Breaking-Up Should Not be Hard to Do! Designing Contracts to Avoid Wars of Attrition*

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Abstract

This paper introduces a general model of dissolving a partnership where both the decision to dissolve the partnership and the roles the partners play in the exit mechanism are endogenously determined. We show that certain pairings of triggering rules and exit mechanisms can lead to a war of attrition. This inefficiently prolongs the dissolution process. Our framework suggests simple guidelines for avoiding such outcomes. We test our predictions with an experiment. Treatments explore break-up incentives using several different exit mechanisms and triggering rule pairings. The experimental results provide strong support for the robustness of the underlying theoretical predictions.

Keywords: Dissolving a Partnership, Exit Mechanisms, Shoot-Out Mechanism, War of Attrition

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1 Introduction

In the dissolving a partnership literature, we typically examine the fairness or efficiency properties of exit mechanisms taking some other features of the process as given.[1] For example, the literature almost always takes the roles played by the partners in the mechanism as a parameter — i.e., which partner is player 1 in the mechanism or which partner is player 2. This is natural. However, in practice, these details are determined in the partnership contract.

A contract specifies the circumstances under which an exit mechanism is "triggered" as well as details the process in which the parameters of the exit mechanism are to be determined. Thus, exit mechanisms are generally part of a larger "break-up" game played by the partners and this game is induced by the language of the initial partnership contract. A potential consequence of ignoring this larger game is that we may prematurely endorse an exit mechanism for the properties it exhibits in isolation only to discover later the perverse incentives it created in the larger game. The goal of this paper is to study this larger game both theoretically and experimentally.

We begin the paper by defining a simple game in order to study the incentives provided by different kinds of partnership contracts. We call this game the "break-up game." Specifically, a break-up game is induced by the composition of a *triggering rule* and an *exit mechanism*. An exit mechanism is the set of rules for assigning the company to one player and deciding how the other player is compensated. The triggering rule, in contrast, defines the conditions under which the partnership will be dissolved. It takes the break announcements of the partners, sets a date (if any) for dissolving the partnership, and determines any parameters needed to implement the exit mechanism. At a minimum, for every type of exit mechanism, the triggering rule must set a date of dissolution and assign partners to "player roles" in the exit mechanism.

To illustrate this game consider the first of two examples.

Example 1 *Two partners belong to an ineffective partnership and must decide when they would like to dissolve. Waiting to dissolve is costly, but at any time either of the partners can decide to trigger the exit mechanism. After the*

[1] See, for example, Cramton et al. (1987), Güth and Van Damme (1986), McAfee (1992), or Van Essen and Wooders (2016). In each of these papers, the focus on the game induced by an exit mechanism and the properties of equilibrium play.

mechanism is triggered, the two parties conduct their due diligence and privately learn their value for the company. The partnership is then dissolved as follows: the partner who first triggers the exit mechanism is required to name a price per share for the whole company and the other partner is compelled to either purchase or sell his own shares at the named price.

The combination of the exit mechanism and the triggering rule induces a break-up game. The partners lose money until one of them indicates the desire to dissolve (i.e., makes a break announcement), this announcement determines the partners' roles in the exit mechanism and the date of dissolution. Finally, the partners split the company according to the rules of the exit mechanism. In Example 1, the exit mechanism is a version of divide and choose called a "Texas Shoot-Out." This procedure has two roles: a "divider" and a "chooser." The triggering rule in the example uses a "break" announcement by one of the players to determine a dissolution date (in this case at the same date as the break announcement) and to assign roles to the players. Specifically, the divider role is assigned to the first person to announce the break and the chooser role is assigned to the other player.

The Texas Shoot-Out is the most commonly used exit mechanism in practice and is popular due to the well known fairness properties of divide and choose. In the paper, we show that this mechanism when viewed in the larger break-up game setting may have the unintended consequence of prolonging an ineffective partnership. In particular, we show that there is a symmetric equilibrium in the Example 1 break-up game where the partners choose to wait to dissolve the company even though waiting is costly and that it is transparently efficient for them to dissolve immediately. In other words, the partnership contract induces a *war of attrition.*

The intuition for the result is as follows. In either an independent or common values environment, if partners are risk neutral and each have an equal-share in the partnership, then it is beneficial to be assigned to the second player in the shoot-out mechanism.[2] According to the rules of the game, the partner who first triggers the exit mechanism is assigned to be the first mover. Thus, the game provides a "prize" for *not* being the first person to signal their desire to dissolve. This structure induces a war of attrition.[3]

[2]This is a result first shown by McAfee (1992) in an independent private values model. Morgan (2004) extends the result to a common values setting. In a complete information model, the reverse ranking holds.

[3]The war of attrition was originally studied by Smith (1974) and has since been used to

This is a bleak result since there is evidence that many real-life partnership contracts fit the framework of this simple break-up game. For example, Fleischer and Schneider (2012), in a legal summary of shoot-out clauses and how they are triggered in practice, state that most buy-sell clauses are triggered as follows: "Party A (the party wishing to leave or take over the company) initiates the procedure by making an offer..." There is also reason to be unhappy with the Texas Shoot-Out as an exit mechanism—although it is used in practice. McAfee (1992) established that the Texas Shoot-Out mechanism is ex-post *inefficient* when partners have private information. We do not dispute this observation. However, the main insight of our example is that the way in which the exit mechanism is triggered matters and can generate *additional inefficiency*. We find that the example points to a design problem in potentially many types of partnership contracts.

Fortunately, as bleak as this outcome appears, it has several simple solutions. The war of attrition disappears if we reverse the roles specified by the triggering rule, use a coin flip to assign roles, or use a symmetric auction. This is illustrated in the next example.

