the OSDP.

(b) Now we need to verify that \( D \)'s acceptance rule satisfies the OSDP. If \( D \) fights
upon getting an unacceptable offer (this would be a one-shot deviation from his equilibrium strategy), then his payoff is $EU_D(F) = \frac{p-c_D}{1-\delta}$. If he peacefully rejects and mobilizes (which is his equilibrium strategy), then war occurs in period $t+1$ and his payoff is $EU_D(M) = q + \frac{\delta[(p+m)-c_D]}{1-\delta}$. If he和平fully rejects without mobilizing (which would be a one-shot deviation from his equilibrium strategy), then by sticking to his equilibrium strategy in the future, agreement is reached in period $t+1$ and his payoff is $EU_D(NM) = q + \frac{\delta[(1-\delta)+\delta[(p+m)-c_D]]}{1-\delta}$, which is strictly less than $EU_D(M)$. Thus, mobilizing gives him his best continuation value as long as $EU_D(M) \geq EU_D(F)$, which simplifies to $\delta \geq \frac{(p-c_D)-q}{(p+m)-c_D-q} \in (0,1)$, which we have stipulated to hold in this proposition. Thus, $D$’s acceptance rule must be to accept any offer such that $\frac{p}{1-\delta} \geq EU_D(M)$, which simplifies to $y \geq q(1-\delta) + \delta[(p+m)-c_D]$.

State $m_1$:

(a) Does $S$’s strategy of attacking satisfy the OSDP? By attacking, her payoff is $\frac{1-p-m-c_S}{1-\delta}$. If she makes a one-shot deviation by making an acceptable proposal in the current period, then the best (for herself) acceptable offer that she can make, given $D$’s acceptance rule, is $y = q(1-\delta^2) + \delta^2[(p+3m)-c_D]$, leaving $S$ with a payoff of $\frac{1-q(1-\delta^2)-\delta^2[(p+3m)-c_D]}{1-\delta}$. If $S$ adopts a one-shot deviation by making an unacceptable offer in the current period (and then reverts to her equilibrium strategy), agreement is reached in period $t+2$ (at state $m_3$), giving $S$ a payoff of $(1-q) + \delta(1-q) + \frac{\delta^2[1-(p+3m)+c_D]}{1-\delta} = \frac{1-q(1-\delta^2)-\delta^2[(p+3m)-c_D]}{1-\delta}$, which is the same. Therefore, attacking satisfies the OSDP as long as $\frac{1-p-m-c_S}{1-\delta} \geq \frac{1-q(1-\delta^2)-\delta^2[(p+3m)-c_D]}{1-\delta}$, which is never satisfied when $\delta^2 \leq \frac{4}{3}$, and which can be simplified to $m \geq \frac{(1-\delta^2)(p-q)+\delta^2c_D+c_S}{3\delta^2-1}$ when $\delta^2 > \frac{1}{3}$. Therefore, in this proposition we stipulate that both $m \geq \frac{(1-\delta^2)(p-q)+\delta^2c_D+c_S}{3\delta^2-1}$ and $\delta^2 > \frac{1}{3}$ hold. (Note that as $\delta \to 1$, this lower bound on $m$ approaches $m \geq \frac{c_D+c_S}{2}$.)

(b) Now we need to verify that $D$’s acceptance rule satisfies the OSDP. If $D$ fights upon getting an unacceptable offer (this would be a one-shot deviation from his equilibrium strategy), then his payoff is $EU_D(F) = \frac{p+m-c_D}{1-\delta}$. If he peacefully rejects and mobilizes (which is his equilibrium strategy), then by sticking to his equilibrium strategy in the future, agreement is reached in period $t+2$ and his payoff is $EU_D(M) = q + \delta q + \frac{\delta^2[(p+3m)-c_D]}{1-\delta}$. If

$\text{To see this, note that the original inequality can be rearranged to obtain } (1-\delta^2)(p-q) + \delta^2c_D + c_S \leq m(3\delta^2-1). \text{ LHS} > 0 \text{ always, and RHS} \leq 0 \text{ for } \delta^2 \leq \frac{1}{3}. \text{ When } \delta^2 > \frac{1}{3}, \text{ the inequality can be simplified to } m \geq \frac{(1-\delta^2)(p-q)+\delta^2c_D+c_S}{3\delta^2-1} (> 0).$
he peacefully rejects without mobilizing (which would be a one-shot deviation from his equilibrium strategy), then war occurs in the next period and his payoff is $EU_D(NM) = q + \frac{\delta(p + m - c_D)}{1 - 3}$, which is strictly less than $EU_D(F)$. Thus, mobilizing gives him his best continuation value as long as $EU_D(M) \geq EU_D(F)$, which simplifies to $\delta^2 \geq \frac{[p + m - c_D] - q}{[p + 2m - c_D] - q} \in (0, 1)$, which we have stipulated to hold in this proposition. Thus, $D$’s acceptance rule must be to accept any offer such that $\frac{y}{1 - \delta} \geq EU_D(M)$, which simplifies to $y \geq q(1 - \delta^2) + \delta^2[p + 3m] - c_D$.

