Making a Risky Proposal in Crisis Bargaining

Ahmer Tarar
Department of Political Science
Texas A&M University
email: ahmertarar@tamu.edu

January 22, 2017

Abstract

Fearon (1995) used an ultimatum model of crisis bargaining to show that when country A is uncertain about country B's cost of war, then under very broad conditions A's optimal proposal entails a risk of rejection and hence war. This result is foundational in establishing private information as a rationalist explanation for war. I relax one assumption that Fearon makes and find that A's optimal proposal is risk-free under broader conditions than previous work has indicated. I do the same for when the uncertainty is about the military balance instead. The results do not undermine private information as a rationalist explanation for war, but do suggest that private information is not as conducive to war breaking out as previous work has indicated. I also discuss the reasonableness, in incomplete information models of crisis bargaining, of using various probability distributions to capture the likely empirical distribution of war costs or the probability of winning a war.

1 Introduction

International relations theorists have long posited that misperceptions and uncertainty are pervasive sources of conflict (e.g., Blainey 1988; Jervis 1976; Van Evera 1999; Waltz 1979). Early game-theoretic work showed how private information can lead to war between rational actors (e.g., Brito and Intriligator 1985; Bueno de Mesquita and Lalman 1992; Morrow 1989). Fearon (1995) analyzed a crisis bargaining model in which the disputed good is fully divisible and established that private information about military capabilities or resolve, along with incentives to misrepresent it, is a rationalist explanation for costly, inefficient war that both sides would like to avoid. Private information, along with commitment problems (Fearon 1995; Powell 2006), is now one of the two well-established rationalist explanations for war.

To demonstrate the private information explanation, Fearon analyzed an ultimatum model in which country A makes a take-it-or-leave-it proposal to country B, with rejection of the proposal resulting in war. Under complete information, country A makes an acceptable proposal and war is avoided. However, when country B's cost of war is private information and comes from a continuum of types, then under very broad conditions A's optimal proposal entails a risk of rejection and hence war (the "risk-return tradeoff"; Powell 1999). As Powell (2004, 348; 2006, 170, 174) puts it, under incomplete information the optimal proposal "typically"/"usually"/"generally" carries a risk of rejection and thus war.

In this research note, I examine these conditions in greater detail. Fearon assumes that the lower bound of the set of B's possible costs of war is 0, i.e., that the probability distribution over B's type assigns positive probability density to B having a 0 (as well as any arbitrarily small) cost of war. I relax this assumption, which is arguably against the spirit of the rationalist approach's assumption that war is costly, and find that the conditions under which A makes a risky (interior) proposal are less broad than in Fearon's analysis. And as this lower bound increases, the risk-free (corner) proposal becomes more attractive, and hence is more likely to be made. This leads to a consequent decrease in the risk that war breaks out.

I then derive an analogous result for when the uncertainty is about the military balance (probability of winning a war) rather than B's cost of war. Here too it turns out that A makes a risk-free proposal under broader conditions than previously appreciated.

Thus, the results call into question the common idea that A's optimal proposal under uncertainty "typically"/"usually"/"generally" entails a risk of war. The conditions under which A makes a risk-free proposal are broader than previously appreciated. This certainly does not undermine private information as a rationalist explanation for war, but does suggest that previous work has somewhat overstated the risk of war under uncertainty. I show that the optimal solution to the risk-return tradeoff often involves no risk. The analysis also provides some empirically testable predictions about when a risk-free proposal is more likely to be made, and hence when crisis bargaining is less likely to result in war.

In a related vein, Leventoğlu and Tarar (2008) analyze an infinite-horizon offer-counteroffer crisis bargaining model in which with sufficient patience (i.e., a high discount factor), private information (about the cost of war) simply leads to delay in reaching a negotiated settlement rather than a risk of war. That is, in infinite-horizon crisis bargaining, the risk-return tradeoff need not even exist. In this note, I show that even within the ultimatum model, in which a delayed agreement is not possible and the risk-return tradeoff therefore *must* exist, its optimal solution often involves no risk. Together, these results suggest that uncertainty is not nearly as conducive to war breaking out as previous work has implied,[1] and perhaps provide a theoretical justification for the field's recent focus on commitment problems rather than informational problems as a cause of war.[2]

2 Uncertainty About the Cost of War

I use the exact same model as Fearon (1995). Two countries, A and B, have to divide a divisible good of value 1 between them, or go to war to decide who gets all of it. If war occurs, A wins with probability $p \in (0,1)$ and B wins with probability $1-p$. The costs of war are $c_A, c_B > 0$. Then, $EU_A(war) = (p)(1) + (1-p)(0) - c_A = p - c_A$ and $EU_B(war) = (p)(0) + (1-p)(1) - c_B = 1 - p - c_B$.