Example 2 *Two partners belong to an ineffective partnership and must decide when they would like to dissolve. Waiting to dissolve is costly, but at any time either of the partners can decide to trigger the exit mechanism—an auction. After the mechanism is triggered, the two parties conduct their due diligence and privately learn their value for the company. The partnership is then dissolved as follows: the triggering rule specifies that once a partner triggers the exit mechanism each player is assigned a bidder role in the auction, each bidder submits a bid for the company, the high bid wins, and the high bidder pays the other partner the winning bid.*

In this example, the exit mechanism is an auction called the "Winner's Bid" auction. It has two roles: a "Bidder 1" and a "Bidder 2." The triggering rule uses a "break" announcement by one of the players to determine a dissolution date (in this case at the same date as the break announcement) and to assign the players to roles. The player roles in the auction are symmetric so any assignment of partners to roles is strategically equivalent.

model a number of important economic environments. Hendricks et al. (1988) and Bulow and Klemperer (1999) provide a general War of Attrition models and survey much of this literature. Oprea et al. (2013) examine wars of attrition in a laboratory setting.

In either an independent private values model or a common values model, as we later demonstrate, the partners should dissolve immediately and proceed to the auction. Thus, the break-up game in Example 2 illustrates one method for avoiding the war of attrition outcomes—symmetry. The use of a symmetric auction as the exit mechanism paired with a trigger rule which respects an individual partner's desire to leave the partnership mitigates incentives for prolonging the dissolution process. The symmetric nature of the auction removes any benefits to artificially prolonging the dissolution process. Moreover, since the auction is efficient we eliminate the other source of inefficiency observed with the Texas Shoot-Out.

The ideas presented in the examples are highly suggestive of a general theory of break-up. In the paper we formalize the notion of a break-up game and then use this framework to characterize the pairings of exit mechanisms and triggering rules that create/avoid wars of attrition. The theoretical characterizations provide us with some sharp testable predictions. In particular, they suggest that we can create or eliminate a war of attrition by making small changes to the exit mechanism or triggering rule or both. We develop a series of experiments to test the robustness of these predictions. The three different break-up games we chose to test are similar to the games in Examples 1 and 2. Two of the games use the Texas Shoot-Out mechanism and the third uses a symmetric auction. The two Texas Shoot-Out treatments vary only in their triggering rule. One of the Texas Shoot-Out treatments uses a triggering rule similar to Example 1. The theoretical prediction in this game is that partners will wait to dissolve—i.e., there will be a war of attrition. The other Texas Shoot-out treatment uses a different triggering rule. In this game, once the partners makes a break announcement, a coin flip is used to allocate partners to their roles in the exit mechanism. The theoretical prediction now changes and we expect that the partners should want to dissolve immediately. Finally, in last game, we use an auction mechanism similar to the one in Example 2. The prediction in this game is also for the partners to dissolve immediately. The experimental data largely conforms to our theoretical predictions. War of attritions are observed or not observed in the data in precisely the break-up games that the theory predicts they would or would not be observed.

1.1 Related Literature

There is a large literature on mechanism design concerned with the dissolution of partnerships. The studies of Cramton et al. (1987), Güth and Van Damme (1986), McAfee (1992), Moldovanu (2002), Morgan (2004), Turner (2013), Van Essen (2013), and Van Essen and Wooders (2016) all discuss specific mechanisms for dissolving a partnership either efficiently or fairly.[4] Kittsteiner et al. (2012) and Brown and Velez (2016) experimentally examine the performance of some of these exit mechanisms. None of the aforementioned studies consider the process in which these mechanisms are triggered.

In contrast, de de Frutos and Kittsteiner (2008); Brooks et al. (2010); and Landeo and Spier (2013) all examine models of a break-up process where a shoot-out mechanism is used as the exit mechanism. The closest of these studies to ours is de Frutos and Kittsteiner (2008), hereafter DFK. The papers by Brooks et al. (2010) and Landeo and Spier (2013) both examine an environment where only one of the partners has private information. We therefore focus attention on DFK.

DFK consider a private values model where the roles in the Texas Shoot-Out are determined endogenously by partners bidding to be the chooser.[5] They establish the existence of an *efficient* equilibrium and show that this equilibrium is similar to the equilibrium in a war of attrition game where partner's bid time they are willing to wait before making the first buy-sell offer. In the particular equilibrium they propose, bidders follow "u-shaped" bid functions when determining how long they will wait, the first mover sets a price equal to their valuation, and the second mover always accepts the best deal. Both procedures have an efficient assignment of the company, but there is waste in war of attrition game. Hence, players would prefer the bidding mechanism to the war of attrition.

The DFK study is different from ours for two primary reasons. First, we develop a general model for endogenously determining roles in exit mechanisms. Our framework applies to settings other than ones that employ the Texas Shoot-Out mechanism. Second, we make a different assumption about

[4] Brams and Taylor (1996) survey the fair division literature which has application to the problem of dissolving a partnership.

[5] The idea of having players bid to be the divider or the chooser dates back to Crawford (1977). He shows that in a complete information environment having players bid to be the divider creates a Nash equilibrium that is efficient and envy free. See, also, Young (1995).

the timing of the revelation of private information to the partners. In DFK's war of attrition game, the players are privately informed at the beginning of the game. In our simple Break-Up game, the partner's are initially uninformed and only become informed after the exit mechanism is triggered. We use this assumption for several reasons. First, in many cases, we believe that the signal received by partners after doing their due diligence will completely outweigh the prior held by the partners. For example, it is not uncommon for partners to seek the aide of independent consultants to determine the value of a company after steps to dissolve the partnership have started. Second, we note that partners may dissolve a company for a variety of reasons and not all of these reasons are driven by differences in information about the value of the company. For example, two partners may discover shortly after forming the partnership that they not like each other or can't work together. The model we employ allows us to view partners symmetrically.[6] This symmetry lets us focus on deriving results driven by differences in trigger rules and mechanisms rather than differences in information.

2 Model

In this section, we formally define the Break-Up Game. The key idea in our model is that a Break-Up Game is defined by the combination of an *exit mechanism* and a *triggering rule.* While we define these ideas formally below, the ideas behind the concepts are simple. An exit mechanism is the set of rules which determines which partner gets the company and how the other partner is compensated. The triggering rule, in contrast, defines the conditions under which the partnership will be dissolved. It sets a date (if any) for dissolving the partnership and determines any parameters needed to actually implement the exit mechanism.

The combination of these two items will give us a break-up game. The goal of this section will be to identify the properties of mechanisms and triggering rules that when combined avoid the war of attrition illustrated in the motivating example. We now formally define an exit mechanism and a triggering rule.