State $m_2$:

The proof is exactly the same as in Proposition 5, as the strategies of the two players in states $m_2$ and $m_3$ (the only two states that matter once state $m_2$ is reached) are exactly the same as in Proposition 5. Therefore, the conditions of Proposition 5 (for state $m_2$) are needed, namely that (i) $m \leq \frac{(1 - \delta)(p - q) + \delta c_D + c_S}{3m - 2}$ when $\delta > \frac{2}{3}$, and (ii) $\delta \geq \frac{[p + m - c_D] - q}{[p + 2m - c_D] - q} \in (0, 1)$, both of which we have stipulated to hold in this proposition. (Note that as $\delta \to 1$, the first condition approaches $m \leq c_D + c_S$.)

State $m_3$:

The proof is exactly the same as in Proposition 5, as the strategies of the two players in state $m_3$ are exactly the same as in Proposition 5. Q.E.D.

Proposition 8 If the following four lower bounds on $\delta$ hold (note that all four are satisfied as $\delta \to 1$): (i) $\delta \geq \frac{(p - c_D) - q}{(p + m - c_D) - q} \in (0, 1)$, (ii) $\delta \geq \frac{(p + m) - c_D - q}{(p + 2m) - c_D - q} \in (0, 1)$, (iii) $\delta > \frac{2}{3}$, and (iv) $\delta \geq \frac{[p + 2m - c_D] - q}{[p + 3m] - c_D - q} \in (0, 1)$, and the following three lower bounds on $m$ hold (note that as $\delta \to 1$, the binding condition becomes $m \geq c_D + c_S$): (i) $m \geq \frac{(1 - \delta)(p - q) + \delta c_D + c_S}{\delta}$ ($> 0$), (ii) $m \geq \frac{(1 - \delta)(p - q) + \delta c_D + c_S}{3m - 1}$ ($> 0$ under our above stipulation that $\delta > \frac{2}{3}$), and (iii) $m \geq \frac{(1 - \delta)(p - q) + \delta c_D + c_S}{3m - 2}$ ($> 0$ under our above stipulation that $\delta > \frac{2}{3}$), then the following is a SPE in which $S$ attacks in the first period:

In state $m_0$:

(a) $S$ always attacks.

Note that the numerators of all three inequalities for $m$ are the same and the denominator of inequality (iii) is strictly the smallest (for $\delta < 1$), and hence inequality (iii) is the binding one (i.e., when it is satisfied, the other two are satisfied strictly). Therefore, we could simplify the statement of the proposition by just requiring that (iii) hold, but presenting it as above makes the proof a little more clear.
(b) \( D \) always accepts any offer such that \( y \geq q(1 - \delta) + \delta[(p + m) - c_D] \), and always peacefully rejects and mobilizes for any lower offer.

In state \( m_1 \):

(a) \( S \) always attacks.

(b) \( D \) always accepts any offer such that \( y \geq q(1 - \delta) + \delta[(p + 2m) - c_D] \), and always peacefully rejects and mobilizes for any lower offer.

In state \( m_2 \):

(a) \( S \) always attacks.

(b) \( D \) always accepts any offer such that \( y \geq q(1 - \delta) + \delta[(p + 3m) - c_D] \), and always peacefully rejects and mobilizes for any lower offer.

In state \( m_3 \):

(a) \( S \) always proposes \( y''' = (p + 3m) - c_D \).

(b) \( D \) always accepts any offer such that \( y \geq (p + 3m) - c_D \), and always fights for any lower offer.

PROOF:

State \( m_0 \):

(a) Does \( S \)'s choice of attacking satisfy the OSDP? By attacking, \( S \)'s payoff is \( \frac{1 - p - c_S}{1 - \delta} \).

If \( S \) chooses to make a one-shot deviation by making an acceptable proposal in the current period, then the best (for herself) offer that she can make, given \( D \)'s acceptance rule, is \( y' = q(1 - \delta) + \delta[(p + m) - c_D] \), leaving \( S \) with a payoff of \( \frac{1 - q(1 - \delta) - \delta[(p + m) - c_D]}{1 - \delta} \). If \( S \) makes a one-shot deviation by making an unacceptable offer, \( D \) will reject it by mobilizing and war will occur in the next period (given the future equilibrium strategies), giving \( S \) a payoff of \( \frac{1 - q (1 - \delta) - \delta [(p + m) - c_D]}{1 - \delta} \), which is strictly lower than her payoff for offering \( y' \). Therefore, attacking satisfies the OSDP as long as \( \frac{1 - q (1 - \delta) - \delta [(p + m) - c_D]}{1 - \delta} \leq \frac{1 - p - c_S}{1 - \delta} \), which simplifies to \( m \geq \frac{(1 - \delta)(p - q) + \delta c_D + c_S}{\delta} (> 0) \), which we have stipulated to hold in this proposition. (Note that as \( \delta \to 1 \), this inequality approaches \( m \geq c_D + c_S \).)