The bargaining protocol is a simple ultimatum proposal: A makes a take-it-or-leave-it proposal $(x, 1-x)$, where $x \in [0,1]$ is A's proposed share. B's two choices are to (i) accept this proposal, resulting in each side's payoff being its proposed share (i.e., assuming risk-neutrality), or (ii) go to war, in which case each side gets its expected payoff for war. The following is the standard result under complete information.

Proposition 1 *This game has a unique subgame-perfect equilibrium (SPE) in which B accepts any proposal such that* $x \leq \min\{p + c_B, 1\}$*, and A proposes* $x = \min\{p + c_B, 1\}$.

[1]Fey and Ramsay (2011) present a very general analysis of the link between uncertainty and war, but focus on different questions than the present note.

[2]Powell (2006) provides an empirical justification, noting that long wars present a problem for informational explanations of war. A number of recent works combine informational and commitment problems (e.g., Debs and Monteiro 2014; Wolford, Reiter, and Carrubba 2011).

2.1 Two Types

Now suppose that A is uncertain about c_B, and there are two possible types of B. One type has cost c_{B_l} and the other has cost c_{B_h}, with $0 < c_{B_l} < c_{B_h}$. Also assume that $p + c_{B_l} < 1$. Nature chooses B's cost to be c_{B_l} (the "strong" type) with probability $l \in (0, 1)$, and c_{B_h} with probability $1 - l$. A does not observe this move but knows the probabilities. Then the following is the standard result.

Proposition 2 *Define the following threshold:* $l_{crit} = \frac{\min\{p+c_{B_h},1\}-(p+c_{B_l})}{\min\{p+c_{B_h},1\}-(p-c_A)} \in (0, 1)$. *Then the following is the unique perfect Bayesian equilibrium (PBE) of the game:*

(i) Type c_{B_l} *accepts any proposal* $x \leq p + c_{B_l}$*, and type* c_{B_h} *accepts any proposal* $x \leq \min\{p + c_{B_h}, 1\}$.

(ii) If $l > l_{crit}$*, then* A *makes the safe, limited proposal of* $x = p + c_{B_l}$*, which both types of* B *accept.*

(iii) If $l < l_{crit}$*, then* A *makes the risky, large proposal of* $x = \min\{p + c_{B_h}, 1\}$*, which only type* c_{B_h} *accepts.*

This result shows that with two types, whether A chooses to make a safe or risky proposal depends on the prior probability l that B is the strong (low-cost) type. When this probability exceeds a certain threshold, A makes a safe proposal; otherwise, she makes a risky proposal. With two types, there exist reasonable conditions under which the optimal solution to the risk-return tradeoff involves no risk.

It may be argued that a continuum of types is a more general setting than two (or any finite number of) types (Powell 2004, 345, 348; Wittman 2009, 590), and perhaps the appropriate setting in which to examine the effects of incomplete information on crisis bargaining.

However, at least two responses can be made to justify using two types (or a finite number of types), beyond just simplicity considerations.

First, while it is true that a continuum of types is a *conceptual* generalization over two types in that we are allowing for more types, it is not really a *mathematical* generalization: Proposition 2 cannot be derived from Proposition 3 below (for a continuum of types) just by plugging in certain values, and has to be derived on its own. Discrete random variables are not special cases of continuous random variables.

Second, and more importantly, to the extent that we want game-theoretic models to capture the decision-making of actors with limited cognitive abilities, it may be that the two-type case (or finite-type case more generally) is a more descriptively accurate model. That is, it is plausible that actors in the real world deal with uncertainty by conceptualizing a few possible types of the opponent that they might be facing. It is not clear that actors deal with uncertainty by conceptualizing continuums of possibilities, and it seems that conceptualizing continuums may impose computational burdens that individuals may cognitively wish to avoid.[3] In short, it is plausible that under uncertainty, A would simplify things by conceptualizing two (or a few) possible opponents that she might be facing, one who needs a big share of the disputed good to avoid war because he does not perceive war to be very costly, and another who can be satisfied with a small share.

This is not to argue that a finite-type analysis is preferable to using a continuum of types, but just to point out that reasonable justifications for the finite-type case can be given, beyond just simplicity considerations.

[3]Gintis (2009, xv-xvi) makes a slightly related argument, about finiteness versus infiniteness in game-theoretic models.