[6]This "initially uninformed" assumption is also used in the entry in auctions literature. See Levin and Smith (1994) or Moreno and Wooders (2011).

2.1 Exit Mechanism

An *exit mechanism* is defined by the following four components: First, an exit mechanism must have a list of *player roles* $R = \{r_1, r_2\}$ and, for each $r \in R$, it must also specify a set of possible *messages* M_r that each player r can use in the mechanism. Second, while it is clear that the player roles will need to be filled by the different partners, the exact process in which partners are placed in different roles is not specified in the exit mechanism. As a consequence, we take the assignment of partners to player roles as parametric. In general, the items that must be decided by the partners before the mechanism can be implemented are called *initializing parameters*. We denote the set of all such parameters by Θ and restrict attention to lists of the form $\theta = (t, \theta_{r_1}, \theta_{r_2})$, where $t \in \mathbb{R}_+$ is the date in the Break-Up game where the exit mechanism will be played and θ_r denotes the name of the partner placed in role $r = r_1, r_2$ such that $\theta_{r_1} \neq \theta_{r_2}$. Third, the mechanism must specify an *allocation rule* $A : M_{r_1} \times M_{r_2} \to \Delta\{r_1, r_2\}$, where $\Delta\{r_1, r_2\}$ is the set of probability distributions over the players roles r_1 and r_2. This rule determines how ownership of the company is determined. Fourth, the mechanism must specify a *payment rule* for each player role r_j—i.e., $P_{r_j} : M_{r_1} \times M_{r_2} \to \mathbb{R}$ for $j = 1, 2$.

2.2 Triggering Rule

In the break-up game each partner i will choose a $t_i \in T$. This will be called i's break announcement—i.e., the time period in which she will call break. The *triggering rule* is a mapping which takes break announcements from all of the partners as input and assigns a probability distribution over the initializing parameters of the exit mechanism—i.e.,

$$\tau : T \times T \to \Delta\Theta,$$

where $\Delta\Theta$ is the set of probability distributions on Θ.

While this definition of a triggering rule is quite general, the class of rules that seem most natural is the one that respects an individual partner's desire to dissolve the partnership. We call any rule τ that sets the date of dissolution t equal to the $\min\{t_1, t_2\}$ a "first mover" triggering rule. We utilize this rule for the majority of our results.

We now have the machinery to define a generalized Break-Up Game.

2.3 A Break-Up Game

There are two equal share partners. Initially, neither partner knows their valuation for the whole company, but it is common knowledge that the partners' values are distributed according to distribution F on $[0, \bar{x}] \times [0, \bar{x}]$, where $f = F'$ is the density. Denote partner i's value by x_i.The partners are risk neutral.

In the Break-Up game, Partner 1 and Partner 2 simultaneously choose break announcements t_1 and t_2 in T respectively signaling the time they want to dissolve the partnership. Once the break announcements have been submitted, the initializing parameters for the exit mechanism $\theta = (t, \theta_{r_1}, \theta_{r_2})$ are then determined as a draw from distribution $\tau(t_1, t_2)$ and shown to the partners. The partnership is dissolved at time t. At this time, each partner i privately observes his type x_i, pays a cumulative loss of $C(t) = lt$ such that $l > 0$, and assumes his assigned player role in the mechanism (either r_1 or r_2). The partners then dissolve the partnership according to the rules of the exit mechanism.

2.4 Theoretical Predictions

We now provide some general observations about the pairing of a first-mover triggering rule and exit mechanisms with certain properties. These observations help establish sufficient conditions for seeing/avoiding wars of attrition in the break-up game.

Let $m^* = (m^*_{r_1}(\cdot), m^*_{r_2}(\cdot))$ be a pure strategy Bayesian Nash Equilibrium of the game induced by the exit mechanism.[7] Given an equilibrium, we may compute expected payoffs for each role. If partner i is placed in role r_1 and has type x_i, then his interim expected payoff at this equilibrium is $\pi_{r_1}(x_i)$ whereas his interim expected payoff in role r_2 is the ex-ante expected payoff for roles r_1 and r_2, v_{r_1} and v_{r_2}.

We classify equilibria of the exit mechanism by their ex-ante payoffs. An exit mechanism is *ex-ante payoff symmetric* if the player roles all yield the same ex-ante expected payoffs—i.e., $v_{r_1} = v_{r_2}$—and *ex-ante payoff asymmetric* if the player roles yield different ex-ante expected payoffs—i.e., either $v_{r_1} > v_{r_2}$ or $v_{r_1} < v_{r_2}$. If the exit mechanism is ex-ante payoff asymmetric and the player r role yields a strictly higher ex-ante expected payoff than the other player role, then we say that the role r is *ex-ante payoff dominant.*

[7]The use of a pure strategy equilibrium is simply for notational convenience.

Proposition 1 *Suppose the break-up game is defined by an ex-ante payoff symmetric exit mechanism and uses any first-mover triggering rule, then it is always optimal for each player to choose $t = 0$ as the date to dissolve the partnership.*

Proof. Since the mechanism is ex-ante payoff symmetric we have that $v = v_{r_1} = v_{r_2}$. Thus, any probability distribution over roles induced by τ yields an ex-ante expected payoff of v from the dissolve stage regardless of (t_1, t_2). Next, since we have a first mover trigger rule, the end date for the dissolve stage is $t = \min\{t_1, t_2\}$. If partner j chooses a break date of t_j, then i's expected payoff of choosing to stop at time t_i is

$$v - C(\min\{t_i, t_j\})$$

This is clearly maximized at $t_i = 0$ for all t_j. ∎

Thus, there is no war of attrition if we use an exit mechanism which is ex-ante payoff symmetric and a first-mover triggering rule. There are many exit mechanisms with this property including most of the auction mechanisms suggested in the literature. It is worth point out, however, that the use of an ex-ante payoff symmetric mechanism is not necessary to avoid the war of attrition.

