Technical Supplement to:

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In this technical supplement, I provide proofs for all of the perfect Bayesian equilibria (in pure as well as mixed strategies) of the main model analyzed in the article.\(^1\) I then analyze the case where the challenger (new leader) comes from a weaker pool of candidates than does the incumbent leader. Finally, I analyze a variant of the model in which the incumbent can actually choose what type of target she wants, if she decides to engage in an aggressive foreign policy.

### 1 Pure Strategy Equilibria

The pure strategy equilibria are summarized in Table 1.

First note that there are no separating equilibria, i.e., equilibria of the type in which the competent type chooses, say, \(A\), when the economy is, say, good, and the incompetent type chooses \(\overline{A}\). This is because upon observing a good economy and \(A\) (\(\overline{A}\)), the voter would know for certain that the incumbent is competent (incompetent), and would therefore reelect (depose) her with certainty. Thus, the incompetent type could profitably deviate from her strategy by mimicking the action of the competent type, in this example by choosing \(A\) rather than \(\overline{A}\), thereby ensuring her reelection. As shown in the next section, there do exist mixed strategy equilibria in which the two types of incumbent choose \(A\) with different probabilities — however, the difference is never large enough to allow the voter to meaningfully distinguish between the two types based on their strategies alone, i.e., the incompetent type never deviates “too much” from the behavior of the competent type.

\(^1\)Note that, in a game in which each player has at most two types, every perfect Bayesian equilibrium is also a sequential equilibrium (Fudenberg and Tirole, 1991:346). Therefore, this is also the set of sequential equilibria.
1.1 Good Economy

**GE1:** Both types of incumbent choose nonaggression ($a = b = 0$), and the incumbent is reelected by choosing nonaggression.

Proof: I want to look for all equilibria of the form $a = b = 0$. When $a = b = 0$, $\nu_{GE/A} = \frac{pq}{pq + (1-p)q'}$. Note that $\frac{pq}{pq + (1-p)q'} > p$ can be simplified to obtain $p < 1$, which is true by assumption. Therefore, when $a = b = 0$, $\nu_{GE/A} > p$, and so the incumbent is reelected by choosing nonaggression. Since the incumbent is being reelected (and hence obtaining her highest payoff) by choosing nonaggression, this is an equilibrium for any off-the-equilibrium path beliefs $\nu_{GE/S(A)}^*$ and $\nu_{GE/US(A)}^*$ (the asterisk denotes off-the-equilibrium path beliefs, which the analyst specifies, since Bayes’ Rule does not apply). Q.E.D.

If $\nu_{GE/S(A)}^* \geq p$ and $\nu_{GE/US(A)}^* \geq p$, then the incumbent expects to be reelected even if she chooses aggression, whether or not it is successful. If these optimistic off-the-equilibrium path beliefs (that the incumbent is competent) are held by the voter, then this is an “indifference” equilibrium in the sense that the incumbent expects to be reelected even if she deviates from her equilibrium strategy by choosing $A$. However, if either $\nu_{GE/S(A)}^* < p$ or $\nu_{GE/US(A)}^* < p$, then the incumbent expects to be deposed with positive probability by choosing $A$, and so this is a “strict” equilibrium in which the incumbent has a strict incentive to not deviate from her equilibrium strategy.

**GE2:** Both types of incumbent choose aggression ($a = b = 1$), and the incumbent is reelected regardless of the outcome of the aggression.

**GE3:** Both types of incumbent choose aggression ($a = b = 1$), and the incumbent is reelected if it is successful, and is not reelected if it fails (and is not reelected if she does not choose aggression).
Proof: I want to look for all equilibria of the form $a = b = 1$. When $a = b = 1$, $\nu_{GE/S(A)} = \frac{pq^s}{pq^s + (1-p)q^ps'}$. Note that $\frac{pq^s}{pq^s + (1-p)q^ps'} > p$ can be simplified to obtain $p < 1$, which is true by assumption. Therefore, when $a = b = 1$, $\nu_{GE/S(A)} > p$ and so the incumbent is reelected if the aggression is successful.

The next question is whether $\nu_{GE/US(A)}$ is smaller or bigger than $p$ when $a = b = 1$. When $a = b = 1$, $\nu_{GE/US(A)} = \frac{pq(1-s)}{pq(1-s) + (1-p)q'(1-s')}$, Setting $\frac{pq(1-s)}{pq(1-s) + (1-p)q'(1-s')} \geq p$ and simplifying, we see that this is true if and only if $\frac{q}{q'} \geq \frac{1-s'}{1-s}$. Therefore, when this condition holds, $\nu_{GE/US(A)} \geq p$ and so the incumbent is reelected even if the aggression fails. This is the GE2 equilibrium. Since the incumbent is being reelected by choosing aggression, this is an equilibrium for any off-the-equilibrium path belief $\nu^*_A$.

If $\frac{q}{q'} < \frac{1-s'}{1-s}$, then $\nu_{GE/US(A)} < p$ and so the incumbent is deposed if the aggression is unsuccessful. This is the GE3 equilibrium. Since the incumbent is being deposed with positive probability by choosing aggression, this equilibrium requires that $\nu_{GE/A}^* < p$ so that the incumbent would also be deposed by choosing nonaggression. Q.E.D.

These are all of the pure strategy equilibria when the economy is good.

1.2 Bad Economy

**BE1:** Both types of incumbent choose aggression ($c = d = 1$), and the incumbent is reelected if the aggression succeeds and is deposed if it fails (and would be deposed if she did not undertake aggression).

**BE2:** Both types of incumbent choose aggression ($c = d = 1$), and the incumbent is deposed regardless of whether it succeeds or fails (and would be deposed if she did not undertake aggression).
Proof: I want to look for all equilibria of the form $c = d = 1$. When $c = d = 1$, 
\[ \nu_{BE/US}(A) = \frac{p(1-q)(1-s)}{p(1-q) + (1-p)(1-q')} \]. Note that \[ \frac{p(1-q)(1-s)}{p(1-q) + (1-p)(1-q')} < p \] can be simplified to obtain $p < 1$, which is true by assumption. Therefore, when $c = d = 1$, 
\[ \nu_{BE/US}(A) < p \] and so the incumbent is deposed if the aggression fails.

