Breaking-Up Should Not be Hard to Do! Designing Contracts to Avoid Wars of Attrition*

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Abstract

The partnership dissolution literature has almost entirely focused on the properties of exit mechanisms in isolation taking other features of the break-up process as given. We consider a simple, two-stage model of the dissolution process where both the decision to dissolve and the roles the partners play in the exit mechanism are endogenously determined according to a "triggering rule." We find certain pairings of triggering rules and exit mechanisms can lead to a war of attrition which inefficiently prolongs the dissolution process. However, since these theoretical predictions require sophisticated reasoning and backward induction, it is unclear whether the theory has any empirical validity. We therefore conduct a laboratory experiment to explore this question. Treatments are selected to test the main predictions of the model using combinations of exit mechanisms and triggering rules commonly seen in practice. The experimental results are largely supportive of the underlying theoretical predictions.

Keywords: Dissolving a Partnership, Exit Mechanisms, Shoot-Out Mechanism, War of Attrition

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1 Introduction

A canonical game in game theory, the war of attrition involves players competing by continually paying costs until they drop out; the player that endures the longest wins the "prize." Introduced by Smith (1974) to model biological conflicts between animals, the game has a variety of applications in economics, most notably market-exit decisions between competing firms (see Hendricks et al., 1988; Bulow and Klemperer, 1999, for surveys). While theory has largely characterized the game's equilibria in a variety of different environments, experimental results are mixed as to whether individuals persist too little, too much, or generally adhere to equilibrium predictions in this game.[1]

A further disconnect between theory and experiments is that existing experimental designs all value prizes as explicit payoffs. Theory may model such a prize more implicitly; a prize could be a strategic advantage in a future subgame rather than a distinct monetary reward. Given economic literature has shown individuals struggle in correctly accounting for future payoffs in both strategic and non-strategic contexts,[2] theoretical predictions of these multi-stage wars of attrition may deviate even more from actual behavior.

Partnership dissolution provides an applicable economic environment where the answers to these issues have consequential policy implications. When a partnership is formed, partners are advised to form a legal contract that specifies what to do if the partnership needs to be dissolved. There are many dissolution procedures or exit mechanisms that define rules to determine which partner should get the company and how the other partner should be compensated. Thus, exit mechanisms are the final subgame of a larger "break-up" game played by the partners. Exit mechanisms that favor a certain partner over another based on actions in a previous subgame can potentially generate a war of attrition (e.g., de Frutos and Kittsteiner, 2008). Hence, any endorsement of an exit mechanism cannot solely consider the properties it exhibits in isolation; one must examine the incentives it creates in the larger game and whether these potential incentives ultimately influence behavior. The purpose of this paper is to provide a generalized model for these two-stage break-up games and then to test the model's predictions over a two-stage experiment.

We begin the paper by defining a simple game in order to study the incentives provided by different kinds of partnership contracts. We call this game the "Break-Up Game." Specifically, a

[1] In contrast, it is a general result that individuals overbid in the isomorphic all-pay auction (see Dechenaux et al., 2015, for a survey), with two notable exceptions (Potters et al., 1998; Stephenson and Brown, 2021).

[2] Economic literature is rather pessimistic on whether individuals are able to plan ahead correctly in any type of dynamic optimization or savings applications (e.g., Bone et al., 2009; Brown et al., 2009; Lusardi, 2001). Similarly, evidence suggests agents are not able to fully backward induct in strategic, game-theoretic environments (Dufwenberg and Van Essen, 2018; Krockow et al., 2016).

Break-Up Game is induced by the composition of a *triggering rule* and an *exit mechanism.* An exit mechanism is the set of rules for assigning the company to one player and deciding how the other player is compensated. The triggering rule, in contrast, defines the conditions under which the partnership will be dissolved. It takes the break-up announcements of the partners, determines a date for dissolving the partnership, and determines any other parameters needed to implement the exit mechanism, including assigning partners to "player roles" in the exit mechanism.

To illustrate this game consider the first of two examples.

Example 1 *Two partners belong to an ineffective partnership and must decide when they would like to dissolve. Waiting to dissolve is costly, but at any time either of the partners can decide to trigger the exit mechanism. After the mechanism is triggered, the two parties conduct their due diligence and privately learn their value for the company. The partnership is then dissolved as follows: the partner who first triggers the exit mechanism is required to name a price per share for the whole company and the other partner is compelled to either purchase or sell his own shares at the named price.*

The combination of the exit mechanism and the triggering rule induces a Break-Up Game. The partners lose money until one of them indicates the desire to dissolve (i.e., makes a break announcement), this announcement determines the partners' roles in the exit mechanism and the date of dissolution. Finally, the partners split the company according to the rules of the exit mechanism. In Example 1, the exit mechanism is a version of the Divide and Choose where one party names a price and the other decides to buy at that price.[3] This procedure has two roles: a "Divider" and a "Chooser." The triggering rule in the example uses a "break" announcement by one of the players to determine a dissolution date (in this case at the same date as the break announcement) and to assign roles to the players. Specifically, the Divider role is assigned to the first person to announce the break and the Chooser role is assigned to the other player.

Divide and Choose is the most commonly used exit mechanism in practice and is popular due to its well-known fairness properties. However, this mechanism when viewed in the larger Break-Up Game setting may have the unintended consequence of prolonging an ineffective partnership through a war of attrition. There is a symmetric equilibrium in the Example 1 Break-Up Game where the partners choose to wait to dissolve the company even though waiting is costly and that it is transparently efficient for them to dissolve immediately.

[3] Other names for this procedure include the "buy-sell clause," "Russian Roulette," and "Texas Shoot-Out." We refer to this mechanism throughout the paper as the "Divide and Choose."

The intuition for the result is as follows. In either an ex-ante symmetric, independent or common values environment, if partners are risk neutral and each have an equal-share in the partnership, then it is beneficial to be assigned to the Chooser role in Divide and Choose.[4] According to the rules of the game, the partner who first triggers the exit mechanism is assigned to be the first mover. Thus, the game provides a "prize" for *not* being the first person to signal their desire to dissolve.

This is a bleak result since there is evidence that many real-life partnership contracts fit the framework of this simple break-up game. Fleischer and Schneider (2012), in a legal summary of shoot-out clauses and how they are triggered in practice, state that most buy-sell clauses are triggered as follows: "Party A (the party wishing to leave or take over the company) initiates the procedure by making an offer..." Fortunately, as bleak as this outcome appears, it has several simple solutions. The war of attrition disappears if we reverse the roles specified by the triggering rule, use a coin flip to assign roles, or use a symmetric auction.[5] This is illustrated in the next example.

Example 2 *Two partners belong to an ineffective partnership and must decide when they would like to dissolve. Waiting to dissolve is costly, but at any time either of the partners can decide to trigger the exit mechanism—an auction. After the mechanism is triggered, the two parties conduct their due diligence and privately learn their value for the company. The partnership is then dissolved as follows: the triggering rule specifies that once a partner triggers the exit mechanism each player is assigned a bidder role in the auction, each bidder submits a bid for the company, the high bid wins, and the high bidder pays the other partner the winning bid.*

In this example, the exit mechanism is an auction called the "Winner's Bid Auction."[6] It has two roles: a "Bidder 1" and a "Bidder 2." The triggering rule uses a "break" announcement by one of the players to determine a dissolution date (in this case at the same date as the break announcement) and to assign the players to roles. The player roles in the auction are symmetric so any assignment of partners to roles is strategically equivalent. As a result, unhappy partners should move to dissolve immediately. Thus, the break-up game in Example 2 illustrates one method

[4]McAfee (1992) was the first to characterize this symmetric equilibrium in an independent private values model. Morgan (2004) extends the result to a common values setting. In a complete information model, the reverse ranking holds.

[5]Another symmetric mechanism would be an auction for the role of second mover in Divide and Choose.

[6]Winner's Bid Auction also has other names, including the "Mexican Shoot-Out," and confusingly, the "Texas Shoot-Out." We use "Winner's Bid Auction" or WBA throughout the paper.

for avoiding the war of attrition outcome—symmetry. The partners, in this case, should dissolve immediately and proceed to the auction. Moreover, the auction is efficient (in theory, see below), eliminating another potential source of inefficiency observed with Divide and Choose.

Of course, theoretical incentives may not always play out in reality. As previously mentioned, it is unlikely that agents correctly estimate the future strategic advantage in the exit mechanism and backward induct to the break-up stage. Further, predictions of allocative efficiency of exit mechanisms also do not necessarily conform with experimental results. Equilibrium strategies under the Winner's Bid Auction may be more complicated than under Divide and Choose and the bounded rationality of subjects may remove the former mechanism's allocative efficiency advantage (Kittsteiner et al., 2012).

We develop a two-stage experiment to test the predictions of our model. In the first stage, two subjects simultaneously observe a countdown clock, mutually aware that every second that passes reduces their joint earnings. Either subject may stop the clock at any time, the one that does triggers the breakup. In the second stage, subjects allocate a commonly-owned good and transfer payments using either Divide and Choose or the symmetric Winner's Bid Auction. There are two treatments with Divide and Choose: in one treatment the party that initiates the breakup moves first; in the other, the roles are determined by chance.

Our results are largely consistent with the main comparative statics of our theory. When the exit mechanism is the symmetric Winner's Bid Auction or a Divide and Choose with exogenously determined roles, subjects exit the first stage immediately, exhibiting no war of attrition. However, in Divide and Choose with endogenously determined roles—where the subject who triggers the first-stage exit will be assigned the role of first mover—there is a significant delay in the breakup process, reducing total subject earnings. Thus, we provide the first experimental evidence of a war of attrition generated by the choice of exit mechanism and triggering rule. Inconsistent with theory, we also note exit times in this war of attrition are less than the risk-neutral subgame equilibrium prediction. Allocative efficiency is also not higher under Winner's Bid Auction than Divide and Choose. Both results have some precedent in previous experimental literature.

Admittedly, we have made a specific assumption on information structures that differs from much of the literature on partnership dissolution. We consider what happens if partners only gain the knowledge of their valuation after a break-up has been triggered. Our main reason for doing this was to produce the most transparent theoretical and experimental environment possible to test this two-stage war of attrition. That being said—while there is tremendous heterogeneity in the information structures underlying partnerships in the field—we believe this model may apply quite well to certain circumstances. A common disconnect between theory and economic behavior is how readily and accurately agents can generate valuations (see Ariely et al., 2003, for a discussion). When external, market valuations must be additionally considered, the process becomes even more challenging. It is not uncommon for partners to seek the aid of costly, independent, consultants to determine the value of a company after steps to dissolve the partnership have started. In such cases, each partner will presumably receive a signal after doing their due diligence that will completely outweigh the prior held before the break-up. Secondly, we note that partners may dissolve a company for reasons independent of differences in information about the partnership's value. Much like a marriage, two partners may discover that they no longer like each other or can't work together. Under this new assumption, our model demonstrates a Divide and Choose with endogenously determined roles will generate a prolonged war of attrition with no benefits toward allocative efficiency. In contrast, we characterize other mechanisms that have equal or better allocative properties and do not create such inefficiencies. In our final section, we note whether these results would be qualitatively different under different information structures, and we speculate why mechanisms that may often promote inefficiency are so commonly observed in litigation and contracts.