2.2 Continuum of Types

The following proposition presents the result when c_B comes from a continuum, and is distributed either on the interval $[c_{B_l}, c_{B_h}]$ or on the interval $[c_{B_l}, \infty)$, where $0 \leq c_{B_l}$ and $p + c_{B_l} < 1$. This is a generalization of Fearon's (1995, 411) analysis because he restricts attention to the case where c_B is distributed on $[0, \infty)$.[4] The random variable c_B has a cumulative distribution function $F(\cdot)$ that is differentiable on the interior of the above interval, with probability density function $f(\cdot) = F'(\cdot)$ that is positive everywhere on the above interval. As Fearon does, I assume that the hazard rate $\frac{f(\cdot)}{1-F(\cdot)}$ is non-decreasing, which is satisfied for many commonly-used distributions including the uniform distribution, the most commonly-used one in works that assume a specific continuous distribution (Fudenberg and Tirole 1991, 267).

Proposition 3 *This game has a unique perfect-Bayesian equilibrium (PBE), in which any type c_B of B accepts any proposal such that $x \leq \min\{p + c_B, 1\}$, and A's proposal is as follows:*

(i) If $f(c_{B_l}) \geq \frac{1}{c_A + c_{B_l}}$, then A makes the risk-free, limited proposal of $x = p + c_{B_l}$.

(ii) If c_B is distributed on $[c_{B_l}, c_{B_h}]$ and $f(c_{B_l}) < \frac{1}{c_A + c_{B_l}}$, then A makes a risky, large proposal $x^* > p + c_{B_l}$, where (a) x^* is the unique solution to $\frac{f(x-p)}{1-F(x-p)} = \frac{1}{x-(p-c_A)}$ if $[p + c_{B_h} \leq 1]$ or $[p + c_{B_h} > 1$ and $\frac{f(1-p)}{1-F(1-p)} > \frac{1}{1-(p-c_A)}]$, and (b) $x^* = 1$ if $[p + c_{B_h} > 1$ and $\frac{f(1-p)}{1-F(1-p)} \leq \frac{1}{1-(p-c_A)}]$.

(iii) If c_B is distributed on $[c_{B_l}, \infty)$ and $f(c_{B_l}) < \frac{1}{c_A + c_{B_l}}$, then A makes a risky, large proposal $x^* > p + c_{B_l}$, where (a) x^* is the unique solution to $\frac{f(x-p)}{1-F(x-p)} = \frac{1}{x-(p-c_A)}$ if $\frac{f(1-p)}{1-F(1-p)} >$

[4] Fey, Meirowitz, and Ramsay (2013, 42) analyze the case where c_B is distributed on $[0, 1-p]$, i.e., $c_{B_l} = 0$ and $c_{B_h} = 1 - p$. This is also a special case of the current analysis.

$\frac{1}{1-(p-c_A)}$, *and (b)* $x^* = 1$ *if* $\frac{f(1-p)}{1-F(1-p)} \leq \frac{1}{1-(p-c_A)}$.

The result is similar to Fearon's (1995), except that he derives that when c_B is distributed on the interval $[0, \infty)$, then the condition under which A makes the risk-free proposal is $f(0) > \frac{1}{c_A}$, whereas I derive the more general condition $f(c_{B_l}) > \frac{1}{c_A + c_{B_l}}$ (which simplifies to Fearon's condition when $c_{B_l} = 0$, which he effectively assumes).[5]

This is a non-trivial difference, because Fearon (1995, 394, 411) concludes from his condition that "Under very broad conditions, if A cannot learn B's private information and if A's own costs are not too large, then state A's optimal grab produces a positive chance of war... the *ex ante* risk of war is always positive for small enough c_A greater than zero." This follows straightforwardly from the fact that the right-hand-side of $f(0) > \frac{1}{c_A}$ approaches infinity as $c_A > 0$ approaches 0, and hence for any fixed $f(0)$ the condition will not hold for c_A small enough. This has been interpreted as implying that with a continuum of types (as opposed to two types), the optimal proposal under uncertainty "typically"/"usually"/"generally" entails a positive probability of war. Or to put it another way, the optimal solution to the risk-return tradeoff usually involves risk.[6]

However, when $c_{B_l} > 0$, which is more consistent with the rationalist approach's foundational assumption that war is costly than is $c_{B_l} = 0$, then this is not the case. From the

[5] Fey, Meirowitz, and Ramsay (2013, 42) also assume that $c_{B_l} = 0$ and also derive the condition $f(0) > \frac{1}{c_A}$.