(b) Now we need to verify that \( D \)'s acceptance rule satisfies the OSDP. If \( D \) fights upon getting an unacceptable offer (this would be a one-shot deviation from his equilibrium
strategy), then his payoff is \( EU_D(F) = \frac{r-c_D}{1-\delta} \). If he peacefully rejects and mobilizes (which is his equilibrium strategy), then war occurs in period \( t+1 \) and his payoff is \( EU_D(M) = q + \frac{\delta[(p+m)-c_D]}{1-\delta} \). If he peacefully rejects without mobilizing (which would be a one-shot deviation from his equilibrium strategy), then war occurs in period \( t+1 \) and his payoff is \( EU_D(NM) = q + \frac{\delta(p-c_D)}{1-\delta} \), which is strictly less than \( EU_D(F) \). Thus, mobilizing gives him his best continuation value as long as \( EU_D(M) \geq EU_D(F) \), which simplifies to \( \delta \geq \frac{(p-c_D)-q}{[(p+m)-c_D]-q} \) \( \in (0,1) \), which we have stipulated to hold in this proposition. Thus, \( D \)'s acceptance rule must be to accept any offer such that \( \frac{q}{1-\delta} \geq EU_D(M) \), which simplifies to \( y \geq q(1-\delta) + \delta[(p+m)-c_D] \).

State \( m1 \):

(a) Does \( S \)'s choice of attacking satisfy the OSDP? By attacking, \( S \)'s payoff is \( \frac{1-p-m-c_S}{1-\delta} \). If \( S \) chooses to make a one-shot deviation by making an acceptable proposal in the current period, then the best (for herself) offer that she can make, given \( D \)'s acceptance rule, is \( y" = q(1-\delta) + \delta[(p+2m)-c_D] \), leaving \( S \) with a payoff of \( \frac{1-q(1-\delta)-\delta[(p+2m)-c_D]}{1-\delta} \). If \( S \) makes a one-shot deviation by making an unacceptable offer, \( D \) will reject it by mobilizing and war will occur in the next period (given the future equilibrium strategies), giving \( S \) a payoff of \( (1-q) + \frac{\delta(1-p-2m-c_S)}{1-\delta} = \frac{1-q(1-\delta)-\delta[(p+2m)+c_S]}{1-\delta} \), which is strictly lower than her payoff for offering \( y" \). Therefore, attacking satisfies the OSDP as long as \( \frac{1-q(1-\delta)-\delta[(p+2m)-c_D]}{1-\delta} \leq \frac{1-p-m-c_S}{1-\delta} \), which is never satisfied when \( \delta \leq \frac{1}{2} \), and which can be simplified to \( m \geq \frac{(1-\delta)(p-q)+6c_D+c_S}{2(1-\delta)} \) when \( \delta > \frac{1}{2} \). Our stipulation in this proposition that \( \delta > \frac{2}{3} \) (derived below, in state \( m2 \)) ensures that \( \delta > \frac{1}{2} \), and so we merely require that \( m \geq \frac{(1-\delta)(p-q)+6c_D+c_S}{2(1-\delta)} \), which we have stipulated to hold in this proposition. (Note that as \( \delta \to 1 \), this lower bound on \( m \) approaches \( m \geq c_D + c_S \).)

(b) Now we need to verify that \( D \)'s acceptance rule satisfies the OSDP. If \( D \) fights upon getting an unacceptable offer (this would be a one-shot deviation from his equilibrium strategy), then his payoff is \( EU_D(F) = \frac{b+m-c_D}{1-\delta} \). If he peacefully rejects and mobilizes (which is his equilibrium strategy), then war occurs in period \( t+1 \) and his payoff is \( EU_D(M) = \frac{(1-\delta)(p-q)+6c_D+c_S}{2(1-\delta)} \) (> 0).