The next question is whether $\nu_{BE/S}(A)$ is bigger or smaller than $p$ when $c = d = 1$. When $c = d = 1$, 
\[ \nu_{BE/S}(A) = \frac{p(1-q)s}{p(1-q) + (1-p)(1-q')s'} \]. Setting \[ \frac{p(1-q)s}{p(1-q) + (1-p)(1-q')s'} \geq p \] and simplifying, we see that this is true if and only if \[ \frac{s}{s'} \geq \frac{1-q'}{1-q} \]. Therefore, when this condition holds $\nu_{BE/S}(A) \geq p$ and so the incumbent is reelected if the aggression is successful. This is the BE1 equilibrium. Since the incumbent is being deposed with positive probability by choosing aggression, this equilibrium requires that $\nu^*_{BE/A} < p$ so that the incumbent would also be deposed by choosing nonaggression.

If \[ \frac{s}{s'} < \frac{1-q'}{1-q} \], then $\nu_{BE/S}(A) < p$ and so the incumbent is deposed even if the aggression is successful. This is the BE2 equilibrium. Since the incumbent is being deposed with certainty by choosing aggression, this equilibrium requires that $\nu^*_{BE/A} < p$ so that the incumbent would also be deposed by choosing nonaggression. Q.E.D.

BE3: Both types of incumbent choose nonaggression ($c = d = 0$), and the incumbent is deposed (and would be deposed if she undertook aggression, regardless of whether it succeeds or fails).

Proof: I want to look for all equilibria of the form $c = d = 0$. When $c = d = 0$, 
\[ \nu_{BE/A} = \frac{p(1-q)}{p(1-q) + (1-p)(1-q')} \]. Note that \[ \frac{p(1-q)}{p(1-q) + (1-p)(1-q')} < p \] can be simplified to obtain $p < 1$, which is true by assumption. Therefore, when $c = d = 0$, $\nu_{BE/A} < p$ and so the incumbent is deposed by choosing nonaggression. Since the incumbent is being deposed by choosing nonaggression, this equilibrium requires that $\nu^*_{BE/S(A)} < p$ and $\nu^*_{BE/US(A)} < p$ so
that the incumbent is also deposed by choosing aggression, regardless of whether it succeeds or fails. Otherwise, the incumbent could profitably deviate by choosing aggression, since she would thereby expect to be reelected with positive probability.\footnote{Note that intuitively we would expect $\nu_{BE/S(A)}^* \geq \nu_{BE/US(A)}^*$, since competent types are more likely to be successful in aggression than are incompetent types.} Q.E.D.

These are all of the pure strategy equilibria when the economy is bad.

2 Mixed Strategy Equilibria

Because they are not particularly interesting substantively, I have relegated the description of the mixed strategy equilibria to this technical supplement. Note that in all of the good economy mixed strategy equilibria, the incumbent is always reelected regardless of the outcome, and is always deposed in all of the bad economy mixed strategy equilibria, regardless of the outcome. Also note that because these equilibria place positive probability on all outcomes, no off-the-equilibrium path beliefs have to be specified and hence these equilibria satisfy every possible refinement.

2.1 Good Economy

GE4: $a \in (0, \frac{q-q'}{q}]$ and $b = 0$ is an equilibrium in which the incumbent is reelected regardless of the outcome.

Proof: I want to look for all partially mixed equilibria of the form $a \in (0, 1)$ and $b = 0$. Since the CI (competent incumbent) is the only one choosing $A$ with positive probability, the voter reelects the incumbent for certain if he observes a good economy and $A$. Therefore, for II’s choice of $\overline{A}$ to be incentive-compatible, it is necessary that $\nu_{GE/\overline{A}} \geq p$, so that the incumbent will also be reelected by choosing $\overline{A}$. (This will also justify CI mixing, since she will be indifferent.) Setting $b = 0$, this inequality can be rearranged to obtain $a \leq \frac{q-q'}{q}$. Note
that $0 < \frac{q - q'}{q} < 1$. Q.E.D.

This equilibrium resembles GE1, but with $CI$ choosing $A$ with small positive probability $a$. As long as $a$ is small enough, the voter will still reelect the incumbent when the outcome is $\bar{A}$ (with a good economy), because he is sufficiently confident that the incumbent is $CI$.

GE5: If $\frac{q}{q'} > \frac{1 - s'}{1 - s}$, then $a \in \left[\frac{q'(1 - s')}{q(1 - s)}, 1\right)$ and $b = 1$ is an equilibrium in which the incumbent is reelected regardless of the outcome.

Proof: I want to look for all partially-mixed equilibria of the form $a \in (0, 1)$ and $b = 1$. Since $CI$ is the only one choosing $\bar{A}$ with positive probability, the voter reelects the incumbent when the outcome is $\bar{A}$ (with a good economy). Therefore, for $II$’s choice of $A$ to be incentive compatible, it is necessary that $\nu_{GE/S(A)} \geq p$ and $\nu_{GE/US(A)} \geq p$, so that the incumbent will be reelected even by choosing $A$. (This will also justify $CI$ mixing, since she will be indifferent.) Since $\nu_{GE/S(A)} > \nu_{GE/US(A)}$, we just need to ensure that $\nu_{GE/US(A)} \geq p$. Setting $b = 1$, this inequality can be rearranged to obtain $a \geq \frac{q'(1 - s')}{q(1 - s)}$. This is possible if and only if $\frac{q'(1 - s')}{q(1 - s)} < 1$, which can be rearranged to obtain $\frac{q}{q'} > \frac{1 - s'}{1 - s}$. Q.E.D.

This equilibrium resembles GE2 (for which $\frac{q}{q'} \geq \frac{1 - s'}{1 - s}$ is also a necessary and sufficient condition), but with $CI$ choosing $\bar{A}$ with small positive probability $1 - a$. As long as $a$ is high enough, the voter will still reelect the incumbent upon observing $A$ (even when the outcome is $US(A)$), because he is sufficiently confident that the incumbent is competent.

Before deriving the fully-mixed equilibrium for when the economy is good, I now show that there don’t exist partially-mixed equilibria of the form $a = 0$ and $b \in (0, 1)$, or $a = 1$ and $b \in (0, 1)$.