[6] For example, Powell (2004, 348) writes: "The optimal offer that resolves this trade off typically entails some risk that the dissatisfied state will reject the offer and go to war." Similarly, Powell (2006, 170, 174) writes: "The optimal solution to this trade-off usually entails making an offer that carries some risk of rejection and war... The optimal offer that resolves this trade-off generally entails some risk of rejection, and this is the way that asymmetric information can lead to war."

condition $f(c_{B_l}) > \frac{1}{c_A + c_{B_l}}$, we see that even as $c_A > 0$ becomes arbitrarily small, there is a discrete threshold $\frac{1}{c_{B_l}}$ (> 0) such that if the probability density of facing the lowest-cost ("strongest") type exceeds it, then A makes a risk-free proposal. And as c_{B_l} increases, this threshold decreases, meaning that the risk-free proposal is more likely to be made. The intuition here is that the risk-free proposal of $x = p + c_{B_l}$ is increasing in c_{B_l}, and hence becomes more attractive as c_{B_l} increases.

This indicates that it is not correct to say that the optimal proposal under uncertainty with a continuum of types "typically"/"usually"/"generally" carries a positive probability of war: that conclusion depends on the assumption that the lower bound of the set of B's possible costs of war is 0. When this assumption is relaxed, then the condition under which the optimal proposal is risky is not trivially satisfied. And as either c_A or c_{B_l} increases, the risk-free proposal is more likely to be made. The prediction that crisis bargaining (if no commitment problems are present) has a 0 probability of war when either A's cost of war, or A's perception of the lower bound of the set of B's possible costs of war, or both, are sufficiently large, is potentially amenable to empirical testing. (Admittedly, when incomplete-information crisis bargaining does not result in war, it will be difficult to establish whether this is because a risk-free proposal was made, or because a risky proposal was made but B's cost of war was high enough that it decided to accept the proposal.)

It is also worthwhile to briefly consider the intuition behind the risk-free condition $f(c_{B_l}) > \frac{1}{c_A + c_{B_l}}$. This condition states that A makes the risk-free proposal $x = p + c_{B_l}$ when $f(c_{B_l})$ exceeds a certain threshold. Why? If $f(c_{B_l})$ is sufficiently high, then by making an even slightly risky (interior) proposal, the probability of rejection is too high for the small gain in territory, if the proposal is accepted, to be worthwhile. On the other hand, if

$f(c_{B_l})$ is sufficiently low, then making a sufficiently small interior (risky) proposal is always worthwhile because the probability of rejection will be sufficiently low, and making an even larger interior proposal may be worthwhile as well, depending on the shape of $f(\cdot)$.

To see the condition in practice, suppose that c_B is uniformly distributed on $[c_{B_l}, c_{B_h}]$. The uniform distribution, because of its simplicity, is the most commonly used one in works that assume a specific continuous distribution. The uniform density function is $f(c_B) = \frac{1}{c_{B_h} - c_{B_l}}$ for all $c_B \in [c_{B_l}, c_{B_h}]$, and plugging this into the condition $f(c_{B_l}) > \frac{1}{c_A + c_{B_l}}$, we get $c_{B_h} - c_{B_l} < c_A + c_{B_l}$: if the width of the uniform distribution, $c_{B_h} - c_{B_l}$, is less than a certain threshold, then A makes a risk-free proposal. The intuition is that the uniform density function is constant over $[c_{B_l}, c_{B_h}]$, and hence narrowing the interval of types pushes the density function up (to maintain the area under the density function at 1), eventually ensuring that the risk-free condition $f(c_{B_l}) > \frac{1}{c_A + c_{B_l}}$ will hold. With a uniform distribution of types, the interval of types has to be sufficiently wide for A to make a risky proposal. And as either c_A or c_{B_l} increases, the wider it has to be for the proposal to carry risk.

3 Uncertainty About the Probability of Winning

3.1 Two Types

Now suppose that A knows c_B but is uncertain about p. Suppose that there are two possible values of p, p_l and p_h, with $0 < p_l < p_h < 1$, and $p_l + c_B < 1$. Nature chooses p to be p_l with probability $l \in (0, 1)$ and p_h with probability $1 - l$. B observes this move and hence knows the value of p (thus, we will call p_l the strong "type" of B, and p_h the weak "type"), whereas A does not observe the move but knows the probabilities (e.g., A's military capabilities are known to both sides but only B knows B's). The following is the result.

Proposition 4 *Define the following threshold:* $l_{crit} = \frac{\min\{p_h+c_B,1\}-(p_l+c_B)}{\min\{p_h+c_B,1\}-(p_l-c_A)} \in (0,1)$. *Then the following is the unique perfect Bayesian equilibrium (PBE) of the game:*

(i) "Type" p_l *of* B *accepts any proposal* $x \leq p_l + c_B$*, and "type"* p_h *accepts any proposal* $x \leq \min\{p_h + c_B, 1\}$.