1.1 Related Literature

Our environment departs from the experimental war of attrition literature by having the prize of the war be endogenously determined in a second stage, as opposed to being set by the experimenter. This makes payoff information quite opaque; subjects only infer the strategic advantage of moving second either with a tremendous deal of strategic foresight or through actual game play.

We know of four previous experimental investigations of the war of attrition. While the designs are quite diverse with few commonalities, no design requires subjects to use backward induction and strategic inference to infer the payoffs for the "prize" to be won. Bilodeau et al. (2004) provide the first experimental examination of the war of attrition, in which subjects—randomly rematched over 12 periods—play a three-person, asymmetric-cost, complete information "volunteers' dilemma." Each round subjects were given endowments that declined each second in a 90 second war of attrition. Under this setup, a subgame perfect equilibria would have the low cost subject volunteer immediately, ending the war of attrition. Only 41% of observations followed this prediction. While comparative statics generally held—lower cost subjects were much more likely to ultimately volunteer—on aggregate, subjects overbid/overpersist in this particular "war."

Oprea et al. (2013) examine equilibrium predictions in a war of attrition in a market exit context based off Fudenberg and Tirole (1986). Two subjects with private payoff information earn instantaneous negative payoffs in a duopoly until one player exits. The resulting monopoly produces instantaneous positive payoffs for the remaining subject. Each game has no finite end; there is a 1% probability the game will end every second. Subjects are randomly rematched for 20 periods. Equilibrium predictions, which feature positive waiting times, are generally consistent with observed behavior.

Hörisch and Kirchkamp (2010) examine a two-person war of attrition as a descending clock auction for a prize with known value. Bidders pay different, privately-known, asymmetric costs each second. The time limit on the auction is deliberately chosen to be non-binding; all wars are ended by a bidder. Subjects play this game repeatedly for 24 periods with random-rematching every six periods. The design is deliberately chosen to be isomorphic to a static all-pay auction with identical equilibrium predictions. Subjects drop out too early (relative to the equilibrium prediction) in the dynamic war of attrition but overbid (relative to the equilibrium prediction) in the static all-pay auction. Thus, the authors demonstrate that results from all-pay auctions, despite their theoretical equivalence, may not generalize to dynamic war of attrition environments.

A final study related to the war of attrition is Embrey et al. (2015) who test the two-stage bargaining model of Abreu and Gul (2000). Should two subjects not reach agreement in the first stage, they enter a continuous war of attrition where the surviving player earns his first-stage bid, discounted for each second, t, of the war by $e^{-0.01t}$. Subjects play 15 periods of this game and are randomly rematched each period. For the purposes of testing this specific bargaining theory, players are sometimes matched with a computer player that will not concede in the second stage. A time limit is implied to subjects (i.e., the period can end when the discounted value has "reached zero"), but does not appear to be binding. Though the design features six treatments with various sized pies, differing strategy spaces, and various specifications of computer players, in every case a theoretical upper bound on second-stage persistence based on first-stage behavior can be calculated. Across all treatments, mean subject persistence exceeds that bound by 3–7.5 times.

Another related area of literature is on partnership dissolution. A great deal of literature has characterized the strategic incentives within these procedures. With few exceptions, the allocative properties of exit mechanism are examined in isolation, taking some other features of the process as given. The roles played by the partners in the mechanism are almost always taken as a parameter (c.f. Cramton et al., 1987; Güth and Van Damme, 1986; McAfee, 1992; Moldovanu, 2002; Morgan, 2004; Turner, 2013; Van Essen, 2013; Van Essen and Wooders, 2016).

We are aware of only three other papers that provide models of the larger break-up process in dissolution: de Frutos and Kittsteiner (2008); Brooks et al. (2010) and Landeo and Spier (2013). The first to utilize a two-stage break-up game structure, de Frutos and Kittsteiner (2008), features an independent private values model and is most similar to our work. Brooks et al. (2010) and Landeo and Spier (2013) concern asymmetric information and, for this reason, are considerably less applicable to our environment. All three papers share two key assumptions: i) at least one party is informed of their private valuation before the decision to dissolve ii) Divide and Choose is the exit mechanism. While these papers note the perverse incentives of the latter mechanism will generate a war of attrition, surprisingly, as de Frutos and Kittsteiner (2008) suggest, the implications are not so dire: either an informed party directly negotiates, bypassing the dissolution procedures, or the war will serve to improve allocative efficiency under the exit mechanism. This paper departs from the aforementioned literature in several key ways: first, we consider what happens if partners only gain the knowledge of their valuation after a break-up has been triggered; and second, we allow for a wide variety of different exit mechanisms and triggering rules to be studied in the same model. As previously discussed, these changes produce a structure where efficiency greatly depends of the use of an ex-ante symmetric exit mechanism. We return to consider our results under the differing information structures of all three papers in our concluding section.

Finally, experimental analyses on partnership dissolution have only focused on the allocative properties of exit mechanisms. (For example, the two-stage game of de Frutos and Kittsteiner (2008) has never been experimentally tested.) Both Kittsteiner et al. (2012) and Brown and Velez (2016) experimentally examine the allocative performance Divide and Choose and Winner's Bid Auction under incomplete and complete information, respectively. These experiments both reveal that dissolution mechanisms often fail to conform to theoretical comparative statics. For instance, Kittsteiner et al. (2012) find similar efficiency levels in the Winner's Bid Auction and Divide and Choose due to subject bounded rationality; theory predicts higher efficiency in Winner's Bid Auction. We also observe suboptimal play by our subjects in both stages of our game, and replicate this specific result, however, it does not interfere with the main predictions of our theory.

2 Model

The Break-Up Game is a two-player game defined by an exit mechanism and a triggering rule.

An *exit mechanism* ξ is the set of rules that determines which partner gets the company and how the other partner is compensated. It is defined by the following: First, a list of player roles $R = \{r_1, r_2\}$. Second, for each $r \in R$, there is a set of possible messages M_r that each player role r can send. Third, an *allocation rule that* determines which player role gets the company $A : M_{r_1} \times M_{r_2} \to \Delta\{r_1, r_2\}$, where $\Delta\{r_1, r_2\}$ is the set of probability distributions over the players roles r_1 and r_2. Fourth, a *payment rule* for each player role r_j —i.e., $P_{r_j} : M_{r_1} \times M_{r_2} \to \mathbb{R}$ for $j = 1, 2$. Finally, we assume that every exit mechanism depends on three parameters: the date $t \in \mathbb{R}_+$ when the exit mechanism is played; and, for each r, θ_r is the name of the partner placed in role r where $\theta_{r_1} \neq \theta_{r_2}$. A specific list of these parameters is denoted $\theta = (t, \theta_{r_1}, \theta_{r_2})$ and the set of all θ is given by Θ . Thus, an exit mechanism is given by

$$\xi = (R, (M_r, P_r)_{r \in R}, A, \theta).$$

The *triggering rule* τ defines the conditions under which the partnership will be dissolved. In the Break-Up Game each partner i will first choose a $t_i \in \mathbb{R}_+$. This will be called i's break announcement—i.e., the time period in which she will call break. The triggering rule is a mapping which takes break announcements from all of the partners as input and assigns a probability distribution over the initializing parameters of the exit mechanism—i.e., $\tau : \mathbb{R}^2_+ \to \Delta\Theta$, where $\Delta\Theta$ is the set of probability distributions on Θ. The class of rules that seem most natural is the one that respects an individual partner's desire to dissolve the partnership. We call any rule τ that sets the date of dissolution t equal to the $\min\{t_1, t_2\}$ a "first-mover" triggering rule. We utilize this rule for the majority of our results.

The Break-Up Game is induced by the composition of the triggering rule with the exit mechanism. The triggering rule specifies values for the parameters θ needed by the exit mechanism; and the exit mechanism determines an allocation.

2.1 A Break-Up Game

In the Break-Up Game, there are two equal share, risk-neutral partners: Partner 1 and Partner 2. Initially, neither partner knows their valuation for the whole company, but it is common knowledge that the partners' values are distributed according to distribution F on $[0, \bar{x}] \times [0, \bar{x}]$, where $f = F'$

is the density. The exit mechanism and the triggering rule are known to the two partners.

The timing of the game is as follows: First, Partner 1 and Partner 2 simultaneously choose break announcements t_1 and t_2 in $\mathbb{R}_+$ respectively signaling the time they want to dissolve the partnership. Once the break announcements have been submitted, the initializing parameters for the exit mechanism $\theta = (t, \theta_{r_1}, \theta_{r_2})$ are determined as a draw from distribution $\tau(t_1, t_2)$ and shown to the partners. Second, the partnership is dissolved at time t. At this time, each partner i privately observes his type x_i, pays a cumulative loss of $C(t) = lt$ such that $l > 0$, and assumes his assigned player role in the mechanism (either r_1 or r_2).[7] Finally, the partners then dissolve the partnership according to the rules of the exit mechanism ξ.

2.2 Predictions

The Break-Up Game is a two-stage game of imperfect information. The appropriate prediction concept is therefore subgame perfect Nash equilibrium. In this section, we develop several equilibrium predictions for a Break-Up Game where a "first-mover" triggering rule is paired with various types of exit mechanisms.[8]

We begin with behavior in the subgame following the break announcements in the first stage.

An exit mechanism induces a subgame between the two partners to be played out in the second stage of the Break-Up game. Let $m^* = (m^*_{r_1}(\cdot), m^*_{r_2}(\ \cdot))$ be a pure strategy Nash Equilibrium of this *subgame,* where $m^*_{r_i}(x)$ specifies the action to be taken by the partner in the r_i role of the exit mechanism when his value for the company is given to be x.[9] Given a particular equilibrium m^* of this subgame and the symmetry of the payoff structure, we can compute the expected payoffs for each role. This is the payoff that either player would expect should he be placed in that role in the exit mechanism. Specifically, the ex-ante expected payoff for roles r_1 and r_2, under m^*, are denoted $v^{m^*}_{r_1}$ and $v^{m^*}_{r_2}$ respectively.

The relative magnitude of $v^{m^*}_{r_1}$ and $v^{m^*}_{r_2}$ drive our main results. As such, it is useful to develop the following definitions. An exit mechanism is *ex-ante payoff symmetric* under m^* if the player roles all yield the same ex-ante expected payoffs—i.e., $v^{m^*}_{r_1} = v^{m^*}_{r_2}$. Similarly, the exit mechanism under m^* is *ex-ante payoff asymmetric* if the player roles yield different ex-ante expected payoffs—

[7] We are explicitly assuming the partnership is inefficient. The increasing costs in time are the opportunity costs of lost profits from not being owned by a single individual. This is not to say all partnerships are inefficient in this way, rather the interesting cases where dissolution mechanisms are meaningful are those where the partnership should be dissolved.