CLAIM: There doesn’t exist an equilibrium of the form $a = 0, b \in (0, 1)$.

Proof: Suppose the incumbent is adopting such a strategy. Since only $II$ is choosing $A$
with positive probability, the voter deposes the incumbent upon observing $A$ (with a good economy). Therefore, for $II$’s choice of playing $A$ with positive probability to be incentive-compatible, it is necessary that $\nu_{GE/A} < p$, so that the incumbent will be deposed even by choosing $A$. Setting $a = 0$, this inequality can be rearranged to obtain $b < \frac{q' - q}{q'}$, which is impossible since $\frac{q' - q}{q'} < 0$. (The intuition here is that when $CI$ is choosing $A$ with certainty, the voter’s belief $\nu_{GE/A}$ can never fall below $p$, no matter how high the probability with which $II$ also chooses $A$.) Q.E.D.

**CLAIM:** There doesn’t exist an equilibrium of the form $a = 1, b \in (0, 1)$.

**Proof:** Suppose the incumbent is adopting such a strategy. Since only $II$ is choosing $A$ with positive probability, the voter deposes the incumbent upon observing $A$ (with a good economy). Therefore, for $II$’s choice of playing $A$ with positive probability to be incentive-compatible, it is necessary that $\nu_{GE/S(A)} < p$ and $\nu_{GE/US(A)} < p$, so that the incumbent will be deposed even by choosing $A$. Setting $a = 1$, the former inequality can be rearranged to obtain $b > \frac{q'q}{q'}$. This is impossible, since $\frac{q'q}{q'} > 1$. (The intuition here is that when $CI$ is choosing $A$ with certainty, the voter’s belief $\nu_{GE/S(A)}$ can never fall below $p$, no matter how high the probability with which $II$ also chooses $A$. It is, however, possible for $b$ to be high enough that $\nu_{GE/US(A)} < p$.) Q.E.D.

**GE6:** $a \in \left[ \frac{q'q}{q(1-s)}, \frac{q-q'(1-b)}{q} \right]$ and $b \in (0, \frac{(1-s)(q-q')}{q'(s-s')}]$, in which the incumbent is reelected regardless of the outcome, is the only fully mixed-strategy equilibrium.

**Proof:** I want to look for all equilibria of the form $a \in (0, 1)$ and $b \in (0, 1)$. For mixing to be incentive-compatible, it must be the case that either (i) $\nu_{GE/A} \geq p$ and $\nu_{GE/US(A)} \geq p$ (the latter implies $\nu_{GE/S(A)} > p$) so that the incumbent will be reelected regardless of the outcome, or (ii) $\nu_{GE/A} < p$ and $\nu_{GE/S(A)} < p$ (the latter implies $\nu_{GE/US(A)} < p$) so that the
incumbent will be deposed regardless of the outcome. I will show that case (i) is possible when the specified conditions on $a$ and $b$ hold, but that case (ii) is impossible.

(i) $\nu_{GE/A} \geq p$ can be rearranged to obtain $a \leq \frac{q-q'(1-b)}{q}$. $\nu_{GE/US(A)} \geq p$ can be rearranged to obtain $a \geq \frac{q'b(1-s')}{q(1-s)}$. Thus, we require $a \in \left[ \frac{q-q'(1-b)}{q}, \frac{q'b(1-s')}{q(1-s)} \right]$. Note that $\frac{q'b(1-s')}{q(1-s)} > 0$ and $\frac{q-q'(1-b)}{q} < 1$. For such values of $a$ to be possible, it is necessary that $\frac{q-q'(1-b)}{q} \geq \frac{q'b(1-s')}{q(1-s)}$.

This can be rearranged to obtain $b \leq \frac{(1-s)(q-q')}{q'(s-s')}$. Thus, we require that $b \in (0, \frac{(1-s)(q-q')}{q'(s-s')}]$.

Note that $\frac{(1-s)(q-q')}{q'(s-s')} > 0$, and $\frac{(1-s)(q-q')}{q'(s-s')} \geq 1$ can be rearranged to obtain $\frac{q}{q'} \geq \frac{1-s'}{1-s}$. Thus, when the latter condition holds, the allowable set of values for $b$ becomes $b \in (0, 1)$.

(ii) $\nu_{GE/A} < p$ can be rearranged to obtain $a > \frac{q-q'(1-b)}{q}$. $\nu_{GE/S(A)} < p$ can be rearranged to obtain $a < \frac{q'sb}{q's'}$. For values of $a$ to exist that can satisfy both of these conditions, it is necessary that $\frac{q'sb}{q's'} > \frac{q-q'(1-b)}{q}$. This can be rearranged to obtain $b < -\frac{s(q-q')}{q'(s-s')}$, which is impossible since $\frac{-s(q-q')}{q'(s-s')} < 0$. Hence, there doesn’t exist a fully mixed-strategy equilibrium in which the incumbent is deposed regardless of the outcome (when the economy is good).

Q.E.D.

2.2 Bad Economy

**BE4:** $c = 0$ and $d \in (0, \frac{q-q'}{1-s})$ is an equilibrium in which the incumbent is deposed regardless of the outcome.

*Proof:* I want to look for all partially-mixed equilibria of the form $c = 0$ and $d \in (0, 1)$. Since the $II$ (incompetent incumbent) is the only one choosing $A$ with positive probability, the voter deposes the incumbent for certain if he observes a bad economy and $A$. Therefore, for $II$’s strategy of choosing $A$ with positive probability to be incentive-compatible, it is necessary that $\nu_{BE/A} < p$, so that the incumbent will also be deposed by choosing $A$ (this will also justify $II$’s use of a mixed strategy). Setting $c = 0$, this inequality can be rearranged
to obtain $d < \frac{q - q'}{1 - q}$. Note that $0 < \frac{q - q'}{1 - q} < 1$. Q.E.D.

This equilibrium resembles BE3, but with II choosing $A$ with small positive probability $d$. If $d$ is too high, then the voter will reelect upon observing $\overline{A}$, and so II’s choice of $A$ with positive probability wouldn’t be incentive-compatible. Thus, $d$ has to be sufficiently small.