(ii) If $l > l_{crit}$*, then* A *makes the safe, limited proposal of* $x = p_l + c_B$*, which both "types" of* B *accept.*

(iii) If $l < l_{crit}$*, then* A *makes the risky, large proposal of* $x = \min\{p_h + c_B, 1\}$*, which only "type"* p_h *accepts.*

This is very similar to Proposition 2: with two types, whether A makes a safe or risky proposal depends on whether the prior probability that B is the strong type exceeds a certain threshold. With two types, there exist reasonable circumstances in which the optimal solution to the risk-return tradeoff involves no risk.

3.2 Continuum of Types

Now suppose that p is distributed on the interval $[p_l, p_h]$, with $0 < p_l < p_h < 1$ and $p_l + c_B < 1$. The random variable p has a cumulative distribution function $F(\cdot)$ that is differentiable on the interior of the above interval, with probability density function $f(\cdot) = F'(\cdot)$ that is positive everywhere on the above interval. Assume that the hazard rate $\frac{f(\cdot)}{1-F(\cdot)}$ is non-decreasing.[7]

Proposition 5 *This game has a unique perfect-Bayesian equilibrium (PBE), in which any*

[7] Reed (2003) also analyzes an ultimatum model with uncertainty about the probability of winning and a continuum of types, but does not present a result as below on when A will make a risky versus a risk-free proposal. Instead, he assumes that the parameters are such that the equilibrium proposal is risky (interior) and examines the effect of increasing the variance of the distribution of types on the risky proposal and the likelihood of war.

"type" p of B accepts any proposal such that $x \leq \min\{p + c_B, 1\}$, and A's proposal is as follows:

(i) If $f(p_l) > \frac{1}{c_A+c_B}$, then A makes the risk-free, limited proposal of $x = p_l + c_B$.

(ii) If $f(p_l) < \frac{1}{c_A+c_B}$, then A makes a risky, large proposal $x^* > p_l + c_B$, where (a) x^* is a (possibly non-unique; unique if we impose the marginally stronger requirement that the hazard rate is strictly increasing) solution to $\frac{f(x-c_B)}{1-F(x-c_B)} = \frac{1}{c_A+c_B}$ if $[p_h+c_B \leq 1]$ or $[p_h+c_B > 1$ and $\frac{f(1-c_B)}{1-F(1-c_B)} > \frac{1}{c_A+c_B}]$, and (b) $x^* = 1$ if $[p_h + c_B > 1$ and $\frac{f(1-c_B)}{1-F(1-c_B)} < \frac{1}{c_A+c_B}]$.

With a continuum of types, the condition under which A's optimal proposal is risk-free is $f(p_l) > \frac{1}{c_A+c_B}$, i.e., the probability density of facing the strongest possible type of B exceeds a certain threshold. As before, the intuition is that when this probability density is sufficiently high, then making an even slightly risky (interior) proposal carries too high a probability of rejection for the small gain in territory, if the proposal is accepted, to be worthwhile.

Note from the risk-free condition $f(p_l) > \frac{1}{c_A+c_B}$ that even as $c_A > 0$ becomes arbitrarily small, if $f(p_l) > \frac{1}{c_B}$, then A makes a risk-free proposal. Hence, just like with uncertainty about c_B when we allow $c_{B_l} > 0$, with uncertainty about p we cannot conclude along Fearon's (1995) lines that for c_A small enough A's optimal proposal is definitely risky. The condition under which A's optimal proposal is risky is not trivially satisfied. And as either c_A or c_B increases, it is more likely that the risk-free condition will be satisfied, and hence that the risk-free proposal will be made. For c_A, the logic behind this is straightforward, and for c_B, the intuition is that the risk-free proposal $x = p_l + c_B$ is strictly increasing in c_B and hence becomes more attractive as c_B increases.

To see the condition in practice, suppose that p is uniformly distributed on $[p_l, p_h]$. Substituting the uniform density $f(p) = \frac{1}{p_h - p_l}$ into the safe-proposal condition $f(p_l) > \frac{1}{c_A + c_B}$ gives $c_A + c_B > p_h - p_l$. That is, if the width of the interval of types, $p_h - p_l$, is less than a certain threshold (in this case, the total costs of war), then A makes a risk-free proposal. As earlier, the intuition is that if the uniform distribution is sufficiently narrow, then the constant density function will be high enough that the risk-free condition is definitely satisfied. Whether the uncertainty is about the cost of war or the probability of winning, narrow non-informative priors (i.e., narrow uniform distributions) are conducive to a risk-free proposal being made. This is another prediction that is potentially amenable to empirical testing.