[8] The proofs can be found Appendix Section A.1.

[9] The use of a pure strategy equilibrium is simply for notational convenience.

i.e., either $v_{r_1}^{m^*} > v_{r_2}^{m^*}$ or $v_{r_1}^{m^*} < v_{r_2}^{m^*}$. In the later case, the role which achieves the higher payoff in the exit mechanism is referred to as the *ex-ante payoff dominant role.*

In the following propositions, we use these payoff definitions to generate predictions about the break-up date t chosen in the beginning of the game.

Proposition 1 *Suppose the Break-Up Game is composed of any first-mover triggering rule and an exit mechanism ξ that is ex-ante payoff symmetric under m^*, then there is a subgame perfect Nash equilibrium where the players always follow m^* in the second stage and choose $t_1 = t_2 = 0$ in the first stage.*

In other words, for a given exit mechanism, if players always expect to coordinate on the profile m^* in the subgame which is ex-ante payoff symmetric, then there is no war of attrition in the Break-Up Game if we use a first-mover triggering rule. This result is intuitive. If the exit mechanism is *ex-ante payoff symmetric* under m^*, then there is no "prize" associated with being assigned to one role or the other. Since players lose money by delaying the dissolution process there is no incentive for them to choose any other date of dissolution other than zero. There are many exit mechanisms with this property including most of the auction mechanisms suggested in the literature. However, the use of an ex-ante payoff symmetric mechanism is not necessary to avoid the war of attrition. Proposition 2 states that if we divorce the relationship between the triggering rule and the determination of player roles we also get dissolution at the efficient time period.

Proposition 2 *Suppose the Break-Up Game is defined using a first-mover triggering rule whose assignment of the partners to roles is independent of the break announcements (t_1, t_2), then in any subgame perfect Nash equilibrium we have that players select $t_1 = t_2 = 0$ in the first stage.*

The previous two results illustrated ways in which the war of attrition can be avoided. The next result formalizes our observations from the motivating Example 1. The main point is that the payoffs of the break-up game identified in the proposition exactly match the War of Attrition game studied by Smith (1974).

Proposition 3 *Suppose the Break-Up Game is defined by an exit mechanism ξ that is ex-ante payoff asymmetric under m^*, and a first-mover triggering rule which assigns the first mover a fixed probability $q < \frac{1}{2}$ of being placed in the payoff dominant role, then there is a "war of attrition" subgame perfect Nash equilibrium where the players always follow m^* in the second stage and where the partnership is dissolved at some $t > 0$ with probability 1.*

These propositions, while simple, generate several sharp, testable predictions about play in different types of break-up games. In particular, they help us identify pairings of triggering rules and exit mechanisms that can lead to, or avoid, a war of attrition.

Next, we present two detailed examples of the Break-Up Game using Divide and Choose and Winner's Bid Auction exit mechanisms where values for the partners are drawn *i.i.d.* uniform on support $[50, 150]$ after one partner has triggered the exit mechanism. These examples are useful for two reasons: First, they provide concrete illustrations of the predictions of the Break-up Game. Second, we will use the predictions generated by these two examples to form the research hypotheses for the experiment. The full computational details can be found in the Appendix.

2.2.1 Example 1: Breaking Up with Divide and Choose

Our first example considers the Break-Up Game with a first-mover triggering rule, Divide and Choose as the exit mechanism, and the first mover is selected to be put in the Divider role. In this exit mechanism, there are two player roles: the Divider and the Chooser. The Divider chooses a price. The Chooser observes this price and then decides whether to buy the company at price p (giving p to the Divider) or sell the company to the Divider for a price of p. In equilibrium, a risk neutral Divider chooses his price offer according to the function $p^*(x_D) = \frac{1}{4}x_D + 25$ and the Chooser decides whether to buy or sell according to the function

$$S^*(p; x_C) = \begin{cases} \text{Buy} & \text{if } x_C - p \geq p \\ \text{Sell} & \text{if } x_C - p < p \end{cases} .$$

The exit mechanism always favors the Chooser for any type (McAfee, 1992). The Divider's ex-ante expected payoff is $v_D = 52.083$; and the Chooser's ex ante expected payoff is $v_C = 63.542$.

We would therefore label Divide and Choose as *ex-ante payoff asymmetric* at this equilibrium. The "prize" for being placed in the Chooser role equal to $v_C - v_D = 11.459$. Proposition 3 tells us that there is an associated "war of attrition" subgame perfect equilibrium of the Break-Up Game where the players wait to dissolve the partnership with probability one. Proposition 3 assumes an infinite time horizon. This assumption simplifies the analysis and illustrates the war of attrition induced by certain types of exit mechanisms in the clearest manner. In the experiment, however, we use a fixed and finite time horizon T. While the use of finite horizon T does not change the prediction of a war of attrition, the mixed strategy used in the first stage of the Break-up Game is different than in the infinite horizon case. For completeness, we now present the equilibrium for

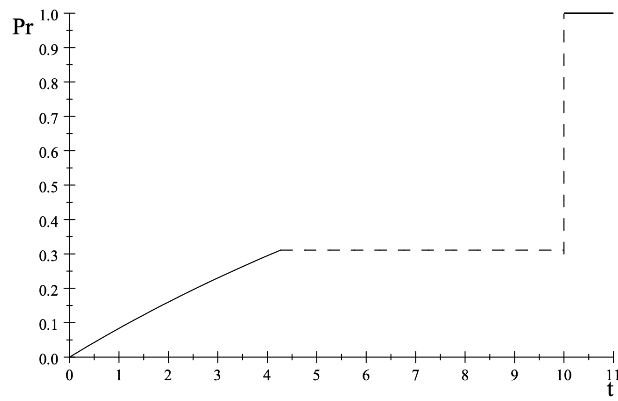

Figure 1: First Stage Mixed Strategy Distribution for War of Attrition Break-Up Game with Divide and Choose Exit Mechanism

the finite time horizon case.

In the experiment, the end date is $T = 10$. The first-stage equilibrium mixed strategy in the associated Break-Up Game is to draw t from

$$B(t) = 1 - \exp\{-\frac{t}{11.459}\}$$

on $[0, 4.271]$ and with probability $\exp\{-\frac{4.271}{11.459}\} \approx 0.69$ on time $\{10\}$.[10] The expected dissolution time in this equilibrium is $t = 5.729$.[11] A graph of this distribution function is provided in Figure 1.

2.2.2 Example 2: Breaking Up with a Simple Auction

Our second example considers the Break-Up Game with a first-mover triggering rule, a Winner's Bid Auction as the exit mechanism, and where the first mover is selected to be put in the Bidder 1 role. There are two player roles: Bidder 1 and Bidder 2. These two simultaneously submit bids to an auctioneer. The high bidder wins the company and pays the losing bidder the high bid. If bidders are risk neutral, then this auction admits a symmetric equilibrium in increasing bidding strategies where a bidder with type x chooses his bid according to the function[12]

$$\beta(x) = \frac{1}{3}x + \frac{25}{3}.$$

[10] The details of this equilibrium construction can be found in Proposition 4 of Appendix Section A.1.

[11] This is the expected minimum order statistic of two draws from the equilibrium mixture.

[12] If bidder's are risk averse, then the maximum that a bidder with value x can guarantee himself is $\frac{1}{2}x$. This maxmin payoff is achieved by bidding $\frac{1}{2}x$.

The resulting equilibrium outcome is always efficient.[13] Since the auction is symmetric, it doesn't favor one bidder role over the other and both players have identical ex-ante expected payoffs of participating in either role. We would therefore label the Winner's Bid Auction as *ex-ante payoff symmetric* at this equilibrium. Proposition 1 tells us that dissolution should be immediate at $t = 0$.

3 The Experiment

This paper features an experiment environment designed to capture the key features of the theoretical model. Treatments vary by both exit mechanism and triggering rule to test the model's major implications.

3.1 Experimental Design

The experiment utilized three treatments under two different exit mechanisms and triggering rules. Two treatments involved the Divide and Choose mechanism. In the Divide and Choose, Endogenous Assignment treatment (henceforth, DCE) the first-mover in the triggering game would become the first-mover (the payoff dominated role) in the subsequent exit mechanism. In the Divide and Choose, Random Assignment treatment (henceforth, DCR) the roles in the Divide and Choose exit mechanism were assigned randomly. The third treatment utilized Winner's Bid Auction (henceforth, WBA) as an exit mechanism. With such a mechanism there is only one possible triggering rule, as both roles in WBA are identical.[14] Table 1 summarizes this design with these three treatments in table form.

The "Break-Up Game," described in Section 2, was implemented. Subjects, randomly selected into groups of two, determined how to allocate one indivisible item with possible transfer payments. In all possible allocations, only one subject could receive the item. Subjects received points for acquiring an item, equal to their value of that item. Receiving no item was associated with a value of 0.[15] Thus, the values for each item are induced values. Subjects' values of the item were

[13] We provide a general expression for the equilibrium bid function in Appendix Section A.3.

[14] In other words, this design could also be thought of as 2×2 as the WBA treatment trivially satisfies both Endogenous and Random Assignment triggering rules.

[15] Receiving no item was described to subjects as receiving item A, an item without value. This distinction may reduce the possibility that subjects would be motivated by the non-monetary desire to "win" an item (e.g., Cooper and Fang, 2008; Roider and Schmitz, 2012), and makes our results comparable with previous literature (i.e., Brown and Velez, 2016). While such joy of winning (as well as any number of other non-monetary desires) may exist in a variety of field partnership dissolution situations, they are not particularly relevant to this evaluation of theoretical predictions across mechanisms. Further, they reduce our control over subjects by complicating the relation between induced values and actual subject valuations.

Treatment Name	Triggering Rule	Exit Mechanism
Winner's-Bid Auction (WBA)	n/a, symmetric roles in exit mechanism	Winner's-Bid Auction
Divide-And-Choose, Endogenous (DCE)	triggerer moves first in exit mechanism	Divide-And-Choose
Divide-And-Choose, Random (DCR)	random assignment in exit mechanism	Divide-And-Choose

Table 1: Description of the Three Treatments

independently drawn from the uniform distribution on the interval [50,150].

Subjects would receive points equal to their value of any items acquired plus or minus any points they transferred to the other subject plus any additional points they might receive from the initial, break-up stage of the game. To avoid incentives associated with repeated play, subjects were randomly re-assigned to each other at the beginning of each period.

3.1.1 Break-up stage

In the break-up stage, each pair observed a clock count down from 10 and had the option to push a button. The stage would end when either subject pushed the button or the counter reached zero. Both subjects would receive a "bonus," added to their point totals, equal to the number on the clock when the stage ended. The subject that pushed the button would be considered the first mover in the break-up stage. The corresponding triggering rule would determine her role in the subsequent exit mechanism. Subjects learned the outcome of the break-up stage immediately after its completion.