Proposition 2 *Suppose the break-up game is defined by any exit mechanism and a first-mover triggering rule which assigns the partners to the two roles according to any probability distribution that is independent of the break announcements (t_1, t_2), then it is always optimal for each player to choose $t = 0$ as the date to dissolve the partnership.*

Proof. Suppose the triggering rule assigns player i to the roles r_1 and r_2 according to probabilities q_i and $1-q_i$ respectively. Let $v_i = q_i v_{r_1} + (1-q_i) v_{r_2}$. Thus, whenever the partnership is dissolved, i has an ex-ante expected payoff of v_i from the dissolve stage regardless of (t_1, t_2). Next, since we have a first mover trigger rule, the end date for the dissolve stage is $t = \min\{t_1, t_2\}$. If partner j chooses a break date of t_j, then i's expected payoff of choosing to stop at time t_i is

$$v_i - C(\min\{t_i, t_j\}).$$

and, again, is clearly maximized at $t_i = 0$ for all t_j. ∎

In short, Proposition 2 states that if we divorce the relationship between the triggering rule and the determination of player roles we get dissolution at the efficient time period.

The previous two results illustrated ways in which we can avoid the war of attrition. The next result formalizes our observations from the motivating Example 1.

Proposition 3 *Suppose the break-up game is defined by an exit mechanism that is ex-ante payoff asymmetric, and a first-mover triggering rule which assigns the first mover a fixed probability* $q < \frac{1}{2}$ *of being placed in the payoff dominant role, then there is a symmetric "war of attrition" equilibrium where the partnership is dissolved, with positive probability, at some* $t > 0$*.*

Proof. First, without loss of generality we can assume that the payoff dominant role in the mechanism is r_1. Thus, we have that $\pi_{r_1} > \pi_{r_2}$ and we can define the expected payoff of the first mover and second mover to be

$$\begin{aligned} v_F &= q\pi_{r_1} + (1-q)\pi_{r_2} \\ v_S &= (1-q)\pi_{r_1} + q\pi_{r_2} \end{aligned}$$

respectively. Since $q < \frac{1}{2}$ we have $v_S > v_F$.

Now suppose partner j chooses his stopping time according to the distribution G on $[0, \infty)$ with density g. Then the expected payoff to i of choosing to stop at time t is

$$u^i = \int_0^t (v_S - C(z))g(z)dz + \int_t^\infty (v_F - C(t))\, g(z)dz.$$

The marginal benefit of choosing to increase t is

$$\begin{aligned} \frac{du^i}{dt} &= (v_S - v_F)\, g(t) - l \int_t^\infty g(z)dz \\ &= (v_S - v_F)\, g(t) - l\, [1 - G(t)] \end{aligned}$$

Now, if G is such that $\frac{du^i}{dt} = 0$, then any t is a best response for i. This occurs when G satisfies

$$g(t) + \frac{l}{(v_S - v_F)} G(t) = \frac{l}{(v_S - v_F)}.$$

Let $a = \frac{l}{v_S - v_F}$ so $g(t) + aG(t) = a$. Using an integrating factor $\exp(at)$ we have that the above differential equation can be re-written as

$$\frac{d}{dt}\left(G(t)\exp(at)\right) = a\exp(at)$$

So, applying the Fundamental Theorem of Calculus, the solution is

$$G(t) = 1 - \exp(-at) + C,$$

where C is a constant. Since $\lim_{t\to\infty} G(t) = 1$ we have that $C = 0$ or $g(t) = a\exp(-at)$. Thus, if each partner chooses their break time according to the distribution $G(t)$, then each player is indifferent between stopping at any time t. It is therefore also optimal choose a stop time according to G. Hence, we have a "the war of attrition" equilibrium. ■

The Texas Shoot-Out discussed in Example 1 fits Proposition 3 and therefore induces the war of attrition.

Finally, given a payoff asymmetric mechanism, we can always eliminate the war of attrition by rewarding the first mover instead of punishing him.

Proposition 4 *Suppose the break-up game is defined by an exit mechanism that is ex-ante payoff asymmetric, and a first-mover triggering rule which assigns the first mover a fixed probability $q > \frac{1}{2}$ of being placed in the payoff dominant role and settles ties with the flip of a fair coin, then it is always optimal for each partner to choose to dissolve the company at $t = 0$.*

Proof. Again, without loss of generality we can assume that the payoff dominant role in the mechanism is r_1. Thus, we have that $\pi_{r_1} > \pi_{r_2}$ and we can define the expected payoff of the first mover and second mover to be

$$\begin{aligned} v_F &= q\pi_{r_1} + (1-q)\pi_{r_2} \\ v_S &= (1-q)\pi_{r_1} + q\pi_{r_2} \end{aligned}$$

respectively. Since $q > \frac{1}{2}$ we have $v_F > v_S$. There is therefore no benefit to waiting to call break. Hence, a player should always choose $t_i = 0$. ■

3 The Experiment

This paper features an experiment environment designed to capture the key features of the theoretical model. Treatments vary by both exit mechanism and triggering rule to test the model's major implications.

Treatment Name	Triggering Rule	Exit Mechanism
Winner's-Bid Auction (WBA)	n/a, symmetric roles in exit mechanism	Winner's-Bid Auction
Divide-And-Choose, Endogenous (DCE)	triggerer moves first in exit mechanism	Divide-And-Choose
Divide-And-Choose, Random (DCR)	random assignment in exit mechanism	Divide-And-Choose

Table 1: Description of the Three Treatments

3.1 Experimental Design

The experiment utilized three treatments under two different exit mechanisms and triggering rules. Two treatments involved the divide-and-choose mechanism. In the Divide-And-Choose, Endogenous Assignment treatment (henceforth, DCE) the first-mover in the triggering game would become the first-mover (the payoff dominated role) in the subsequent exit mechanism. In the Divide-And-Choose, Random Assignment treatment (henceforth, DCR) the roles in the divide-and-choose exit mechanism were assigned randomly. The third treatment utilized the Winner's-Bid-Auction mechanism (henceforth, WBA) as an exit mechanism. With such a mechanism there is only one possible triggering rule, as both roles in the Winner's-Bid Auction are identical. Table 1 summarizes this design with these three treatments in table form.