4 Conclusion

Informational problems have a rich heritage in the conflict literature as a theorized cause of war, and many game-theoretic models of crisis bargaining incorporate private information. Some of these models assume two or a larger finite number of types, others assume a continuous uniform distribution of types, while some assume any continuous distribution of types with a non-decreasing hazard rate. The standard view is that more types are better and comprise a more general analysis (Powell 2004, 345, 348; Wittman 2009, 590), and that with a continuum of types, the optimal proposal "typically"/"usually"/"generally" entails a risk of rejection and hence war (Powell 2004, 348; Powell 2006, 170, 174).

In this research note, I have taken a closer look at these issues. I have argued that the finite-type case has reasonable justifications beyond just simplicity. I have shown that when Fearon's (1995) assumption that the lower bound of the set of B's possible costs of war is 0 is relaxed, a relaxation justified by the rationalist approach's foundational assumption that war is costly, then the optimal proposal is risk-free under broader conditions than in his analysis. When the uncertainty is about the military balance instead, then here too the condition under which the optimal proposal is risky is not trivially satisfied. Whatever the source of uncertainty, the optimal solution to the risk-return tradeoff often involves no risk.

Incomplete-information models of crisis bargaining focus on the interior (risky) solution, for good reason: this is where the "action" is, in that there is a risk of war. In this note, I have sought to rehabilitate its poorer cousin, the corner (risk-free) solution, by showing that the optimal solution to the risk-return tradeoff is the corner solution under broader conditions than previous work has indicated.

While these results certainly do not undermine private information as a rationalist explanation for war, they do suggest that uncertainty is not as conducive to war breaking out as previous work has suggested. This result that the optimal solution to the risk-return tradeoff often involves no risk, especially when combined with Leventoğlu and Tarar's (2008) result that in infinite-horizon crisis bargaining the risk-return tradeoff need not even exist, perhaps provides a theoretical justification for the field's recent focus on commitment problems rather than informational problems as a cause of war.

5 Appendix

Propositions 1, 2, and 4 are straightforward, standard results, and hence the proofs are omitted.

5.1 Proof of Proposition 3

Any proposal $x \in [0, p + c_{B_l}]$ is accepted for sure, resulting in A's payoff being x. Within this range, A's uniquely optimal proposal is thus $x = p + c_{B_l}$. Thus, if c_B is distributed on $[c_{B_l}, \infty)$, or on $[c_{B_l}, c_{B_h}]$ but $p + c_{B_h} \geq 1$, then the optimal value of x occurs in the interval $[p + c_{B_l}, 1]$, i.e., we just have to maximize over this range (and can ignore smaller values of x). On the other hand, if c_B is distributed on $[c_{B_l}, c_{B_h}]$ and $p + c_{B_h} < 1$, then note that A also strictly prefers $x = p + c_{B_l}$ to any $x > p + c_{B_h}$, for the latter is rejected for sure and leads to the strictly smaller payoff of $p - c_A$. Hence, the optimal value of x occurs in the interval $[p + c_{B_l}, \min\{p + c_{B_h}, 1\}]$, i.e., we just need to maximize over this range. Denote this interval by M.

For any $x \in M$, B accepts the proposal if $x \leq p + c_B$, or $c_B \geq x - p$, which occurs with probability $1 - F(x - p)$, where $F(\cdot)$ is the cumulative distribution function for c_B. If the proposal is accepted, A's payoff is x. B rejects the proposal if $x > p + c_B$ or $c_B < x - p$, which occurs with probability $F(x - p)$. If the proposal is rejected, A's payoff is $p - c_A$. Thus, A's expected utility for any $x \in M$ is $EU_A(x) = [1 - F(x - p)](x) + F(x - p)(p - c_A)$. Differentiating, $EU'_A(x) = 1 - F(x - p) - f(x - p)[x - (p - c_A)]$.

$EU'_A(x) \leq 0$ can be re-written as $\frac{f(x-p)}{1-F(x-p)} \geq \frac{1}{x-(p-c_A)}$. LHS is non-decreasing in x and RHS is strictly decreasing in x. Thus, if this inequality holds at $x = p + c_{B_l}$, then it strictly holds for all greater x, meaning that $EU_A(x)$ is strictly decreasing in x on the interval $x \in M$

and hence is uniquely maximized at $x = p + c_{B_l}$. Substituting $x = p + c_{B_l}$ into the inequality, we get $f(c_{B_l}) \geq \frac{1}{c_A + c_{B_l}}$. This gives case (i) in the proposition.