3.1.2 Exit mechanism

Depending on the treatment, subjects would either use a Winner's Bid Auction (WBA) or the Divide and Choose mechanism (DCE and DCR) to allocate their jointly owned item. Under the DCE treatment the subject that pushed the button first in the break-up stage would be the Divider, otherwise the roles of Divider and Chooser were randomly assigned (this was also the case had neither subject pushed the button in the DCE).

Under the WBA treatment, subjects observed their valuation for the period and simultaneously submitted their bids for the item. The subject with the higher bid received the item, and the subject with the lower bid received a transfer equal to the higher bid from the subject who acquired the item. In this way, the WBA is a first-price auction to acquire item B. In the case of equal bids, the item was assigned randomly to either subject. Bids were restricted to the interval [0, 150].

After submitting a bid, each subject was allowed to submit a possible value for the other player's

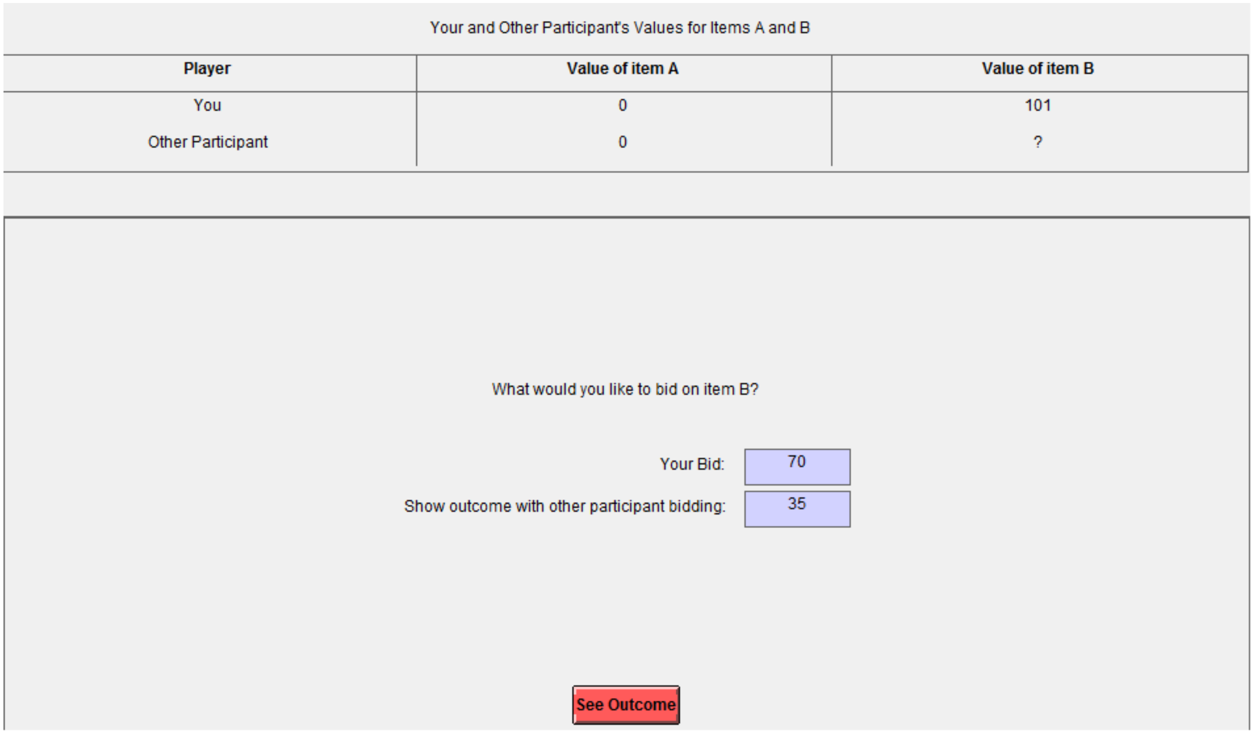

Figure 2: Game Interface in WBA

bid (see Figure 2). The experimental software then displayed the outcome (i.e., who gets which item, what amount is transferred for each player, each players' earnings for that period) that would occur with those two bids as well as a table that showed all possibilities that could happen if the other player's bid were below, equal to, or above the subject's bid.

After a subject viewed these possibilities, she could choose to confirm her bid, or chose an alternate bid. If she chose an alternate bid, the process repeated. The process ended when a subject confirmed her bid.

Under the Divide and Choose exit mechanism (DCE and DCR treatments), the Divider would move first. After observing her valuation for the period, the Divider chose whether the subject who acquires item B should receive or pay a transfer and the amount of that transfer (see Figure 3). Transfers were restricted on the interval [0, 150]. After one subject made a proposal, she saw a table of possible outcomes that displayed the two possible outcomes (i.e., who gets which item, what amount is transferred for each player, each player's earnings for that period) when the other subject choose to take item A or item B.

The subject then had the opportunity to confirm her decision or make another one. If she chose to try another proposal, the process repeated until she confirmed a proposal. Once a proposal was

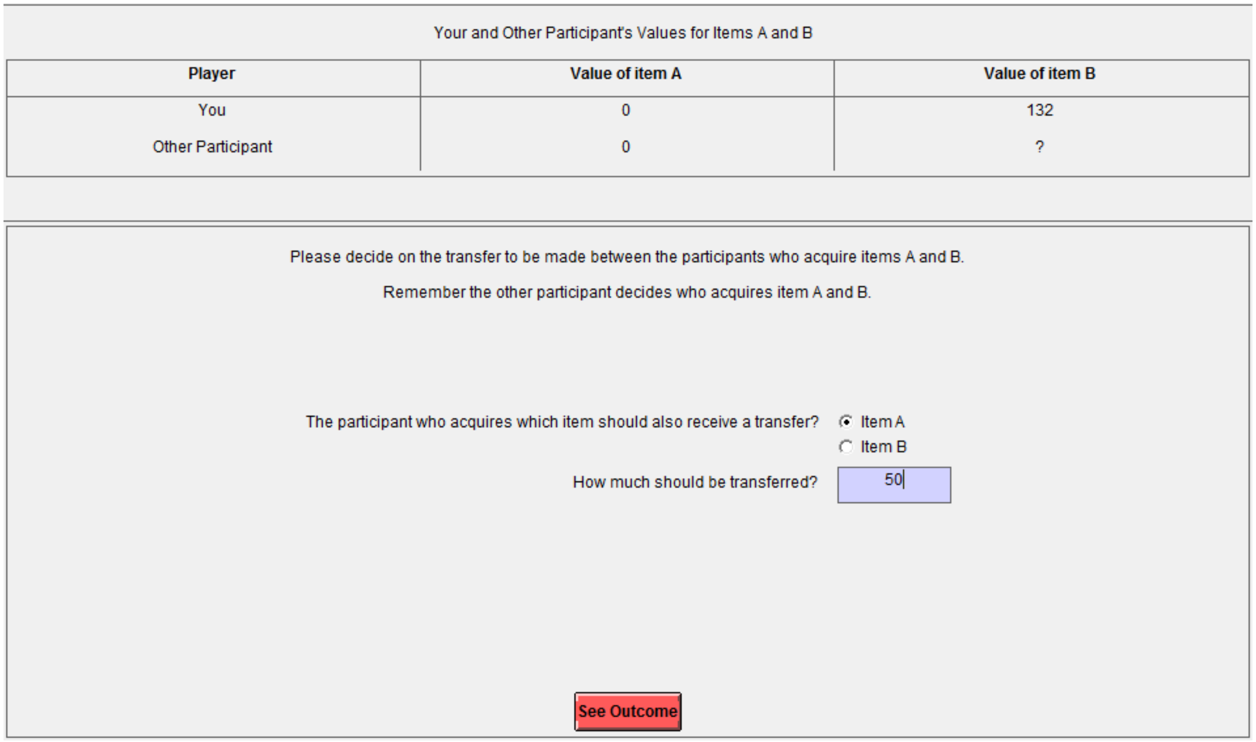

Figure 3: Game Interface in DCE and DCR

confirmed, the other subject viewed the proposal and her valuation. The display showed her two outcomes: both her own and the other subject's total earnings if she chose to take item A or item B. The Chooser then had the opportunity to choose either item.

At the end of the game a feedback screen would describe both players' actions under the mechanism and provide information on each subject's valuation and total points earned that round. This information was revealed to subjects at the end of the game with the intent to aid learning over the course of the session. At the end of each game, subjects would be reassigned to a subject pair and a new Break-Up Game would begin with each subject drawing new valuations. This process would continue for 30 periods.

3.2 Experimental Procedures

Subjects entered the laboratory and sat at computer terminals with dividers to make sure their anonymity was preserved. Before the experiment began instructions were read aloud to ensure that all rules, procedures and payoffs of the relevant two-stage mechanism were common knowledge.[16] With the exception of subject identities and histories (which could induce reputational effects)

[16]Instructions and surveys are available to readers as supplemental materials.

and subject valuations (which were disclosed in the manner specified by the theory), all other payoff-relevant information was common knowledge.

Six sessions were held at the Economic Research Laboratory (ERL) in the Economics Department at Texas A&M University during June 2016. Subjects were recruited using ORSEE software (Greiner, 2015) and made their decisions on software programmed in the Z-tree language (Fischbacher, 2007). Subjects were 131 Texas A&M undergraduates from a variety of majors. At the end of each session, subjects filled out a questionnaire consisting of demographics information, an unincentivized risk-preference task (similar to Eckel and Grossman, 2008), and a Cognitive Reflection Test (Frederick, 2005). Experiments lasted about two hours.

A group of thirty and a group of twenty subjects participated in the two sessions of the Divide and Choose, Endogenous Assignment treatment (DCE).[17] Two groups of twenty subjects each participated in the two sessions of the Divide and Choose, Random Assignment treatment (DCR); twenty-five[18] and sixteen subjects, respectively, participated in the two sessions of WBA. To avoid issues with preferences that involve complementarities across periods (see Azrieli et al., 2018; Brown and Healy, 2018), one period was randomly selected at the end of each experiment to be paid. Subjects received earnings from that round converted to cash at the rate of 1 point=$0.35 plus a $5 show-up payment. Earnings ranged from $5.00 to $77.10 with averages of $30.20, $27.26, $28.03, for the DCE, DCR and WBA sessions, respectively.

3.3 Hypotheses

The research hypotheses are derived from the equilibrium predictions of the theoretical model in Section 2.2.

Our first hypothesis concerns the delay decision subjects make in the first stage of the Break-Up Game. The existence of the war of attrition is our primary area of interest. In the experimental game, the transaction bonus from the break-up stage captures the dissolution time t in the theoretical propositions. Specifically, the bonus is $10 - t$ where t is in seconds. We expect no war of attrition in the Break-Up Game with either the WBA or the DCR mechanisms. This is the content of both Proposition 1 and Proposition 2. In both of these games, in equilibrium, participants are

[17] A time limit was reached in the initial, 30 subject DCE session so we only obtained 17 periods of observations. All other sessions went the full 30 periods. All results presented in this paper are robust to analysis where all data is truncated after 17 periods or period-specific dummy variables are used. We can find no plausible explanation how this truncation would be responsible for any of our results. In fact, because subjects tend to wait longer to hit the button in this treatment in later periods, it is very likely our results would have been stronger had this session gone the full 30 periods.