**BE5:** If $\frac{s}{s'} < \frac{1 - q'}{1 - q}$, then $c = 1$ and $d \in \left(\frac{(1-q)s}{(1-q')s'}, 1\right)$ is an equilibrium in which the incumbent is deposed regardless of the outcome.

**Proof:** I want to look for all partially-mixed equilibria of the form $c = 1$ and $d \in (0, 1)$. Since II is the only one choosing $\overline{A}$ with positive probability, the voter deposes the incumbent when the outcome is $\overline{A}$ (with a bad economy). Therefore, for II’s strategy of choosing $\overline{A}$ with positive probability to be incentive compatible, it is necessary that $\nu_{BE/S(A)} < p$ and $\nu_{BE/US(A)} < p$, so that the incumbent will be deposed even by choosing $A$ (this will also justify II’s use of a mixed strategy). Since $\nu_{BE/S(A)} > \nu_{BE/US(A)}$, we just need to ensure that $\nu_{BE/S(A)} < p$. Setting $c = 1$, this inequality can be rearranged to obtain $d > \frac{(1-q)s}{(1-q')s'} (> 0)$.

This is possible if and only if $\frac{(1-q)s}{(1-q')s'} < 1$, which can be rearranged to obtain $\frac{s}{s'} < \frac{1 - q'}{1 - q}$. Q.E.D.

This equilibrium resembles BE2 (for which $\frac{s}{s'} < \frac{1 - q'}{1 - q}$ is also a necessary and sufficient condition), but with II choosing $\overline{A}$ with small positive probability $1 - d$. If $1 - d$ were too high, then the voter would reelect upon observing $S(A)$ (and eventually even $US(A)$), and so II’s choice of $\overline{A}$ with positive probability wouldn’t be incentive-compatible. Therefore, $d$ has to be sufficiently high.

Before deriving the fully-mixed equilibrium for when the economy is bad, I now show that there don’t exist partially-mixed equilibria of the form $c \in (0, 1)$ and $d = 0$, or $c \in (0, 1)$ and $d = 1$. 

10
CLAIM: There doesn’t exist an equilibrium of the form $c \in (0, 1)$, $d = 0$.

Proof: Suppose the incumbent is adopting such a strategy. Since only $CI$ is choosing $A$ with positive probability, the voter reelects the incumbent upon observing $A$ (with a bad economy). Therefore, for $II$’s choice of $\overline{A}$ to be incentive-compatible, it is necessary that $\nu_{BE/\overline{A}} \geq p$, so that the incumbent will also be reelected by choosing $\overline{A}$ (this will also justify $CI$ mixing). Setting $d = 0$, this inequality can be rearranged to obtain $c \leq \frac{q - q'}{1 - q}$, which is impossible since $\frac{q - q'}{1 - q} < 0$. (The intuition here is that when $II$ is choosing $\overline{A}$ with certainty, the voter’s belief $\nu_{BE/\overline{A}}$ can never reach or exceed $p$, no matter how high the probability with which $CI$ also chooses $A$. ) Q.E.D.

CLAIM: There doesn’t exist an equilibrium of the form $c \in (0, 1)$, $d = 1$.

Proof: Suppose the incumbent is adopting such a strategy. Since only $CI$ is choosing $\overline{A}$ with positive probability, the voter reelects the incumbent upon observing $\overline{A}$ (with a bad economy). Therefore, for $II$’s choice of $A$ to be incentive-compatible, it is necessary that $\nu_{BE/S(A)} \geq p$ and $\nu_{BE/US(A)} \geq p$, so that the incumbent will be reelected even by choosing $A$ (this will also justify $CI$ mixing). Setting $d = 1$, the latter inequality can be rearranged to obtain $c \geq \frac{(1 - q')(1 - s')}{(1 - q)(1 - s)}$. This is impossible, since $\frac{(1 - q')(1 - s')}{(1 - q)(1 - s)} > 1$. (The intuition here is that when $II$ is choosing $A$ with certainty, the voter’s belief $\nu_{BE/US(A)}$ can never reach or exceed $p$, no matter how high the probability $c$ with which $CI$ also chooses $A$. It is, however, possible for $c$ to be high enough that $\nu_{BE/S(A)} \geq p$, if $\frac{s'}{q'} > \frac{1 - q'}{1 - q}$.) Q.E.D.

BE6: $c \in \left(\frac{1 - q - (1 - q')(1 - d)}{1 - q}, \frac{(1 - q')(1 - d) - s'}{(1 - q)(1 - s')}\right)$ and $d \in (0, \frac{s(1 - q')}{(1 - q')(1 - s')})$, in which the incumbent is deposed regardless of the outcome, is the only fully mixed-strategy equilibrium.

Proof: I want to look for all equilibria of the form $c \in (0, 1)$ and $d \in (0, 1)$. For mixing to be incentive-compatible, it must be the case that either (i) $\nu_{BE/\overline{A}} \geq p$ and $\nu_{BE/US(A)} \geq p$ (the
latter implies $\nu_{BE/S(A)} > p$ so that the incumbent will be reelected regardless of whether she chooses $A$ or $A$, or (ii) $\nu_{BE/A} < p$ and $\nu_{BE/S(A)} < p$ (the latter implies $\nu_{BE/US(A)} < p$) so that the incumbent will be deposed regardless of whether she chooses $A$ or $A$. I will show that case (ii) is possible when the specified conditions on $c$ and $d$ hold, but that case (i) is impossible.

(i) $\nu_{BE/A} \geq p$ can be rearranged to obtain $c \geq \frac{1-q-(1-q')(1-d)}{1-q}$. $\nu_{BE/US(A)} \geq p$ can be rearranged to obtain $c \geq \frac{(1-q')(1-s')d}{1-q}$. For values of $c$ to exist that can satisfy both of these conditions, it is necessary that $\frac{1-q-(1-q')(1-d)}{1-q} \geq \frac{(1-q')(1-s')d}{1-q}$. This can be rearranged to obtain $d \leq \frac{(q'-q)(1-s)}{(1-q')(s-s')}$, which is impossible since $\frac{(q'-q)(1-s)}{(1-q')(s-s')} < 0$. Hence, there doesn’t exist a fully mixed-strategy equilibrium in which the incumbent is reelected regardless of the outcome (when the economy is bad).