Now suppose that c_B is distributed on $[c_{B_l}, c_{B_h}]$ and $f(c_{B_l}) < \frac{1}{c_A + c_{B_l}}$, meaning that $EU_A(x)$ is strictly increasing at $x = p + c_{B_l}$. First suppose that $p + c_{B_h} \leq 1$. Note that $EU'_A(x = p + c_{B_h}) = -(c_A + c_{B_h}) f(c_{B_h}) < 0$, i.e., $EU_A(x)$ is strictly decreasing at $x = p + c_{B_h}$, meaning that $EU'_A(x) = 0$ for at least one value of $x \in (p + c_{B_l}, p + c_{B_h})$. $EU'_A(x) = 0$ is equivalent to $\frac{f(x-p)}{1-F(x-p)} = \frac{1}{x-(p-c_A)}$. Since LHS is non-decreasing in x and RHS is strictly decreasing in x, $EU'_A(x) = 0$ for only a single value of $x \in (p + c_{B_l}, p + c_{B_h})$, which establishes that the solution is unique (and is a maximizer rather than a minimizer). Now suppose that $p + c_{B_h} > 1$. $EU'_A(x = 1) < 0$ is equivalent to $\frac{f(1-p)}{1-F(1-p)} > \frac{1}{1-(p-c_A)}$, and hence if the latter holds then $EU_A(x)$ is strictly decreasing at $x = 1$, meaning that $EU_A(x)$ is again uniquely maximized by setting $EU'_A(x) = 0$. On the other hand, if $\frac{f(1-p)}{1-F(1-p)} \leq \frac{1}{1-(p-c_A)}$ then $EU'_A(x = 1) \geq 0$, meaning that $EU_A(x)$ is uniquely maximized at $x = 1$. This completes case (ii), and the exact same arguments following "Now suppose that $p + c_{B_h} > 1$" also cover case (iii). Q.E.D.

5.2 Proof of Proposition 5

Any proposal $x \in [0, p_l + c_B]$ is accepted for sure, resulting in A's payoff being x. Within this range, A's uniquely optimal proposal is thus $x = p_l + c_B$. If $p_h + c_B < 1$, then also note that A's payoff for any $x \geq p_h + c_B$ is constant, since any such proposal is rejected with probability 1, resulting in A's expected payoff being $\int_{p_l}^{p_h} (p - c_A) f(p) dp$. Thus, we just need to maximize over the interval $x \in [p_l + c_B, \min\{p_h + c_B, 1\}]$, i.e., the optimal value of

x occurs in this interval.[8] Denote this interval by M.

For any $x \in M$, B accepts the proposal if $x \leq p + c_B$, or $p \geq x - c_B$, which occurs with probability $1 - F(x - c_B)$, where $F(\cdot)$ is the cumulative distribution function for p. If the proposal is accepted, A's payoff is x. B rejects the proposal if $x > p + c_B$ or $p < x - c_B$, which occurs with probability $F(x - c_B)$. If the proposal is rejected, A's payoff is $p - c_A$. Thus, A's expected utility for any $x \in M$ is $EU_A(x) = \int_{p_l}^{x-c_B}(p - c_A)f(p)dp + \int_{x-c_B}^{p_h}(x)f(p)dp = x-(x+c_A)F(x-c_B)+\int_{p_l}^{x-c_B} pf(p)dp$. Differentiating (and using the Fundamental Theorem of Calculus for the derivative of a definite intergral), $EU'_A(x) = 1-F(x-c_B)-f(x-c_B)[c_A+c_B]$.

$EU'_A(x) < 0$ can be re-written as $\frac{f(x-c_B)}{1-F(x-c_B)} > \frac{1}{c_A+c_B}$. LHS is non-decreasing in x and RHS is constant. Thus, if this inequality holds at $x = p_l + c_B$, then it holds for all greater x, meaning that $EU_A(x)$ is strictly decreasing in x on the interval $x \in M$ and hence is uniquely maximized at $x = p_l + c_B$. Substituting $x = p_l + c_B$ into the inequality, we get $f(p_l) > \frac{1}{c_A+c_B}$. This gives case (i) in the proposition.

Now suppose that $f(p_l) < \frac{1}{c_A+c_B}$, meaning that $EU_A(x)$ is strictly increasing at $x = p_l + c_B$. First suppose that $p_h + c_B \leq 1$. Note that $EU'_A(x = p_h + c_B) = -(c_A + c_B)f(p_h) < 0$, i.e., $EU_A(x)$ is strictly decreasing at $x = p_h + c_B$, meaning that $EU'_A(x) = 0$ for at least one value of $x \in (p_l + c_B, p_h + c_B)$. $EU'_A(x) = 0$ is equivalent to $\frac{f(x-c_B)}{1-F(x-c_B)} = \frac{1}{c_A+c_B}$. Since LHS is non-decreasing in x and RHS is constant, it is possible for there to be an interval