[18] Because of the odd number, one subject was randomly selected not to participate each period.

expected to dissolve at $t = 0$. In the experiment, this translates into subjects earning the full transaction bonus of 10. In contrast, from Proposition 4, we expect there to be a war of attrition in the Break-Up Game when the exit mechanism is the DCE. At the risk neutral equilibrium, the expected delay is $t = 5.729$ seconds which corresponds to a transaction bonus of 4.271. Hypothesis 1 summarizes the expected differences in delay time between the treatments.

Hypothesis 1 (War of Attrition) *There is a higher transaction bonus in the WBA and DCR treatments than in the DCE.*

The next three hypotheses concern predictions relative to the different exit mechanisms: efficiency, conformity of subject's behavior to the equilibrium strategies in the different mechanisms, and comparison of payoffs.

Hypothesis 2 concerns the allocative efficiency of the exit mechanisms. Did the partner who valued the company the most end up with the company at the end of the game? In equilibrium, the WBA is ex-post efficient whereas Divide and Choose is not. In the experimental environment, we expect Divide and Choose to result in an inefficient outcome about 12.5% of the time (or equivalently to be 87.5% efficient). The following hypothesis summarizes this result.

Hypothesis 2 (Allocative Efficiency) *The allocative efficiency in the WBA is higher than the allocative efficiency of DC.*

The next hypothesis concerns predicted behavior. In general, the risk neutral equilibrium of each mechanism forms the basis for our Stage 1 "dissolution" results. However, we recognize that risk aversion may influence bidding decisions. Thus, in both mechanisms, we expect bids to fall in the interval of bids defined by the risk neutral bid and the bidder's maxmin bid.

Hypothesis 3 (Expected Behavior) *In the Divide and Choose, the Divider with value x should bid somewhere between his equilibrium risk neutral bid function $\frac{1}{4}x + 25$ and his maxmin bid of $\frac{1}{2}x$. In contrast, the Chooser will always pick the option that has the greater value. In the WBA, a bidder with value x will bid somewhere between his equilibrium risk neutral bid function $\frac{1}{3}x + \frac{25}{3}$ and his maxmin bid of $\frac{1}{2}x$.*

Finally, at the risk neutral equilibrium it is possible to compare the expected payoffs of the different player roles in the Divide and Choose mechanism and the WBA. In the Divide and Choose, the Divider's ex-ante expected payoff is $v_D = 52.083$ and the Chooser's ex-ante expected

	Divide-And-Choose, Endogenous Assignment (DCE)	Divide-And-Choose, Random Assignment (DCR)	Winner's Bid Auction (WBA)
full bonuses attained	0.088 (0.284)	0.683 (0.466)	0.797 (0.403)
bonus amount	6.321 (3.553)	9.537 (0.887)	9.718 (0.658)
allocative efficiency	0.757 (0.429)	0.765 (0.424)	0.728 (0.445)
average earnings excluding bonus	55.889 (13.186)	55.019 (13.051)	55.752 (13.485)
triggerer earnings differential	-35.500[a] (66.792)	-1.125 (73.010)	4.857 (42.954)
subject pairs	555	600	600
subjects	50	40	41
sessions	2	2	2

[a] excludes 105 pairings where neither subject triggered break up.

Table 2: Summary Statistics of Outcomes Variables by Treatment (Standard Deviations in Parentheses).

payoff is $v_C = 63.542$. In the WBA, a bidder has an ex-ante expected payoff of $v_{WBA} = 58.333$. This is summarized by the following expected ranking of role payoffs:

Hypothesis 4 (Payoff Ranking) *The average payoffs of subjects placed in the Divider role are smaller than the average payoffs of subjects placed in the Chooser role. Furthermore, the average payoffs of subjects participating in the WBA are more than subjects placed in the Divider role, but smaller than the average payoffs of subjects played in the Chooser role.*

In the next section we will present the data from our experiment and test these hypotheses.

4 Results

Table 2 provides summary statistics for the main outcome variables of the experiments. Each observation is at the subject-pair level. The overall statistics are suggestive of the theoretical predictions. Bonuses are very close to maximum levels in the DCR and WBA treatments but not the DCE. There is an observed ex-post earnings disadvantage for having triggered the break-up in the DCE but not the DCR or WBA.

	(1) full bonus attained	(2) bonus amount	(3) allocative efficiency
DCR	0.592***	3.143***	-0.005
	(0.048)[a]	(0.128)	(0.047)
WBA	0.708***	3.325***	-0.034
	(0.033)	(0.117)	(0.032)
observation level	pair	pair	pair
observations	1690[b]	1755	1755
log likelihood	-623.404	-3631.958	-961.713

Notes: All three regressions use separate, crossed, random effects terms for the subject with the high value and the subject with the low value of the item. Period dummy variables are also included in all regressions. Alternate specifications with continuous period variable or no period variable do not appreciably change results.

[a] Standard errors are estimated from 100 cluster bootstraps taken at the session level.

[b] Excludes 65 period 1 observations where no full bonuses were attained.

Table 3: Regression Analysis of Pair-Level Outcomes on Treatment

4.1 Break-up Stage—Bonus Amounts

Theory predicts the break-up stage will immediately conclude in the DCR and WBA treatments—where subjects' role in the exit mechanism is independent of who triggers the break-up—but will be drawn out as the war of attrition in the DCE treatment. Outcome data is largely consistent with this prediction; Table 2, row 1 shows that subjects attained the full bonus amount, the equilibrium prediction, in 68.3% and 79.7% of all subject pairings in the DCR and WBA treatments, respectively, and only 8.8% in the DCE treatment. A linear probability model regression of whether the maximum bonus was attained by the pair on treatment shows these differences are significant at the 1% level (see Table 3, column (1)).[19] Specifically the DCR and WBA treatments are a associated with a 59 and 71 probability point increase of attaining the full bonus relative to the DCE treatment. Though not predicted by theory, the 12 probability point difference between the DCR and WBA is also statistically significant ($p < 0.01$).[20] Figure 4 shows the percent of subject pairs that attained the full bonus over the 30 periods of the experiment. The differences between

[19] Following the specification of Brown and Velez (2016) we use a crossed effects model with separate random effects terms for both the high-value and low-value subject in each pair and period-specific dummy variables.

[20] Because the DCR, unlike the WBA, still has asymmetric roles, a variety of behavioral explanations (e.g., regret aversion, deterministic probability) may explain why subject might hesitate to push the button in this setup. This could be thought of as an additional advantage to the symmetry of the WBA. We are cautious to emphasize this result because it was not hypothesized ex-ante, and the effect is considerably smaller than the difference in the DCE and WBA treatments.

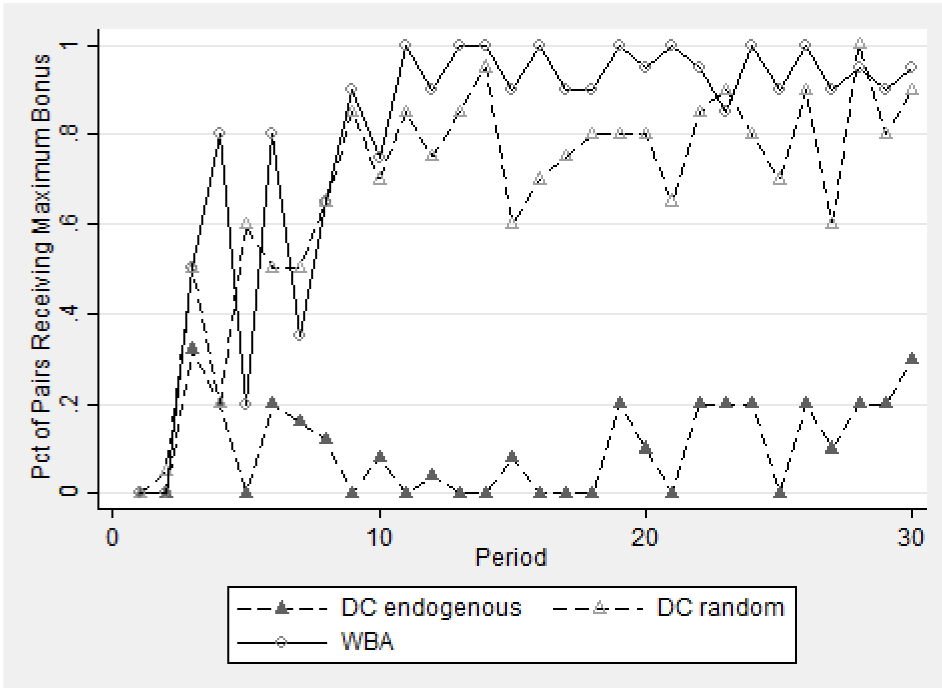

Figure 4: Full Bonus Attainment by Period

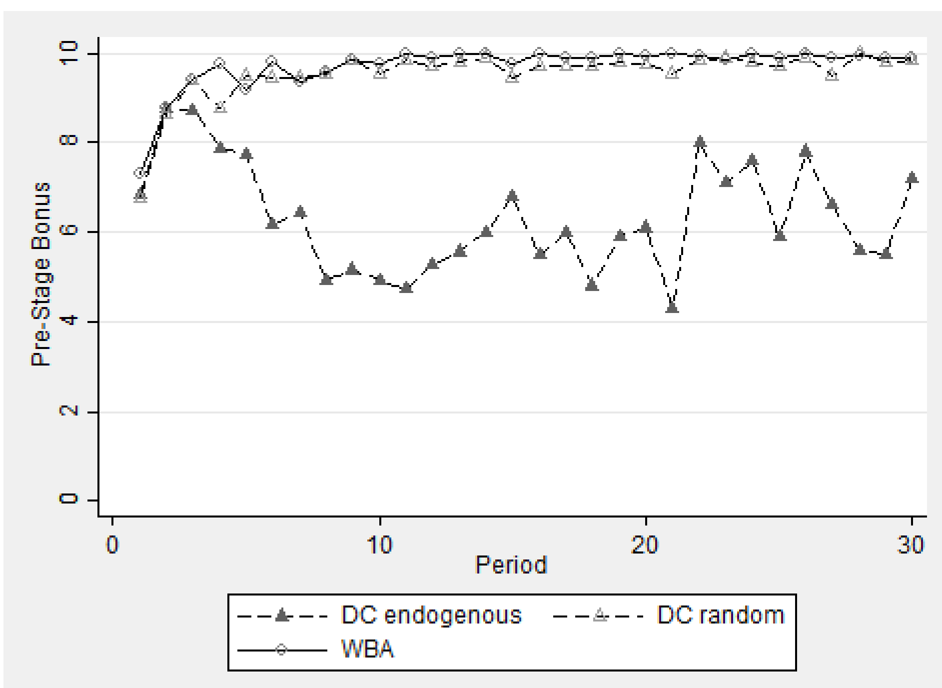

Figure 5: Pre-stage Bonus by Period

the DCE and the other treatments intensify over time. In all periods 8 and after, a greater number of pairs receive the full bonuses amount in either DCR or WBA than under the DCE ($p < 0.05$, for each period 8–30, Fischer exact test).