(ii) $\nu_{BE/A} < p$ can be rearranged to obtain $c > \frac{1-q-(1-q')(1-d)}{1-q}$. $\nu_{BE/S(A)} < p$ can be rearranged to obtain $c < \frac{(1-q')(1-s')d}{1-q}$. For such values of $c$ to be possible, it is necessary that $\frac{1-q-(1-q')(1-d)}{1-q} < \frac{(1-q')(1-s')d}{1-q}$. This can be rearranged to obtain $d < \frac{s(q-q')}{(1-q')(s-s')} (> 0)$. Thus, we require that $d \in (0, \frac{s(q-q')}{(1-q')(s-s')})$.

Note that $\frac{s(q-q')}{(1-q')(s-s')} \geq 1$ can be rearranged to obtain $\frac{s}{s} \leq \frac{1-q'}{1-q}$. Hence, when the latter condition holds, the allowable set of values for $d$ is $d \in (0, 1)$. Also note that, for $d \leq \frac{q-q'}{1-q}$ ($\leq \frac{s(q-q')}{(1-q')(s-s')}$), it is the case that $\frac{1-q-(1-q')(1-d)}{1-q} \leq 0$, and so the acceptable set of values for $c$ becomes $c \in (0, \frac{(1-q')(s-s')}{(1-q)s})$. Q.E.D.
3 Challenger Comes From Weaker Pool than Incumbent

Now suppose that if the voter decides to depose the incumbent, the challenger (new leader) is competent with probability $p'$, where $p' < p$ (previously, we were assuming that the challenger comes from the same pool as the incumbent, namely that $p' = p$). I want to show that there exists an equilibrium in which both types of incumbent choose nonaggression even when the economy is bad, and the incumbent is reelected.

**BE1’**: Both types of incumbent choose nonaggression ($c = d = 0$), and the incumbent is reelected by choosing nonaggression.

**BE2’**: Both types of incumbent choose nonaggression ($c = d = 0$), and the incumbent is deposed by choosing nonaggression (and would be deposed if she chose aggression, regardless of whether it succeeds or fails).

**Proof**: I want to look for all equilibria of the form $c = d = 0$. When $c = d = 0$, $\nu_{BE/A} = \frac{p(1-q)}{p(1-q)+(1-p)(1-q')}. \quad \text{Note that } \frac{p(1-q)}{p(1-q)+(1-p)(1-q')} \geq p'$ can be simplified to obtain $\frac{1-q}{1-q'} \geq \frac{p'}{p}. \quad \text{Therefore, when this condition holds, the incumbent is being reelected by choosing nonaggression. This is the BE1’ equilibrium. Since the incumbent is being reelected (and hence obtaining her highest payoff) by choosing nonaggression, this is an equilibrium for any off-the-equilibrium path beliefs } \nu_{BE/S(A)}^* \text{ and } \nu_{BE/US(A)}^*.$

If $\frac{1-q}{1-q'} < \frac{p'}{p}$, then the incumbent is deposed by choosing nonaggression. This is the BE2’ equilibrium. Since the incumbent is being deposed by choosing nonaggression, this equilibrium requires that $\nu_{BE/S(A)}^* < p'$ and $\nu_{BE/US(A)}^* < p'$ so that the incumbent is also deposed by choosing aggression, regardless of whether it succeeds or fails. Otherwise, the incumbent could profitably deviate by choosing aggression, since she would thereby expect
to be reelected with positive probability. Q.E.D.

Now I want to look for all equilibria in which both types of incumbent choose aggression.

**BE3’**: Both types of incumbent choose aggression \((c = d = 1)\), and the incumbent is reelected only if it is successful (and would not be reelected if she chose nonaggression).

**BE4’**: Both types of incumbent choose aggression \((c = d = 1)\), and the incumbent is reelected whether it succeeds or fails.

**BE5’**: Both types of incumbent choose aggression \((c = d = 1)\), and the incumbent is deposed whether it succeeds or fails (and would be deposed if she chose nonaggression).

Proof: I want to look for all equilibria of the form \(c = d = 1\). When \(c = d = 1\), \(\nu_{BE/S(A)} = \frac{p(1-q)s}{p(1-q)s + (1-p)(1-q')s'}\). Note that \(\frac{p(1-q)s}{p(1-q)s + (1-p)(1-q')s'} \geq p'\) can be simplified to obtain \(\frac{1-q}{1-q'} \geq \frac{p' - s'(1-p)}{p - s(1-p')}.\) Therefore, when this condition holds, the incumbent is being reelected if the aggression is successful.

When \(c = d = 1\), \(\nu_{BE/US(A)} = \frac{p(1-q) - (1-s)}{p(1-q)(1-s)(1-p)(1-q')(1-x')}\). Setting this greater than or equal to \(p'\) and simplifying, we obtain \(\frac{1-q}{1-q'} \geq \frac{p' - s'(1-p)}{p - s(1-p')}.\) Therefore, when this condition holds, the incumbent is being reelected if the aggression is successful.

Note that \(\frac{p'(1-s')(1-p)}{p(1-s)(1-p')} > \frac{p'(1-s')(1-p)}{p(1-s)(1-p')}.\) Therefore, when \(\frac{1-q}{1-q'} \geq \frac{p'(1-s')(1-p)}{p(1-s)(1-p')},\) the incumbent is reelected regardless of whether the aggression succeeds or fails. This is the BE4’ equilibrium. This equilibrium requires that \(\nu^*_{BE/\overline{A}} < p'\) so that the incumbent is deposed if she chooses nonaggression.

When \(\frac{p'(1-s')(1-p)}{p(1-s)(1-p')} \leq \frac{1-q}{1-q'} < \frac{p'(1-s')(1-p)}{p(1-s)(1-p')},\) then the incumbent is reelected if the aggression is successful and is deposed if it is not. This is the BE3’ equilibrium. This equilibrium requires that \(\nu^*_{BE/\overline{A}} < p'\) so that the incumbent is deposed if she chooses nonaggression.

When \(\frac{p'(1-s')(1-p)}{p(1-s)(1-p')} > \frac{1-q}{1-q'}\), then the incumbent is deposed regardless of whether the aggression succeeds or fails. This is the BE5’ equilibrium. This equilibrium requires that
\[ \nu_{BE/A}^* < p' \] so that the incumbent is deposed if she chooses nonaggression. Q.E.D.