The "break-up game," described in Section 2, was implemented. Subjects, randomly selected into groups of two, determined how to allocate one indivisible item with possible transfer payments. In all possible allocations, only one subject could receive the item. Subjects received points for acquiring an item, equal to their value of that item. Receiving no item was associated with a value of 0.[8] Thus, the values for each item are induced values. Subjects' values of the item were independently drawn from the uniform distribution on the interval [50,150]. A subject knew her valuation of the item as well as the uniform distribution from which valuations were drawn.

Subjects would receive points equal to their value of any items acquired (i.e., induced values) plus or minus any points they transferred to the other subject plus any additional points they might receive from the initial, break-up stage of the game.

To avoid incentives associated with repeated play, subjects were randomly

[8]Receiving no item was described to subjects as receiving item A, an item without value. This distinction may reduce the possibility that subjects would be motivated by the non-monetary desire to "win" an item (e.g., Cooper and Fang (2008); Roider and Schmitz (2012), and makes our results comparable with previous literature (i.e., Brown and Velez (2016)).

re-assigned to each other at the beginning of each period. Subjects were instructed that they were to be randomly re-matched each period, and no identifying information (e.g., subject number) was disclosed to a subject about her match in any period.

3.1.1 Break-up stage

In the break-up stage, each pair observed a clock count down from 10 and had the option to push a button. The stage would end when either subject pushed the button or the counter reached zero. Both subjects would receive a "bonus," added to their point totals, equal to the number on the clock when the stage ended. The subject that pushed the button would be considered first-mover in the break-up stage. The corresponding triggering rule would determine her role in the subsequent exit mechanism.

At the end of the break-up stage a screen would inform subjects of which subject hit the button first (if any), the bonus that was given to both subjects, and how the triggering rule would assign subjects in the next stage.

3.1.2 Exit mechanism

Depending on the treatment, subjects would either use a winner's-bid auction (WBA) or divide-and-choose mechanism (DCE and DCR) to allocate their jointly owned item. Under the DCE treatment the subject that pushed the button in the break-up stage would be the divider, otherwise the roles of divider and chooser were randomly assigned (this was also the case had neither subject pushed the button in the DCE).

Under the winner's bid auction exit mechanism (WBA treatment only), subjects observed their valuation for the period and simultaneously submitted their bids for the item. The subject with the higher bid received the item, and the subject with the lower bid received a transfer equal to the higher bid from the subject who acquired the item. In this way, the winner's-bid auction is a first-price auction to acquire item B. In the case of equal bids, the item was assigned randomly to either subject. Bids were restricted to the interval [0, 150].

After submitting a bid, each subject was allowed to submit a possible value for the other player's bid. The experimental software then displayed the outcome (i.e., who gets which item, what amount is transferred for each player, each players' earnings for that period) that would occur with those

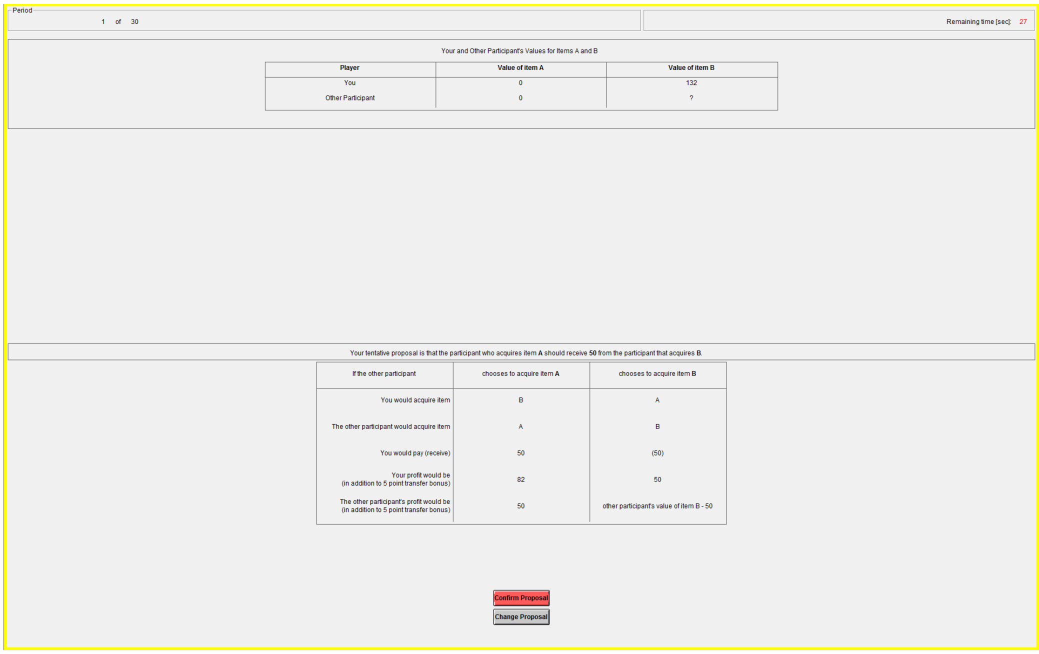

Figure 1: Game Interface in WBA

two bids as well as a table that showed all possibilities that could happen if the other player's bid were below, equal to, or above the subject's bid (see Figure 1).

After a subject viewed these possibilities, she could choose to confirm her bid, or chose an alternate bid. If she chose an alternate bid, the process repeated. The process ended when a subject confirmed her bid.

Under the divide-and-choose exit mechanism (DCE and DCR treatments), the divider would move first. The divider chose whether the subject who acquires item B should receive or pay a transfer and the amount of that transfer. Transfers were restricted on the interval [0, 150]. After one subject made a proposal, she saw a table of possible outcomes (see Figure 2) that displayed the two possible outcomes (i.e., who gets which item, what amount is transferred for each player, each player's earnings for that period) when the other subject choose to take item A or item B.