Table 3, Column (2) provides a similar regression specification of the amount of the bonus attained by each subject-pair. Consistent with the theoretical predictions, both the DCR and WBA treatments are associated with a more than 3 point increase in the bonus relative to the DCE treatment ($p < 0.01$). Figure 5 shows the average bonus amount per period in each of the three mechanisms. The differences between the DCE and the other treatments intensify over time. In all periods 5 and after, the median amount of the bonus in each pair under either DCR or WBA is higher than the DCE ($p < 0.05$, for each period 5–30, Mann-Whitney-Wilcoxon test).

Result 1 (War of Attrition) *Consistent with our model, there is a higher transaction bonus in the WBA and DCR treatments than the DCE.*

While the comparative statics are consistent with the underlying model, a look at the absolute numbers in Figure 5 reveals some slight differences from the absolute predictions of theory. Average bonus amounts in the WBA and DCR differ by 0.3–0.45 seconds from the full bonus amount, a difference that does not appear to be economically meaningful. Bonuses in the DCE treatment have a mean value of 6.321, however, more than two-seconds greater than the predicted mean value of 4.271 in the risk-neutral equilibrium ($p < 0.001$, Table 3, regression (2), no period dummy specification, not shown). Thus, on average, subjects under-persist in this particular war of attrition relative to the equilibrium prediction. Recall that previous experimental tests of the war of attrition are varied on whether subjects persist too long, too little, or in line with theoretical predictions (see Section 1.1).

4.2 Exit Mechanism—Efficiency and Earnings

After the decision to break-up had been made, subjects used an exit mechanism to determine who should receive the valuable good. That mechanism would be considered (ex-post) efficient if the subject who had the higher value for item obtained it. Table 3, column (3) provides a linear probability model regression of this measure of allocative efficiency. The estimates on the treatment dummy variable indicate no statistically significant difference between treatments. This result is not surprising for the DCE and DCR treatments because they use an identical exit mechanism. The fact the WBA does not attain higher levels of efficiency is not consistent with theory, but is entirely consistent with previous research in this area (Kittsteiner et al., 2012, find the same result). In contrast to that previous literature which explained the inefficiency due to subjects' propensity to bid their valuation, we rarely observe that result, if anything, subjects slightly *underbid* relative to the risk-neutral Nash equilibrium level than *overbid* (see Result 3).[21]

Result 2 (Allocative Efficiency) *In contrast to the theoretical prediction, there is no difference in levels of allocative efficiency attained by the WBA versus the two DC treatments.*

Figures 6(a-c) provide a heatmap of all allocative decisions in the three treatments. In the WBA these are the bidding decisions of both players, in the DCE and DCR this is the allocative choice by the Divider. Theory provides bounds of these decisions shown by the two lines, representing the risk-neutral and minmax bidding behavior. The lines form a region on the graphs that represents, for a given valuation draw, all equilibrium-based, allocative behavior that could be explained with risk-preferences alone. The heatmaps show similar patterns in i) these regions feature a higher concentration of subject behavior than other areas of the graph; ii) deviations from these regions are disproportionately below rather than above. This is consistent with underbidding in the WBA mechanism and lower-than-predicted transfer amounts given to the agent without the item in the Divide-and-Choose mechanism.

Table 4 provides a comparison of observed frequencies in the equilibrium-predicted regions and the comparative frequencies should subject decisions be decided uniformly at random. Note that this null hypothesis is equivalent to the heatmap charts being identically colored across Figures 6(a-c), or alternatively by calculating the fraction of the total chart area the equilibrium-region occupies. In all six sessions, the proportion of subject play within the theoretically predicted regions is greater than what random chance would dictate. A non-parametric binomial test—treating each session as an observation—would reject the null hypothesis of subject play being randomly distributed in this region at the 2^{-6} (one-tailed) or 2^{-5} (two-tailed) levels, respectively ($p < 0.05$, in both cases).

Result 3 (Expected Behavior) *Not all subject choices are consistent with the range predicted by the theoretical model: some subjects bid outside the bounds created by the risk-neutral and minmax predictions. In other words, risk preferences cannot account for all subject deviations from equi-*

[21] There are fundamental differences in the auction structures of our WBA from Kittsteiner et al. (2012). In their experiment, subjects are told their valuation for the entire company, but are only bidding on the half of the company they do not own. We simplify things and do not have that distinction: subjects are told their specific value for the item they wish to acquire. This design change may reduce the propensity of subjects to overbid simply by removing the possibility for subjects to conflate these two values.

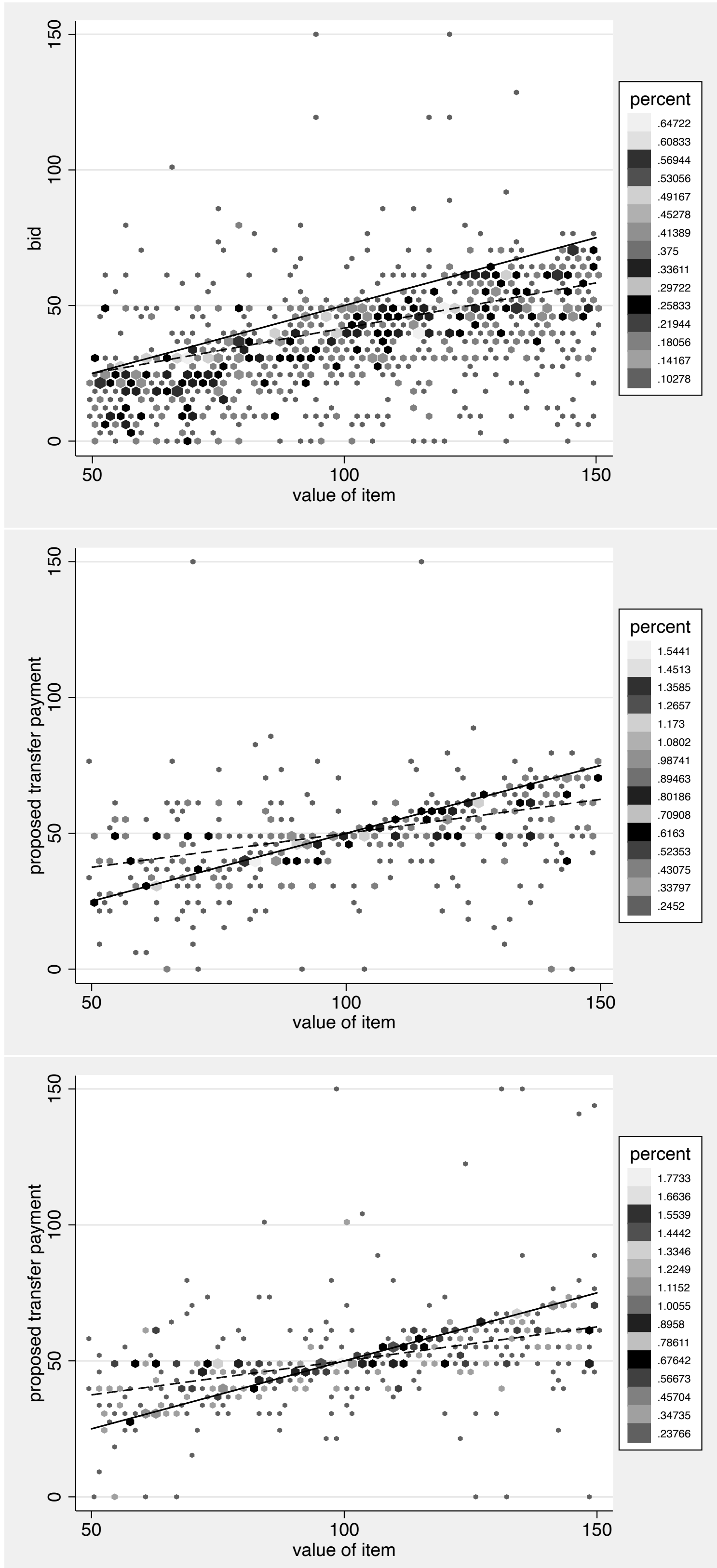

Figure 6: (a, top) Subject bidding decisions in WBA. (b, middle) Divider allocation decisions in DCE. (c, bottom) Divider allocation decisions in DCR.

session	relevant observations	observations that fall in theoretically predicted region	percent	fraction of total strategy space that is rationalizable by theory	exceeds uniform random prediction?
DCE-1	255	58	0.227	0.063	Y
WBA-1	720	183	0.254	0.063	Y
DCR-1	300	113	0.377	0.055	Y
DCE-2	300	92	0.307	0.063	Y
DCR-2	300	70	0.233	0.063	Y
WBA-2	480	116	0.242	0.055	Y

Table 4: Observance of subject play falling in equilibrium-predicted regions by session.

librium strategy. However, across all sessions, the regions of behavior predicted by the theoretical model house a much greater proportion of subject play than uniform random behavior would dictate.

A crucial consideration for testing subjects' ability to backward induct the prize under this model is the existence of an earnings differential for the subjects who triggered the break-up in the exit stage. Recall, subjects would not want to trigger a break-up in the exit stage of the DCE mechanism because it would put them in a disadvantageous strategic situation; there is no similar disadvantageous situation in the DCR or WBA. It is essential we confirm this finding empirically for our model to be valid. Regressions (2) and (3) in Table 4 examine subject earnings on the treatment interactions with the act of triggering the break-up. Consistent with the comparative statics of theory, subjects who triggered the break-up in the DCE mechanism earned 36 less points than subjects who didn't ($p < 0.01$).[22]

While the sign of the number is comforting, the magnitude is a bit surprising. First, the theoretical prediction of first-mover advantage in this game is only 11.459. While Dividers, *on average*, play the risk neutral equilibrium (see Result 3), the dispersion of their choices is responsible for the greater magnitude of the first-mover disadvantage.

In contrast, subjects in the DCR random treatment earned statistically identical amounts to those who did not trigger the break-up ($-36.141 + 34.093 = -2.049$, $p \approx 0.65$). Subjects in the WBA who triggered the break up earned slightly more ($-36.141 + 40.090 = 3.949$, $p < 0.05$), but this effect is roughly an order of magnitude lower than the DCE treatment. All effects remain after controlling for gender and subject survey responses to risk-aversion question and CRT questions.[23]

[22] Note that we have omitted the 105 subject decisions where a subject was assigned the role of Divider in the DCE at random (because no subject triggered the exit). Because these are strategically disadvantageous situations—on average, these 105 Dividers do 42 points worse than their corresponding Choosers—counting them would change the coefficients in regressions (2) and (3) to -31.404 and -31.586 (from -36.141 and -36.432). Because there is no meaningful change in standard errors, the level of statistical significance would remain the same.