### 3.1 Discussion

From our proof of BE1', we see that if \( \frac{1-q}{1-q'} \geq \frac{p'}{p} \), then the incumbent is reelected even by choosing nonaggression. Note that \( \frac{1-q}{1-q'} \) \((< 1)\) is a measure of how much control over the economy the incumbent is perceived to have. The closer \( q \) is to \( q' \), meaning the less control over the economy the incumbent is perceived to have, the bigger this ratio is (it converges to 1 from below as \( q \) converges to \( q' \) from above). That is, the bigger this ratio is, the less control over the economy the incumbent is perceived to have. Thus, the condition \( \frac{1-q}{1-q'} \geq \frac{p'}{p} \) means that when the incumbent is not perceived to have much control over the state of the economy, the incumbent is reelected even if she does not use a diversionary policy, because the bad economy downgrades the voter’s belief about the incumbent’s competence only a little, and the incumbent is still preferred over the challenger, who comes from a weaker pool.

On the other hand, suppose that this condition doesn’t hold, so that the incumbent is deposed if she chooses nonaggression. Then, the proof for BE3' shows that, if \( \frac{1-q}{1-q'} \geq \frac{p'\cdot s' \cdot (1-p)}{p \cdot s \cdot (1-p')} \), then the incumbent will be reelected if she undertakes aggression and it turns out to be successful. (Note that \( \frac{p'}{p} > \frac{p' \cdot s' \cdot (1-p)}{p \cdot s \cdot (1-p')} \).) This condition can be rewritten as \( \frac{s}{s'} \geq \frac{(1-q') \cdot p' \cdot (1-p)}{(1-q) \cdot p \cdot (1-p')} = S_{\text{critical}} \). In other words, \( s \) has to be sufficiently large compared to \( s' \), meaning that the incumbent needs to choose a sufficiently difficult target in order to get reelected (if the aggression is successful; note that for \( p' < p \), this threshold is strictly smaller than the threshold of \( \frac{s}{s'} \geq \frac{1-q'}{1-q} \) in the model where the challenger comes from the same pool as the incumbent, which makes intuitive sense). Note that \( \frac{\partial S_{\text{critical}}}{\partial p'} = \frac{(1-q')(1-p)}{(1-q)p(1-p')} > 0 \), meaning that as \( p' \) becomes smaller (holding \( p \) constant), the right-hand-side of the condition
\[ \frac{s}{s'} \geq \frac{(1-q') (1-p)}{(1-q) p (1-p')} \]

becomes smaller, meaning that \( s \) doesn’t have to be as large relative to \( s' \) in order for this inequality to hold. This means that as \( p' \) becomes smaller compared to \( p \), the incumbent can choose a weaker target (for whom \( s \) and \( s' \) are both high) in order to get reelected (if the aggression turns out to be successful). In other words, she doesn’t have to choose as difficult a target. The weaker the pool from which the challenger comes, the weaker the target that the incumbent can choose in order to get reelected (if the aggression turns out to be successful), because the lower is the competency threshold that she needs to exceed in order to be reelected.

### 4 Endogenous Choice of Target

Now suppose that the incumbent has an array of targets that she can choose if she decides to use a diversionary policy. A target can be characterized by its values of \( s \) and \( s' \), and hence be denoted by the ordered pair \((s, s')\). If the incumbent decides to use an aggressive foreign policy, she chooses which target \((s, s')\) she wants, and the voter observes this decision (there could be a continuum of targets available, or a finite number).

All of the equilibria described previously still hold. In any equilibrium in which the incumbent chooses aggression and \( \frac{s}{s'} \) has to satisfy a certain value for that equilibrium to exist, just specify that both types of incumbent choose a target (they both choose the same one — again, there cannot be separating behavior in an equilibrium, for the same reason discussed earlier — namely, the voter would learn the incumbent’s type from its choice of target alone, and hence the incompetent type has an incentive to mimic the behavior of the competent type) for which that condition holds (assuming that such a target is available — if it doesn’t, the equilibrium doesn’t exist in the model with endogenous choice of targets),
and if they choose any other target (this is off-the-equilibrium path behavior), the voter believes (these are off-the-equilibrium path beliefs) that the probability that the incumbent is competent is less than \( p \) (or \( p' \), when the challenger comes from a weaker pool than the incumbent), regardless of whether the aggression ends in success or failure, and so the incumbent is deposed if she chooses a different target. (As discussed below, for some of the equilibria, we can allow less restrictive off-the-equilibrium path beliefs; however, the “absolutely pessimistic” beliefs specified above will always do the trick and ensure that it is an equilibrium.)

For example, consider the BE1 equilibrium, the model’s main “diversionary war” equilibrium. This equilibrium requires that \( \frac{s}{\bar{r}} \geq \frac{1-q'}{1-q} \). So suppose that both types of incumbent choose a target for which this condition holds. Then, if the aggression turns out to be successful, the incumbent is reelected. If the aggression turns out to be unsuccessful, the incumbent is deposed (as shown in the proof of BE1). Thus, as mentioned in the proof of BE1, this equilibrium requires the off-the-equilibrium path belief to be \( \nu^*_{BE/\mathcal{A}} < p \) so that the incumbent would also be deposed by choosing nonaggression.

To ensure that this is an equilibrium in the model where endogenous choice of targets is allowed, we could just specify that if the incumbent chooses a different target (this is off-the-equilibrium path behavior), the voter believes the incumbent to be competent with some probability less than \( p \), regardless of whether it succeeds or fails, so that the incumbent would be deposed by choosing a different target.