The subject then had the opportunity to confirm her decision or make another one. If she chose to try another proposal, the process repeated until she confirmed a proposal. Once a proposal was confirmed, the other subject

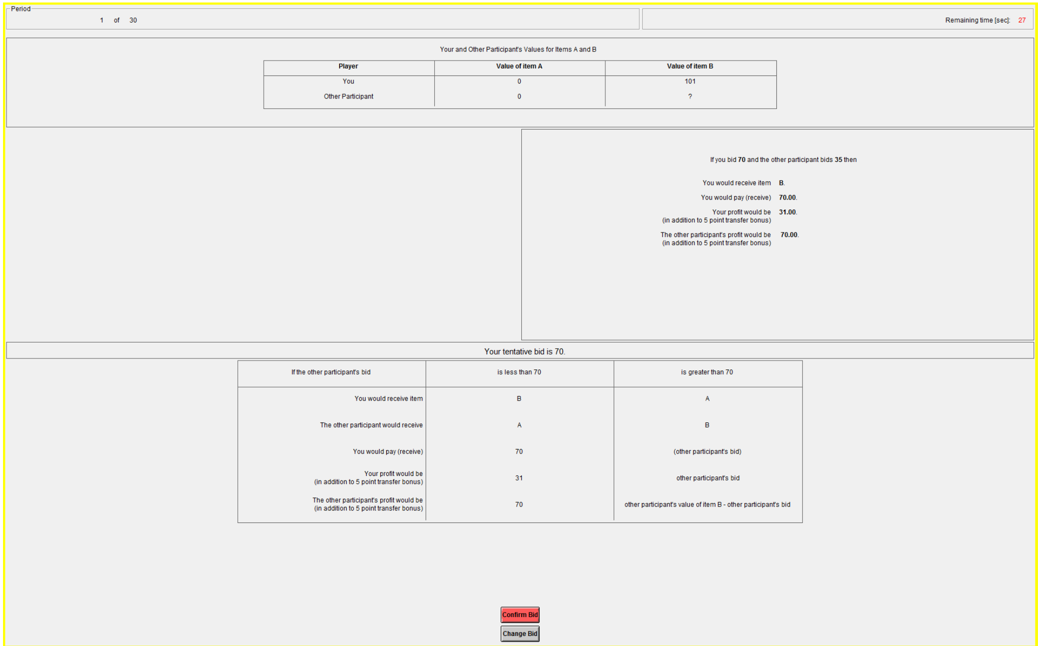

Figure 2: Game Interface in DCE and DCR

viewed the proposal. The display showed her two outcomes: both her own and the other subject's total earnings if she chose to take item A or item B. The chooser then had the opportunity to choose either item.

At the end of the game a feedback screen would describe both players actions under the mechanism and provide information on each subject's valuation and total points earned that round. All information was revealed to subjects at the end of the game as feedback with the intent to aide learning over the course of the session. At the end of each game, subjects would be reassigned to a subject pair and a new break-up game would begin with each subject drawing new valuations. This process would continue for 30 periods.

3.2 Experimental Procedures

Six sessions were held at the Economic Research Laboratory (ERL) in the Economics Department at Texas A&M University during June 2016. Subjects were recruited using ORSEE software (Greiner (2015)) and made their decisions on software programmed in the Z-tree language (Fischbacher (2007)). Subjects sat at computer terminals with dividers to make sure their anonymity was preserved. Subjects were 131 Texas A&M undergraduates from a variety of majors. At the end of each session, subjects filled out a questionnaire consisting of demographics information, an unincentivized risk-preference task (similar to Eckel and Grossman (2008)), and a Cognitive Reflection Test (Frederick (2005)). Experiments lasted about two hours.

Thirty and twenty subjects, respectively, participated in the two sessions of the Divide-And-Choose, Endogenous Assignment treatment (DCE);[9] Twenty and twenty subjects, respectively, participated in the two sessions of the Divide-And-Choose, Random Assignment treatment (DCR); twenty-five[10] and sixteen subjects, respectively, participated in the two sessions of the Winner's Bid Auction (WBA) treatment. To avoid issues with preferences that involve complementarities across periods (see Azrieli et al. (2012)), one period was randomly selected at the end of each experiment to be paid. Subjects received earnings from that round converted to cash at the rate of 1 point=$0.35 plus a $5 show-up payment. Earnings ranged from $5.00 to $77.10 with averages of $30.20, $27.26, $28.03, for the DCE, DCR and WBA sessions, respectively.

3.3 Theoretical Predictions

The theoretical model allows for the possibility that the partners take forever to break-up. This is not possible in the experimental lab so we use a finite end date. However, it is straightforward to see that the content of the propositions remains unchanged if we choose a finite end date for break up. Propositions 1 and 2 remain unchanged. Proposition 3 also remains unchanged, the only difference is the distribution used in the proof alters by a constant. This is since, for a finite end date $T < \infty$, we need $\lim_{t\to T} G(t) = 1$. Nevertheless, there remains a symmetric mixed equilibrium where partners choose $t > 0$ with positive probability. In our experimental game, the transaction bonus from the break-up stage of the break-up game

[9]A time limit was reached in the initial, 30 subject DCE session so only obtained 17 periods of observations were obtained. All other sessions went the full 30 periods. All results presented in this paper are robust to analysis where all data is truncated after 17 periods or period-specific dummy variables are used. We can find no plausible explanation how this truncation would be responsible for any of our results. In fact, because subjects tend to wait longer to hit the button in this treatment in later periods, it is very likely are results would have been stronger had this session gone the full 30 periods.

[10]Because of the odd number, one subject was randomly selected not to participate each period.

captures the theoretical concept of t from Propositions 1–3. Specifically, the bonus is $10-\ t$ where t is in seconds. This leads to clear predictions for each of the treatments.

Prediction 1 (WBA) *The Winner's Bid Auction exit mechanism is symmetric. By Proposition 1, the optimal dropout time is* $t = 0$*, and the corresponding transaction bonus is 10.*

Prediction 2 (DCR) *The Divide-And-Choose, Random Assignment uses a triggering mechanism that is independent of the results of the break-up stage. By Proposition 2, the optimal dropout time is* $t = 0$*, and the corresponding transaction bonus is 10.*

Prediction 3 (DCE) *The Divide-And-Choose, Endogenous Assignment uses a triggering mechanism that assigns the payoff dominated role to the first-mover in the break-up stage with full probability. By Proposition 3, there is a symmetric equilibrium prediction where break-up occurs at* $t > 0$ *with positive probability, and the corresponding transaction bonus is less than 10.*

In short, we should expect higher transaction bonus levels in the WBA and DCR treatments than the DCE.