\( ^{6} \)To see this, note that the original inequality can be rearranged to obtain \( (1-\delta)(p-q) + 6c_D + c_S \leq m(2\delta - 1) \). \( LHS > 0 \) always, and \( RHS \leq 0 \) for \( \delta \leq \frac{1}{2} \). When \( \delta > \frac{1}{2} \), the inequality can be simplified to \( m \geq \frac{(1-\delta)(p-q)+6c_D+c_S}{2(1-\delta)} \) (> 0).
\[ q + \frac{\delta[(p+2m)-c_D] - q}{1-\delta}. \] If he peacefully rejects without mobilizing (which would be a one-shot deviation from his equilibrium strategy), then war occurs in period \( t+1 \) and his payoff is \( EU_D(NM) = q + \frac{\delta(p+m-c_D)}{1-\delta} \), which is strictly less than \( EU_D(F) \). Thus, mobilizing gives him his best continuation value as long as \( EU_D(M) \geq EU_D(F) \), which simplifies to \[ \delta \geq \frac{[(p+m)-c_D] - q}{[(p+2m)-c_D] - q} \in (0,1), \] which we have stipulated to hold in this proposition. Thus, \( D \)'s acceptance rule must be to accept any offer such that \[ \frac{y}{1-\delta} \geq EU_D(M), \] which simplifies to \[ y \geq q(1-\delta) + \delta[(p + 2m) - c_D]. \]

**State m2:**

(a) Does \( S \)'s choice of attacking satisfy the OSDP? By attacking, \( S \)'s payoff is \[ \frac{1-p-2m-c_S}{1-\delta}. \] If \( S \) chooses to make a one-shot deviation by making an acceptable proposal in the current period, then the best (for herself) offer that she can make, given \( D \)'s acceptance rule, is \( y'' = q(1-\delta) + \delta[(p+3m)-c_D] \), leaving \( S \) with a payoff of \[ \frac{1-q(1-\delta)-\delta(p+3m)-c_D}{1-\delta}. \] If \( S \) makes a one-shot deviation by making an unacceptable offer (and then reverting to her equilibrium strategy), \( D \) will reject it by mobilizing and agreement will be reached in the next period, giving \( S \) a payoff of \[ \frac{1-q-\delta}{1-\delta} = \frac{1-q(1-\delta)-\delta(p+3m)-c_D}{1-\delta}, \] which is the same as for offering \( y'' \). Therefore, attacking satisfies the OSDP as long as \[ \frac{1-q(1-\delta)-\delta(p+3m)-c_D}{1-\delta} \leq \frac{1-p-2m-c_S}{1-\delta}, \] which is never satisfied when \( \delta \leq \frac{2}{3} \), and which can be simplified to \( m \geq \frac{(1-\delta)(p-q)+\delta c_D+c_S}{3\delta-2} \) when \( \delta > \frac{2}{3} \). Therefore, in this proposition we stipulate that both \( m \geq \frac{(1-\delta)(p-q)+\delta c_D+c_S}{3\delta-2} \) and \( \delta > \frac{2}{3} \) hold. (Note that as \( \delta \rightarrow 1 \), this lower bound on \( m \) approaches \( m \geq c_D + c_S \).

(b) Now we need to verify that \( D \)'s acceptance rule satisfies the OSDP. If \( D \) fights upon getting an unacceptable offer (this would be a one-shot deviation from his equilibrium strategy), then his payoff is \( EU_D(F) = \frac{p+2m-c_D}{1-\delta} \). If he peacefully rejects and mobilizes (which is his equilibrium strategy), then agreement is reached in the next period and his payoff is \( EU_D(M) = q + \frac{\delta(p+2m-c_D)}{1-\delta} \). If he peacefully rejects without mobilizing (which would be a one-shot deviation from his equilibrium strategy), then war occurs in period \( t+1 \) and his payoff is \( EU_D(NM) = q + \frac{\delta(p+2m-c_D)}{1-\delta} \), which is strictly less than \( EU_D(F) \). Thus, mobilizing gives him his best continuation value as long as \( EU_D(M) \geq EU_D(F) \), which simplifies to

\[ \frac{(1-\delta)(p-q)+\delta c_D+c_S}{3\delta-2} \geq \frac{1-p-2m-c_S}{1-\delta} \]

(> 0).

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\[ \text{To see this, note that the original inequality can be rearranged to obtain} \] \[ (1-\delta)(p-q) + \delta c_D + c_S \leq m(3\delta - 2). \text{ LHS is always positive, and RHS is 0 for} \] \[ \delta \leq \frac{2}{3}. \text{ When} \delta > \frac{2}{3}, \text{ the inequality can be simplified to} \] \[ m \geq \frac{(1-\delta)(p-q)+\delta c_D+c_S}{3\delta-2} \] (>) 0).
\[ \delta \geq \frac{[(p+2m)-c_D]-q}{[(p+3m)-c_D]-q} \in (0, 1), \] 
which we have stipulated to hold in this proposition. Thus, \( D \)'s acceptance rule must be to accept any offer such that \( \frac{y}{1-\delta} \geq EU_D(M) \), which simplifies to \( y \geq q(1-\delta) + \delta[(p+3m) - c_D] \).

**State m3:**

The proof is exactly the same as in Proposition 5, as the strategies of the two players in state \( m3 \) are exactly the same as in Proposition 5. Q.E.D.
References