Therefore, BE1 is an equilibrium for any target for which the condition \( \frac{s}{\bar{r}} \geq \frac{1-q'}{1-q} \) holds. That is, there are a multiplicity of BE1 equilibria (assuming that there is more than one target for which the condition holds), in each of which the incumbent chooses a different
target. Which equilibrium does the incumbent prefer? Ex post, i.e. after nature chooses the incumbent’s type, the competent type would prefer the equilibrium in which, among all targets for which the condition above holds, the actual target chosen is the one for which $s$ is the highest, as this is the probability that the competent incumbent is reelected (when the economy turns out to be weak). The incompetent type would prefer the target for which $s'$ is the highest. These are not necessarily the same (although, I will argue below, from a substantive viewpoint they will always be the same). For example, consider the target for which, among all those for which the condition $s \geq \frac{1-q'}{1-q}$ holds, $s'$ is the highest. Denote this target by $(s, s')$. It may be that there is another target, denoted by $(s_1, s'_1)$, for which $s'_1 < s'$ and $s_1 > s$, and for which the condition therefore also holds. The competent type would strictly prefer the equilibrium in which $(s_1, s'_1)$ is chosen rather than $(s, s')$, whereas the incompetent type is strictly best off in the equilibrium in which $(s, s')$ is chosen.

Ex ante, i.e. before the incumbent’s type is chosen (this is the standard way to discuss which equilibrium an actor prefers, when there are multiple equilibria), the incumbent would prefer the equilibrium in which the target for which $p \cdot (1-q) \cdot s + (1-p) \cdot (1-q') \cdot s'$ is the highest (this is the incumbent’s ex ante expected utility for the economy turning out bad, in the BE1 equilibrium) is chosen. The voter prefers the equilibrium for which $p(1-q)[s(1)+(1-s)(p)]+(1-p)(1-q')[s'(0)+(1-s')(p)]$ is the highest (this is the voter’s ex ante expected utility for the economy turning out bad, in the BE1 equilibrium).

We would like to be able to say from a substantive point of view that the incumbent prefers the equilibrium in which the weakest target for which the condition $s \geq \frac{1-q'}{1-q}$ holds is chosen. However, as discussed above, there is not necessarily such a clearly “weakest” target. Suppose that, among all targets for which the condition holds, there is one for which
as well as \(s'\) are the highest. Then, this can rightly be regarded as the weakest target among those for which the condition holds, as the probabilities that the competent as well as the incompetent type are successful are both maximized. On the other hand, suppose that the target \((s, s')\) is the one for which \(s'\) (but not necessarily \(s\)) is maximized, among all targets for which the condition holds. Then, there may be another target, \((s_1, s_1')\), for which \(s_1 > s\) but \(s_1' < s'\), and for which the condition therefore also holds. Now, it can’t be said that one target is clearly weaker than the other, because the competent type would rather fight \((s_1, s_1')\), whereas the incompetent type would rather fight \((s, s')\). Ex ante, which equilibrium the incumbent prefers depends on how large \(p\), the prior probability that the incumbent turns out to be competent, is. When \(p\) is high, the incumbent attaches more weight to \(s\) being high, and attaches more weight to \(s'\) being high when \(p\) is low. (To be more accurate, the incumbent attaches more weight to \(s\) being high when \(p(1 - q)\) is high, which is the ex ante probability that the incumbent turns out to be competent but winds up with a weak economy, relative to \((1 - p)(1 - q')\), which is the ex ante probability that the incumbent turns out to be incompetent and ends up with a weak economy.)

The discussion above was somewhat theoretical. From a substantive viewpoint, it seems that there will indeed be a clearly weakest target, among all targets for which the condition holds. Again, suppose \((s, s')\) is the target for which \(s'\) (but not necessarily \(s\)) is maximized, among all targets for which the condition holds. Above, we pointed out that theoretically, there may be another target, \((s_1, s_1')\), for which \(s_1 > s\) but \(s_1' < s'\), and for which the condition therefore also holds. However, consider what we are saying by this. We are basically saying that there could exist another target which the incompetent type is less likely to succeed against relative to the original target (which intuitively means that it is a militarily stronger
target than the original one), but the competent type is actually more likely to succeed. This doesn’t make sense at all from a substantive viewpoint. What seems more reasonable is that, for a militarily stronger target, both types’ probabilities of being successful decline. The incompetent type’s probability may decline at a greater rate, and hence the ratio $\frac{s}{s'}$ may be greater for tougher targets (against whom competence plays a greater role relative to weaker targets), but for any target for which $s'$ is lower, $s$ should be lower as well (although maybe not at the same rate). Thus, from a substantive viewpoint, it seems that if $(s, s')$ is the target for which $s'$ is maximized among all targets for which the condition holds, $s$ is also the maximum among all these targets. Therefore, from a substantive viewpoint, there will indeed be a clearly weakest target for which the condition holds, and the incumbent is strictly best off in the equilibrium in which this target is chosen, ex ante as well as ex post. Our intuition that the incumbent likes to choose the weakest target for which the condition holds is correct.

4.1 Specific Off-the-Equilibrium Path Beliefs that can be Allowed

For each specific equilibrium of the main model, I now discuss which off-the-equilibrium path beliefs (if the incumbent chooses a different target) are acceptable for that to be an equilibrium in the model where endogenous choice of targets is allowed.

For GE1 and GE2, any off-the-equilibrium path beliefs are acceptable, since the incumbent is being reelected with certainty. For GE3 (in which the incumbent chooses a competence revealing target and is only reelected if successful), it seems reasonable that if the incumbent deviates and chooses a (non-competence revealing) target for which $\frac{q}{q'} \geq \frac{1-s'}{1-s}$ (as in GE2), then the incumbent should be reelected regardless of whether the aggression
succeeds or fails.\textsuperscript{3} But under these reasonable off-the-equilibrium path beliefs, GE3 is no longer an equilibrium, as the incumbent would have an incentive to deviate and choose such a target (assuming such a target is available). Therefore, GE3 requires perhaps unreasonable off-the-equilibrium path beliefs, e.g. the “absolutely pessimistic” beliefs mentioned in the previous section (GE3 also requires the perhaps unreasonable of-the-equilibrium path belief that $\nu_{GE/A}^* < p$). Therefore, when the economy is good, the emphasis should be on GE1 and GE2 (as is done in the article), in the model where the incumbent can endogenously choose the target.