3.4 Results

Table 2 provides summary statistics for the main outcome variables of the experiments. Each observation is at the subject-pair level. The overall statistics are suggestive of the theoretical predictions. Bonuses are very close to maximum levels in the DCR and WBA treatments but not the DCE. There is an observe ex-post earnings disadvantage for having triggered the break-up in the DCE but not the DCR or WBA.

3.4.1 Break-up Stage—Bonus Amounts

Theory predicts the break-up stage will immediately conclude in the DCR and WBA treatments—where subjects' role in the exit mechanism is independent of who triggers the break-up—but will be drawn out as the war of attrition in the DCE treatment. Outcome data is largely consistent with this prediction; Table 2, row 1 shows that subjects attained the full bonus

	Divide-And-Choose, Endogenous Assignment (DCE)	Divide-And-Choose, Random Assignment (DCR)	Winner's Bid Auction (WBA)
full bonuses attained	0.088 (0.284)	0.683 (0.466)	0.797 (0.403)
bonus amount	6.321 (3.553)	9.537 (0.887)	9.718 (0.658)
allocative efficiency	0.757 (0.429)	0.765 (0.424)	0.728 (0.445)
average earnings excluding bonus	55.889 (13.186)	55.019 (13.051)	55.752 (13.485)
triggerer earnings differential	-35.500[a] (66.792)	-1.125 (73.010)	4.857 (42.954)
subject pairs	555	600	600
subjects	50	40	41
sessions	2	2	2

a. excludes 105 pairings where neither subject triggered break up.

Table 2: Summary Statistics of Outcomes Variables by Treatment (Standard Deviations in Parentheses).

amount, the equilibrium prediction, in 68.3% and 79.7% of all subject pairings in the DCR and WBA treatments, respectively, and only 8.8% in the DCE treatment. A linear probability model regression of whether the maximum bonus was attained by the pair on treatment shows these differences are significant at the 1% level (see Table 3, column (1)).[11] Specifically the DCR and WBA treatments are a associated with a 59 and 71 probability point increase of attaining the full bonus relative to the DCE treatment. Though not predicted by theory, the 12 probability point difference between the DCR and WBA is also statistically significant ($p < 0.01$). Figure 3 shows the percent of subject pairs that attained the full bonus over the 30 periods of the experiment. The differences between the DCR and the other treatments intensify over time. For periods 4 and after, a Fischer Exact test finds the number of full bonuses attained by either DCR or WBA to be significantly different than the DCE at the 5% level.

Table 3, Column (2) provides a similar regression specification of the amount of the bonus attained by each subject-pair. Consistent with the

[11] Following the specification of Brown and Velez (2016) we use a crossed effects model with separate random effects terms for both the high-value and low-value subject in each pair and period-specific dummy variables.

	(1) full bonus attained	(2) bonus amount	(3) allocative efficiency
Divide-And-Choose Random	0.592*** (0.033)	3.290*** (0.277)	-0.005 (0.906)
Winner's Bid Auction	0.708*** (0.033)	3.464*** (0.276)	-0.034 (0.039)
First random effects term?	subject with high value	subject with high value	subject with high value
Second random effects term?	subject with low value	subject with low value	subject with low value
Period dummy variables?[a]	Y	Y	Y
observation level	pair	pair	pair
observations	1690[b]	1755	1755
log likelihood	-631.014	-3631.958	-961.713

[a] Alternate specifications with continuous period variable or no period variable do not appreciably change results.
[b] Excludes 65 period 1 observations where no full bonuses were attained.

Table 3: Regression Analysis of Pair-Level Outcomes on Treatment

theoretical predictions, both the DCR and WBA treatments are associated with a more than 3 point increase in the bonus relative to the DCE treatment ($p < 0.01$). Figure 4 shows the average bonus amount per period in each of the three mechanisms. The differences between the DCR and the other treatments intensify over time. For periods 3 and after, a Mann-Whitney-Wilcoxon test finds the number of full bonuses attained by either DCR or WBA is significantly different than the DCE at the 5% level.

3.4.2 Exit Mechanism—Efficiency and Earnings

After the decision to break-up had been made subjects used an exit mechanism to determine who should receive the valuable good. That mechanism would be considered (ex-post) efficient if the subject who had the higher value for item obtained it. Table 3, column (3) provides a linear probability model regression of this measure of allocative efficiency. The estimates on the treatment dummy variable indicate no statistically significant difference between treatments. This result is not surprising for the DCE and DCR treatments because they use an identical exit mechanism. The fact the WBA does not attain higher levels of efficiency is not consistent with theory, but is entirely

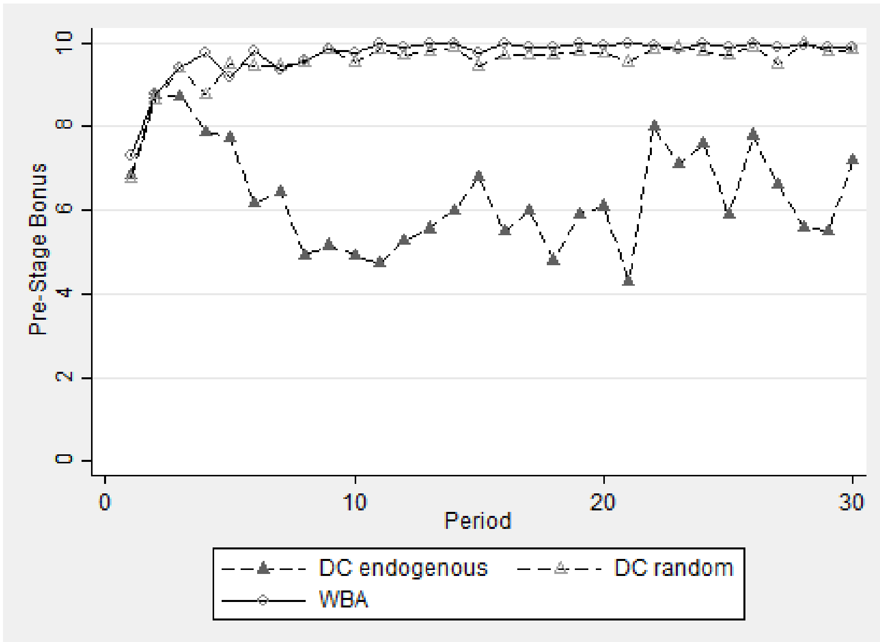

Figure 3: Pre-stage Bonus by Period

consistent with previous research in this area (Kittsteiner et al. (2012) find the same result). As total earnings for a subject-pair only vary by whether efficiency is attained, it is not surprising there are no overall differences in subject earnings across mechanisms (Table 4, column (1)).