[23] Each additional question correct on the CRT (out of 3) is associated with a 2-point increase in subject earnings

	(1) earnings	(2) earnings	(3) earnings
DCR	-1.131	-18.385***	-17.708***
	(0.786)	(1.578)	(1.787)
WBA	-0.393	-20.645***	-19.965***
	(0.392)	(1.364)	(1.269)
Triggered break-up		-36.141***	-36.432***
		(1.772)	(1.422)
DCR ×		34.093***	33.873***
triggered break-up		(4.844)	(4.540)
WBA ×		40.090***	40.196***
triggered break-up		(2.565)	(2.075)
Gender and survey controls?	N	N	Y
observation level[a]	decision	decision	decision
observations	3405	3405	3405
r-squared	0.002	0.083	0.090

Notes: All three regressions use subject-level random effects. Period dummy variables are also included in all regressions. Alternate specifications with continuous period variable or no period variable do not appreciably change results.

[a] All three regression models use cluster-robust standard errors at the session level.

Table 5: Regression Analysis of Subject Period Earnings on Treatment

In short, there is little empirical, payoff-based evidence to support a war of attrition in the first stage of either of these treatments.

Hypothesis 4 provides a strict payoff ranking of all subjects earnings in the allocative stage of the experiments. Average subject earnings should be highest for the Chooser in either DC treatment, next highest in the WBA treatment, then lowest for the Divider in either DC treatment. We have already confirmed that for the DCE treatment that earnings are higher for the Chooser role than Divider. We also see that subjects in the WBA auction earn 20.645 less than the Chooser but 19.445 ($40.090 + -20.645$) more than the Divider in the DCE ($p < 0.001$, both comparisons). Thus payoffs in the WBA fall between payoffs for subjects in the Chooser role and payoffs for the Divider role, inline with the theoretical prediction.

Over time however, as Figure 7 shows, the payoff differential between Dividers and Choosers in both DC treatments is reduced. The mean session-level difference between types in the first half of the experiment is 44 points; it is 27 points over the second half of the experiment. All four sessions show reduced earnings differential in the second half of the experiment ($p < 0.10$ signed-rank test). In contrast, there is not a consistent trend in payoffs over the two halves of the experiment in the two WBA sessions.

($p < 0.01$). Changing one's preferred gamble from the nth to the $(n + 1)$th most risky (out of 6) is associated with a 1-point decrease in subject earnings, though the effect is not significant ($p \approx 0.23$).

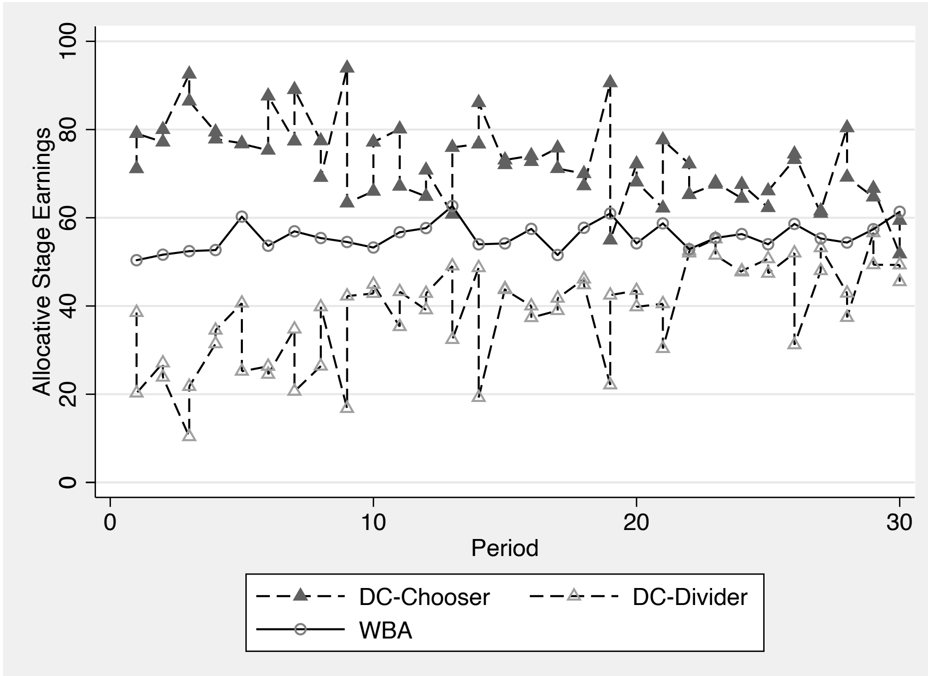

Figure 7: Allocative stage profits for by Period for Dividers (DCR and DCE pooled), Choosers (DCR and DCE pooled), and WBA subjects.

Result 4 *(Payoff Ranking) Consistent with theory, average payoffs for subjects in the Divider role are less than in the Chooser role in both DCR and DCE treatments. Also consistent with theory, average payoffs for subjects in the WBA treatment fall between those in the Chooser role and those in the Divider role.*

To summarize, our results are largely consistent with the predictions of theory. Being a Divider in the Divide and Choose exit mechanism is associated with lower earnings. A triggering rule that assigns that role to the agent who triggers the break-up features more prolonged break-ups, consistent with the idea that the rule disincentivizes break-ups and creates a war of attrition. While there is evidence of suboptimal play, it does not alter the comparative statics of the underlying theory for the break-up stage. The major departures from theory concern the lower efficiency of the auction mechanism and, to a lesser extent, the under-persistence in the war of attrition, both of which have precedence in past experimental literature.

5 Discussion

Partnerships form and sometimes they need to be dissolved. Exit mechanisms play an important role in this process. They can create a transparent set of rules that greatly simplifies the process for dividing joint assets. Ultimately, however, these mechanisms must be built into the partnership contract and partners will need to specify the conditions under which exit mechanisms need to be triggered. There is a very small subset of the partnership dissolution literature that recognizes this issue. All of it makes similar informational assumptions on parties before they trigger a breakup. We depart from this assumption, developing a simple theoretical model for looking at the consequences of pairing certain classes of triggering rules with certain types of exit mechanisms. The model suggested that certain pairings give rise to wars of attrition while other pairings avoid this type of outcome. The experimental results provide strong support for these predictions and illustrate the robustness of these claims.

Our model assumed a particular information structure. It is important to note that the Divide and Choose with an endogenous triggering rule can also promote inefficiencies under different information structures. For instance, a similar war of attrition occurs when parties are informed of private or common-value valuations before rather than after the breakup, though the war may have the perverse benefit of improving allocative efficiency (see de Frutos and Kittsteiner, 2008). A complete information structure would be problematic but for opposite reasons; individuals would be incentivized to break-up even efficient partnerships to gain a first-mover advantage. Both environments could be remedied with ex-ante payoff symmetric mechanisms. For instance, de Frutos and Kittsteiner (2008) give some endorsement to a modified Divide and Choose mechanism where the first stage features an auction to be the second mover. Given the sub-optimality of subject bidding in our results and others and the failure of subjects to fully account for the magnitude of the first-mover disadvantage in the DCE treatment, this mechanism may work better in theory than in practice.

The asymmetric information structure is the most tricky. If the costs of inefficiency are mainly borne on the uninformed party,[24] we may see similar results. It is strategically disadvantageous for the uniformed party to move first (Brooks et al., 2010; Landeo and Spier, 2013), and this may disincentivize an efficient break-up. While the informed party is not so disadvantaged, it may have incentives to prolong the inefficient partnership as well. Brooks et al. (2010) show the informed party can hold-out from triggering dissolution and instead make ultimatum-style buy-out offers to the uninformed, capturing more economic rents than had it triggered the mechanism. It is less clear to what extent ex-ante payoff symmetric mechanisms would help in these cases. Though Landeo

[24]Note that reasons for dissolution may be for reasons independent of their information realizations (see Section 1).

and Spier (2013) propose that a court should exogenously determine who is the informed party; if this determination is made independent of break-up decision, it would also be ex-ante payoff symmetric.

The question remains why the Divide and Choose rule with this specific triggering rule continues to be the predominant break-up mechanism used in partnership-dissolution litigation and contracts.[25] We suspect the development of these policies was focused on the most benign case, an asymmetric information environment where the informed party triggers the break-up.[26] We as well as others argue that this is not the only possible information structure. Further, the combined mechanism and triggering rule may introduce new, perverse, incentives on partners. However, once a mechanism like this becomes the standard, its use can snowball to cases where it is particularly ill-suited.[27]

It is not without precedent for the law based on an idealized example to fail in cases that do not match that example. Fault-based divorce legislation provides an interesting parallel. Divorce once required the consent of both parties, putting the party most desiring divorce at a strategic disadvantage. Presumably, there are cases where putting the breaker-upper at a disadvantage makes sense. The ultimate effect, however, was very different from intended; relative to no-fault divorce, the law preserves destructive marriages (Stevenson and Wolfers, 2006). Though the stakes are obviously different, we urge legal professionals to be wary of ex-ante payoff dominant mechanisms. In comparison to the ex-ante payoff symmetric, such mechanisms may also preserve inefficient partnerships.

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[25] Another possibility is that these inefficiencies function as a commitment device (e.g. Strotz, 1955) for the dynamically inconsistent who wish to leave partnerships early, or as signaling devices for types to reveal their intentions to commit to the partnership (e.g. Spence, 1973). While we cannot exclude these possibilities, we note they have not been discussed in the partnership dissolution literature.

[26] A oft-cited line by Circuit Judge Frank Easterbrook that this mechanism "keeps the first mover honest" (Valinote v. Ballis; 295 F3d. 666; 2002) echoes the sentiment that this specific informational structure was focus of the legal policies.

[27] This herding mentality is exemplified in de Frutos and Kittsteiner (2008). They suggest a "...lawyer who fails to recommend to his clients adopting [the Divide and Choose provision] could be accused of malpractice. (p. 184–185)"

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A Appendix

In this section, we provide the proofs for the propositions and more detailed equilibrium predictions for Divide and Choose and the Winner's Bid Auction.