BE2 and BE3 also require the absolutely pessimistic beliefs in order to be equilibria in the model with endogenous choice of targets. This is because in these equilibria, the incumbent is being deposed with certainty (on the equilibrium path). However, it seems reasonable that if the incumbent deviates and chooses a sufficiently challenging target (i.e., one for which $s' \geq 1 - q' = 1 - q$, as in BE1), the incumbent should be reelected if successful.\textsuperscript{4} But under these reasonable beliefs, BE2 and BE3 would no longer be equilibria, as the incumbent could then profitably deviate by choosing such a target (if one is available). Therefore, BE2 and BE3 require perhaps unreasonable off-the-equilibrium path beliefs in order to be equilibria in the model with endogenous choice of targets. Therefore, when the economy is bad, the emphasis should be on BE1 (as is done in the article), in the model where the incumbent can endogenously choose the target.

On the other hand, when there is no sufficiently challenging target available, then BE2 and BE3 do not need to rely on perhaps unreasonable off-the-equilibrium path beliefs, and

\textsuperscript{3}This is based on the supposition that if such a target is chosen, the voter believes that both types of incumbent chose it with the same probability.

\textsuperscript{4}Again, this is based on the supposition that if such a target is chosen, the voter believes that both types of incumbent chose it with the same probability.
hence they become quite reasonable equilibria. Then, from a substantive point of view we can conclude that, when no sufficiently challenging target is available, the incumbent is deposed with certainty when the economy turns out to be weak (this is in the model where \( p' = p \), i.e. the challenger comes from the same pool of candidates as the incumbent — when \( p' < p \), this is not necessarily the case, as discussed earlier). If we assume that war is just slightly costly (i.e., even for some positive but arbitrarily small amount), it seems reasonable that BE3 (in which nonaggression is chosen) will be preferred over BE2 (in which aggression against a weak, non-competence revealing target is chosen), since the incumbent is going to be deposed anyway. That is, when no sufficiently challenging target is available, it seems reasonable that no aggressive foreign policy will be used, since it will not help the incumbent’s reelection chances anyway.

In BE1, the incumbent chooses a target for which \( \frac{s}{s'} \geq \frac{1-q'}{1-q} \) holds, and is reelected only if it is successful. It seems reasonable that if the incumbent deviates and chooses another target for which the condition also holds, then she should still be reelected if successful.\(^5\) Under these seemingly reasonable off-the-equilibrium path beliefs, the only BE1 equilibrium which is an equilibrium in the model with endogenous choice of targets is the one in which the weakest target among all those for which \( \frac{s}{s'} \geq \frac{1-q'}{1-q} \) holds is chosen, i.e. the one for which both \( s \) and \( s' \) are the highest (recall our discussion earlier; from a substantive viewpoint, if there exist any targets for which the above condition holds, then such a clearly weakest target among them will exist). Any other target would not be an equilibrium under these beliefs, because then both types of incumbent could profitably deviate by choosing the weakest target (among all those that satisfy the condition) instead. That is, any other

\(^5\)Again, this is based on the supposition that if such a target is chosen, the voter believes that both types of incumbent chose it with the same probability.
target requires perhaps unreasonable off-the-equilibrium path beliefs, such as the absolutely pessimistic ones. This is an additional argument for why the weakest target (among those that satisfy the condition) will be chosen: not only does the incumbent strictly prefer this equilibrium among all BE1 equilibria, but it is also the only BE1 equilibrium which does not require perhaps unreasonable off-the-equilibrium path beliefs.

For all of the good economy mixed strategy equilibria (GE4, GE5, and GE6), the incumbent is being reelected with certainty on the equilibrium path, and hence these are equilibria for any off-the-equilibrium path beliefs (if another target is chosen) in the model with endogenous choice of targets. (Note that, in the model with endogenous choice of targets, there is another way in which the incumbent can be mixing, in addition to mixing between aggression and nonaggression; namely it can be mixing in its choice of target, if it chooses aggression. I don’t explore mixed strategy equilibria like these.) In all of the bad economy mixed strategy equilibria (BE4, BE5, and BE6), the incumbent is being deposed with certainty on the equilibrium path, and hence these equilibria require the absolutely pessimistic beliefs, if the incumbent deviates and chooses a different target. Again, the absolutely pessimistic beliefs are not necessarily reasonable, e.g. if the incumbent deviates and chooses a target for which the condition \( \frac{s}{q} \geq \frac{1-q}{1-q} \) holds (if such a target exists), in which case it could reasonably be argued that she should be reelected if successful.

BE1’ and BE4’ can have any off-the-equilibrium path beliefs when endogenous choice of target is allowed, as the incumbent is being reelected with certainty on the equilibrium path. In BE2’ and BE5’, the incumbent is being deposed with certainty, and hence these require the absolutely pessimistic beliefs in order to be equilibria with endogenous choice of targets. Again, the absolutely pessimistic beliefs are perhaps unreasonable, if the incumbent
chooses a sufficiently difficult target (if available) which should get her reelected if successful.

For BE3’, in which the incumbent is being reelected only if the aggression is successful, the same discussion for BE1 above applies. Namely, for the weakest target among all those for which the condition $\frac{s}{s'} \geq \frac{(1-q')p'(1-p)}{(1-q)p(1-p')}$ holds (assuming that any such targets exist), BE3’ is an equilibrium even under the reasonable off-the-equilibrium path beliefs that if some other target such that the condition also holds is chosen, the incumbent is reelected if she is successful. But for any other target such that the condition holds, BE3’ requires perhaps unreasonable off-the-equilibrium path beliefs, such as the absolutely pessimistic ones. Moreover, as in BE1, among all BE3’ equilibria, the incumbent strictly prefers the one in which the weakest target for which the condition holds is chosen.

References

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<th>Incompetent</th>
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<th>$US(A)$</th>
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Table 1: Pure Strategy Equilibria. The column labeled “Competent” (“Incompetent”) indicates the action taken by the competent (incompetent) type of incumbent in that equilibrium. The $\overline{A}$, $S(A)$, and $US(A)$ columns indicate whether the voter reelects or deposes the incumbent when the outcome is nonaggression, successful aggression, or unsuccessful aggression, respectively (“depends” in GE1 and GE2 indicates that whether the incumbent is reelected or deposed depends on the off-the-equilibrium path beliefs, which can be anything in those two equilibria). The last column indicates the necessary and sufficient conditions for that equilibrium to exist (note that GE1 and BE3 always exist, although the latter requires restrictive off-the-equilibrium path beliefs). An asterisk (*) denotes off-the-equilibrium path outcomes and beliefs.