[8] Note that, in contrast to when the uncertainty is about c_B, when the uncertainty is about p then it is not necessarily the case that A prefers to make the best risk-free proposal ($x = p_l + c_B$) to a proposal that is rejected with probability 1 (any $x \geq p_h + c_B$). For example, if p is uniformly distributed on $[p_l, p_h]$, then $EU_A(x \geq p_h + c_B) = \frac{p_l+p_h}{2} - c_A$, which for sufficiently small c_A and c_B strictly exceeds $p_l + c_B$. Nevertheless, as shown below, in equilibrium A never proposes any $x \geq p_h + c_B$.

of values of x (with the interval strictly to the right of $p_l + c_B$ and strictly to the left of $p_h + c_B$) that maximize $EU_A(x)$, and to ensure that there is a *unique* maximizer, we have to impose the marginally stronger restriction that the hazard rate is strictly increasing (just at the smallest value of x that satisfies $\frac{f(x-c_B)}{1-F(x-c_B)} = \frac{1}{c_A+c_B}$) rather than just non-decreasing. Now suppose that $p_h + c_B > 1$. $EU'_A(x = 1) < 0$ is equivalent to $\frac{f(1-c_B)}{1-F(1-c_B)} > \frac{1}{c_A+c_B}$, and hence if the latter holds then $EU_A(x)$ is strictly decreasing at $x = 1$, meaning that $EU_A(x)$ is maximized (again at possibly an interval of values of x) by setting $EU'_A(x) = 0$. On the other hand, if $\frac{f(1-c_B)}{1-F(1-c_B)} < \frac{1}{c_A+c_B}$ then $EU'_A(x = 1) > 0$, meaning that $EU_A(x)$ is uniquely maximized at $x = 1$. This completes case (ii). Q.E.D.

References

[1] Blainey, Geoffrey. 1988. *The Causes of War*. 3rd ed. New York: Free Press.

[2] Brito, Dagobert L., and Michael D. Intriligator. 1985. "Conflict, War, and Redistribution." *American Political Science Review* 79(4):943-57.

[3] Bueno de Mesquita, Bruce, and David Lalman. 1992. *War and Reason*. New Haven: Yale University Press.

[4] Debs, Alexandre, and Nuno P. Monteiro. 2014. "Known Unknowns: Power Shifts, Uncertainty, and War." *International Organization* 68(Winter):1-31.

[5] Fearon, James D. 1995. "Rationalist Explanations for War." *International Organization* 49(3):379-414.

[6] Fey, Mark, Adam Meirowitz, and Kristopher W. Ramsay. 2013. "Credibility and Commitment in Crisis Bargaining." *Political Science Research and Methods* 1(1):27-52.

[7] Fey, Mark, and Kristopher W. Ramsay. 2011. "Uncertainty and Incentives in Crisis Bargaining: Game-Free Analysis of International Conflict." *American Journal of Political Science* 55(1):149-169.

[8] Fudenberg, Drew, and Jean Tirole. 1991. *Game Theory*. Cambridge: MIT Press.

[9] Gintis, Herbert. 2009. *Game Theory Evolving*. 2nd ed. Princeton University Press.

[10] Jervis, Robert. 1976. *Perception and Misperception in International Politics*. Princeton: Princeton University Press.

[11] Leventoğlu, Bahar, and Ahmer Tarar. 2008. "Does Private Information Lead to Delay or War in Crisis Bargaining?" *International Studies Quarterly* 52:533-53.

[12] Morrow, James D. 1989. "Capabilities, Uncertainty, and Resolve: A Limited Information Model of Crisis Bargaining." *American Journal of Political Science* 33(November):941-972.

[13] Powell, Robert. 1999. *In the Shadow of Power: States and Strategies in International Politics*. Princeton: Princeton University Press.

[14] Powell, Robert. 2004. "Bargaining and Learning While Fighting." *American Journal of Political Science* 48(2):344-361.

[15] Powell, Robert. 2006. "War as a Commitment Problem." *International Organization* 60:169-203.

[16] Reed, William. 2003. "Information, Power, and War." *American Political Science Review* 97(4):633-641.

[17] Van Evera, Stephen. 1999. *Causes of War: Power and the Roots of Conflict*. Ithaca, NY: Cornell University Press.

[18] Waltz, Kenneth. 1979. *Theory of International Politics*. New York: McGraw-Hill.

[19] Wittman, Donald. 2009. "Bargaining in the Shadow of War: When is a Peaceful Resolution Most Likely?" *American Journal of Political Science* 53(3):588-602.

[20] Wolford, Scott, Dan Reiter, and Clifford J. Carrubba. 2011. "Information, Commitment, and War." *Journal of Conflict Resolution* 55(4):556-79.