A.1 Proofs of Propositions

Proof of Proposition 1. Since the mechanism is ex-ante payoff symmetric we have that $v = v_{r_1} = v_{r_2}$. Thus, any probability distribution over roles induced by τ yields an ex-ante expected payoff of v from the exit mechanism regardless of (t_1, t_2). Next, since we have a first-mover trigger rule, the date of dissolution is $t = \min\{t_1, t_2\}$. If partner j chooses a break date of t_j, then i's expected payoff of choosing to stop at time t_i is

$$v - C(\min\{t_i, t_j\})$$

This is clearly maximized at $t_i = 0$ for all t_j. ■

Proof of Proposition 2. Suppose the triggering rule assigns player i to the roles r_1 and r_2 according to probabilities q_i and $1 - q_i$ respectively. Let $v_i = q_i v_{r_1} + (1 - q_i) v_{r_2}$. Thus, whenever the partnership is dissolved, i has an ex-ante expected payoff of v_i from the exit mechanism regardless of (t_1, t_2). Next, since we have a first-mover trigger rule, the date of dissolution is $t = \min\{t_1, t_2\}$. If partner j chooses a break date of t_j, then i's expected payoff of choosing to stop at time t_i is

$$v_i - C(\min\{t_i, t_j\}).$$

and, again, is clearly maximized at $t_i = 0$ for all t_j. ■

Proof of Proposition 3. First, without loss of generality we can assume that the payoff dominant role in the mechanism is r_1. Thus, we have that $\pi_{r_1} > \pi_{r_2}$ and we can define the expected payoff of the first mover and second mover to be

$$\begin{aligned} v_F &= q\pi_{r_1} + (1 - q)\pi_{r_2} \\ v_S &= (1 - q)\pi_{r_1} + q\pi_{r_2} \end{aligned}$$

respectively. Since $q < \frac{1}{2}$ we have $v_S > v_F$.

Now suppose partner j chooses his stopping time according to the distribution G on $[0, \infty)$ with density g. Then the expected payoff to i of choosing to stop at time t is

$$u^i = \int_0^t (v_S - C(z))g(z)dz + \int_t^\infty (v_F - C(t))\, g(z)dz.$$

If each partner chooses their break time according to the distribution

$$\begin{aligned} G(t) &= 1 - \exp\left\{-\frac{C(t)}{v_S - v_F}\right\} \\ &= 1 - \exp\left\{-\frac{C(t)}{(1-2q)\,\pi_{r_1} + (2q-1)\pi_{r_2}}\right\} \end{aligned}$$

then each player is indifferent between stopping at any time t.[28] It is therefore also optimal to choose a stop time according to G. ■

In the experiment, we used a finite time horizon T that was known to all of the players. Proposition 4 presents the war of attrition equilibrium for the finite time horizon case.[29]

Proposition 4 *Consider the Break-Up Game where the exit mechanism is divide and choose and where the first mover is selected to be divider. If there is a time* $t^* \in [0, T]$ *where the payoff to being the first mover is equal to the payoff of the partners making simultaneous announcements at the last possible time period (i.e.,* $-t^* = \frac{1}{2}(v_C - v_D) - T$*), then in the first stage of the Break-Up Game it is optimal for both players mix according to the function*

$$B(t) = 1 - \exp\{-\frac{t}{v_C - v_D}\}$$

on $[0, t^*]$ *and with probability* $1 - B(t^*)$ *on time* $\{T\}$.

Proof of Proposition 4. Fix equilibrium behavior in the exit mechanism and suppose that Bidder 2 mixes in the first stage according to B on $[0, t^*]$ and the remainder on $\{T\}$. Player 1's expected payoff from choosing $t \leq t^*$ is

$$\int_0^t [(v_C - v_D) - m]\, B'(m)dm - \int_t^{t^*} tB'(m)dm + (1 - B(t^*))\,(-t)\,.$$

[28] The second equality follows since

$$\begin{aligned} v_S - v_F &= ((1-q)\pi_{r_1} + q\pi_{r_2}) - (q\pi_{r_1} + (1-q)\pi_{r_2}) \\ &= (1-2q)\,\pi_{r_1} + (2q-1)\pi_{r_2} > 0. \end{aligned}$$

[29] The following proposition is an application of Theorem 3 found in Hendricks, Weiss, and Wilson (1988) to our Break-up Game.

The first order condition is

$$\begin{aligned}
\left[(v_C - v_D) - t\right] B'(t) - \int_t^{t^*} B'(m)dm + tB'(t) - (1 - B(t^*)) &= 0 \\
&\rightarrow \\
B'(t) + \frac{1}{v_C - v_D} B(t) &= \frac{1}{v_C - v_D}
\end{aligned}$$

The solution of this differential equation is

$$B(t) = 1 - \exp\{-\frac{t}{v_C - v_D}\}.$$

Hence, if Bidder 2 is mixing according to B player 1 is indifferent between all $t \leq t^*$. In particular, he expects a payoff of

$$\int_0^{t^*} \left[(v_C - v_D) - m\right] B'(m)dm - (1 - B(t^*))t^*.$$

Bidding $t^* < t < T$ is clearly dominated. Now suppose Bidder 1 bids $t = T$, then his expected profit is

$$\int_0^{t^*} \left[(v_C - v_D) - m\right] B'(m)dm + (1 - B(t^*)) \left(\frac{1}{2}(v_C - v_D) - T\right).$$

In order to mix on $[0, t^*]$ and the remainder on $\{T\}$ Bidder 1 must be indifferent between the two expected payoffs – i.e., if

$$-t^* = \left(\frac{1}{2}(v_C - v_D) - T\right)$$

in other words, if the payoff to being the first mover at time t^* is equal to the payoff of being simultaneous at time T. The symmetric argument applies to Bidder 2. ■

A.2 Equilibrium Predictions for Divide and Choose

In Divide and Choose there are two players: the Divider and the Chooser. The Divider chooses a price p. The Chooser observes this price and then decides whether to buy the company at price p (giving p to the Divider) or sell the company to the other player for a price of p. We now derive an equilibrium for the game induced by these rules where we suppose the Divider and Chooser's values are private information and independently drawn according to the distribution F on $[\underline{x}, \bar{x}]$ with pdf f.

We start with the equilibrium behavior of the Chooser. At any price p, the Chooser with value

x_C should choose to buy (sell) if the value he obtains from buying (selling) exceeds the price (payoff) he would receive by selling (buying). This gives the equilibrium strategy

$$S(x_C;p) = \begin{cases} \text{Buy} & \text{if } \frac{1}{2}x_C \geq p \\ \text{Sell} & \text{if } \frac{1}{2}x_C < p \end{cases}$$

which is independent of the distribution F. Working back in the game, the Divider knows the chooser will follow S. Hence, in selecting his price offer, the Divider with value x_D needs to solve

$$\max_p \int_{2p}^{\bar{x}} pf(z)dz + \int_{\underline{x}}^{2p} [x_D - p]\, f(z)dz.$$

Thus, in equilibrium, the Divider should choose p^* to satisfy

$$p^* = \frac{1}{2}x_D + \frac{1 - 2F(2p^*)}{4f(2p^*)}.$$

Example 3 *In the experiment, values were drawn uniform on [50, 150]. So, the cdf is* $F(x) = \int_{50}^{x} \frac{1}{100}dz = \frac{1}{100}x - \frac{1}{2}$ *and the pdf is* $f(x) = \frac{1}{100}$*. The risk neutral equilibrium bid function for the Divider is*

$$\begin{aligned} p^*(x_D) &= \frac{1}{2}x_D + \frac{1 - 2F(2p^*)}{4f(2p^*)} \\ &= \frac{1}{4}x_D + 25 \end{aligned}$$

The Divider's interim expected payoff is

$$\pi_D(x_D) = \frac{1}{400}x_D^2 + 25$$

which yields an ex-ante expected payoff of 52.083 *. The Chooser"s interim expected payoff is*

$$\pi_C(x_C) = \begin{cases} 50 & \text{if } x_C < 75 \\ \frac{1}{100}x_C^2 - \frac{3}{2}x_C + \frac{425}{4} & \text{if } 75 < x_C < 150 \\ x_C - 50 & \text{if } x_C > 125 \end{cases}$$

which yields an ex-ante expected payoff of 63.542*.*

A.3 Equilibrium Predictions in the WBA

In the WBA, bidders simultaneously submit bids to an auctioneer. The high bidder wins the company and pays the losing bidder his bid. We now derive a symmetric equilibrium for the game induced by this auction in increasing bid strategies where the bidders' values are private information and each drawn independently according to the distribution F on $[\underline{x}, \bar{x}]$ with pdf f.

Assume β is a symmetric risk neutral equilibrium in increasing bidding strategies and that Bidder 2 follows this strategy. We consider Bidder 1's best response problem when his type is x

$$\max_b \int_{\underline{x}}^{\beta^{-1}(b)} [x-b]\, f(z)dz + \int_{\beta^{-1}(b)}^{\bar{x}} \beta(z)f(z)dz.$$

The first order condition is

$$\frac{1}{\beta'(\beta^{-1}(b))}\, [x-2b]\, f(\beta^{-1}(b)) - F(\beta^{-1}(b)) = 0$$

In a symmetric equilibrium, Bidder 1 needs to choose $b = \beta(x)$ so the first order condition simplifies to the differential equation

$$\beta'(x) = [x - 2\beta(x)]\, \frac{f(x)}{F(x)}.$$

If we re-write the differential equation and apply an integrating factor, the above differential equation is expressed by

$$\begin{aligned} \beta'(x)F(x)^2 + 2\beta(x)f(x)F(x) &= xf(x)F(x) \\ &\text{or} \\ \frac{d}{dx}\left(\beta(x)F(x)^2\right) &= xf(x)F(x). \end{aligned}$$

Hence, from the Fundamental Theorem of Calculus, we have

$$\beta(x)F(x)^2 = \int_{\underline{x}}^{x} mf(m)F(m)dm + C,$$

where C is a constant. Since the left hand side of the above equation is zero when $x = \underline{x}$ we deduce that $C = 0$ as well. Hence the equilibrium bid function is

$$\beta(x) = \frac{\int_{\underline{x}}^{x} mf(m)F(m)dm}{F(x)^2}.$$

Example 4 *In the experiment, values were drawn uniform on [50, 150]. So, the cdf is* $F(x) = \int_{50}^{x} \frac{1}{100} dz = \frac{1}{100}x - \frac{1}{2}$ *and the pdf is* $f(x) = \frac{1}{100}$*. The risk neutral equilibrium bid function is*

$$\begin{aligned} \beta(x) &= \frac{\int_{50}^{x} m \frac{1}{100}(\frac{1}{100}m - \frac{1}{2})dm}{(\frac{1}{100}x - \frac{1}{2})^2} \\ &= \frac{1}{3}x + \frac{25}{3} \end{aligned}$$

The interim expected payoff of a bidder with type x *in the WBA is*

$$\begin{aligned} &\int_{50}^{x} (\frac{2}{3}x - \frac{25}{3}) \frac{1}{100} dz + \int_{x}^{150} (\frac{z}{3} + \frac{25}{3}) \frac{1}{100} dz \\ = &\ \frac{1}{200}x^2 - \frac{1}{2}x + \frac{325}{6} \end{aligned}$$

This yields an ex-ante expected payoff of

$$\int_{50}^{150} \left(\frac{1}{200}x^2 - \frac{1}{2}x + \frac{325}{6} \right) \frac{1}{100} dx = 58.333.$$

The efficient ex-ante surplus of the bidders is the expected type of the highest bidder or

$$2 \int_{50}^{150} \int_{50}^{x} x \left(\frac{1}{100} \right) \left(\frac{1}{100} \right) dy dx = 116.67$$

which is twice the ex-ante expected payoff of the WBA or $(58.333)\,2 = 116.67$*.*