More crucial to the model is the earnings differential for the subjects who triggered the break-up in the exit stage. Recall, subjects would not want to trigger a break-up in the exit stage of the DCE mechanism because it would put them in a disadvantageous strategic situation; there is no similar disadvantageous situation in the DCR or WBA. It is essential we confirm this finding empirically for our model to be valid. Regressions (2) and (3) in Table 4 examine subject earnings on the treatment interactions with the act of triggering the break-up. As predicted by theory, subjects who triggered the break-up in the DCE mechanism earned 36 less points than subjects who didn't ($p < 0.01$).[12] In contrast subjects in the DCR random treatment earned statistically identical amounts to those who did not trigger the break-

[12]Note that we count the 105 subject decisions where a subject was assigned the role of divider in the DCE at random (because no subject triggered the exit) as not triggering the break-up. Because these are strategically disadvantageous situations—on average,

	(1) earnings	(2) earnings	(3) earnings
Divide-And-Choose Random	-1.131	-18.385***	-17.708***
	(2.470)	(2.634)	(2.603)
Winner's Bid Auction	0.393	-20.645***	-19.965***
	(2.115)	(2.188)	(2.250)
Triggered break-up		-36.141***	-36.432***
		(3.069)	(3.049)
Divide-And-Choose Random ×		34.093***	33.873***
triggered break-up		(3.944)	(3.885)
Winner's Bid Auction ×		40.090***	40.196***
triggered break-up		(3.345)	(3.291)
Gender and survey controls?	N	N	Y
Subject random effects?	Y	Y	Y
Period dummy variables?[a]	Y	Y	Y
observation level[b]	decision	decision	decision
observations	3405	3405	3405
r-squared	0.002	0.083	0.090

[a] Alternate specifications with continuous period variable or no period variable do not appreciably change results.
[b] All three regressions use robust standard errors clustered on subject.

Table 4: Regression Analysis of Subject Period Earnings on Treatment

up ($-36.141 + 34.093 = -2.048$, $p \approx 0.30$). Subjects in the WBA who triggered the break up earned slightly more ($-36.141 + 40.090 = 3.949$, $p < 0.05$), but this effect is roughly an order of magnitude lower than the DCE treatment. These effects remain after controlling for gender and subject responses to survey questions.[13]

To summarize, our results are largely consistent with the predictions of theory. Being a divider in the divide-and-choose exit mechanism is associated with lower earnings. A triggering rule that assigns that role to the agent who triggers the break-up features more prolonged break-ups, consistent with the

these 105 dividers do 42 points worse than their corresponding choosers—their inclusion, if anything, will *diminish* our estimate of the strategic disadvantage of triggering the break-up in the DCE.

[13]Each additional question correct on the CRT (out of 3) is associated with a 2-point increase in subject earnings ($p < 0.01$). Changing one's preferred gamble from the nth to the $(n+1)$th most risky (out of 6) is associated with a 1-point increase in subject earnings ($p < 0.10$). While we have no real prior hypothesis on what the magnitude of these effects should be, they do go in the direction one would expect. There is no additional effect of gender.

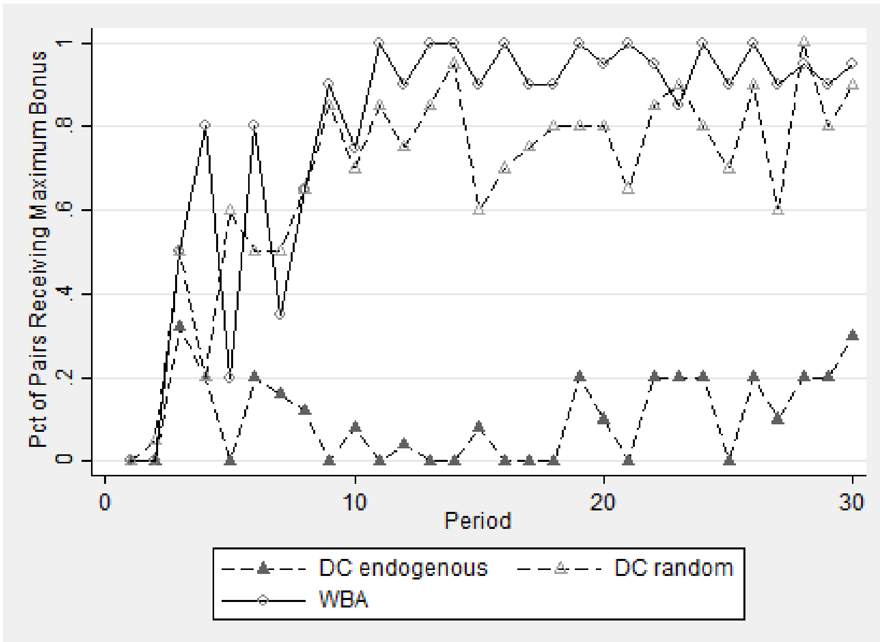

Figure 4: Full Bonus Attainment by Period

idea that the rule disincentivizes break-ups and creates a war of attrition.

4 Summary

Partnerships form and sometimes they need to be dissolved. Exit mechanisms play an important role in this process. They can create a transparent set of rules that greatly simplifies the process for dividing joint assets. Ultimately, however, these mechanisms must be built into the partnership contract and partners will need to specify the conditions under which exit mechanisms need to be triggered. The main purpose of this paper was to illustrate that this process matters. In this regard we developed a simple theoretical model for looking at the consequences of pairing certain classes of triggering rules with certain types of exit mechanisms. The model suggested that certain pairings give rise to wars of attrition (and therefore should be avoided in practice) while other pairings avoid this type of outcome. The experimental results provide strong support for these predictions and illustrate the robustness of these claims.

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