Physics 201, Exam 3 **Name (printed)** Solution

On my honor as a Texas A&M University student, I will neither give nor receive unauthorized help on this exam.

Name (signed) ____________________

The multiple-choice problems carry no partial credit.
Circle the correct answer or answers. **An answer is approximately correct if it is correct to 2 significant figures.**
In the work-out problems, you are graded on your work, with partial credit.
(The answer by itself is not enough, and you receive credit only for your work.)
Be sure to include the correct units in the answers, and give your work in the space provided.

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1. (5) Our Foucault pendulum in the Mitchell Institute Building next door has a mass of 180 kg and a period of 10.2 s. (Go time it yourself after the exam!) It swings with an amplitude of 2 m. Calculate its length.

(a) about 6 m
(b) about 16 m
(c) about 26 m
(d) about 36 m
(e) about 46 m
(f) about 56 m

$$T = 2\pi\sqrt{\frac{L}{g}}$$
$$\Rightarrow \boxed{L =} g\left(\frac{T}{2\pi}\right)^2$$
$$= (9.8)\left(\frac{10.2}{2\pi}\right)^2 m$$
$$= \boxed{25.8\ m}$$

[This photo is from the interesting blog by Texas A&M graduate student Matt Springer at http://scienceblogs.com/builtonfacts/.]

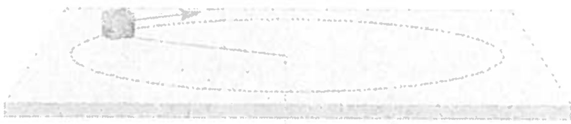
2. (5) In one of our demonstrations in class, brave volunteers (with feet off the floor) seized a rotating bicycle wheel, and then twisted it so that the axis of rotation was shifted by 90°. The effect observed was due to:

(a) an increase in the angular speed of rotation of the wheel
(b) a decrease in the angular speed of rotation of the wheel
(c) an increase in the moment of inertia
(d) a decrease in the moment of inertia
(e) the fact that angular momentum is a scalar quantity
(f) the fact that angular momentum is a vector quantity

3. (5) Suppose that you increase the amplitude of a mass vibrating on a spring with simple harmonic motion. Which of the following statements about this mass are correct?
(There may be more than one correct choice.)

(a) Its maximum speed increases.
(b) Its period of oscillation increases.
(c) Its maximum acceleration increases.
(d) Its maximum kinetic energy increases.
(e) Its maximum potential energy increases.

4. (5) A block of mass m is being whirled around on a frictionless surface (as shown), in a circular path of radius r. Its initial velocity is v. If it is pulled in so that the new radius is $r/2$, what is its new velocity?



(a) $2v$
(b) $4v$
(c) $8v$
(d) $v/2$
(e) $v/4$
(f) $v/8$
(g) $\sqrt{2}\,v$
(h) $v/\sqrt{2}$
(i) v

no torque $\Rightarrow$ ang. mom. constant with $r' = \frac{1}{2} r$

$\Rightarrow m v' r' = m v r$

$\Rightarrow m v' (\frac{1}{2} r) = m v r$

$\Rightarrow \boxed{v' = 2v}$

5. (5) In the preceding problem, the initial tension in the string is T. What is the final tension after the block is pulled in to the new radius of $r/2$?

(a) $2T$
(b) $4T$
(c) $8T$
(c) $T/2$
(d) $T/4$
(e) $T/8$
(f) $\sqrt{2}\,T$
(g) $T/\sqrt{2}$
(h) T

$T = m \frac{v^2}{r}$

$\boxed{T' =} m \frac{v'^2}{r'}$

$= m \frac{(2v)^2}{\frac{1}{2} r}$

$= 8 m \frac{v^2}{r}$

$= \boxed{8T}$

6. (5) For an ultrasonic transducer with a period of oscillation of $0.15\,\mu s = 0.15 \times 10^{-6}\,s$, what is the approximate angular frequency ω? (Recall that $1\text{ MHz} = 10^6\text{ Hz}$.)

(a) 0.94 MHz
(b) 9.4 MHz
(c) 6.7 MHz
(d) 67 MHz
(e) 41.9 MHz
(f) 419 MHz

$\omega = \frac{2\pi}{T} = \frac{2\pi}{0.15 \times 10^{-6}\,s} = 41.9 \times 10^{6}\,Hz = 41.9$ M

7. (12) A steel wire 2.00 m long with a circular cross section must stretch no more than 0.20 cm when a 400 N weight is hung from one of its ends. Calculate the minimum diameter that this wire must have.

Young's modulus of steel $= 2.0 \times 10^{11}$ Pa

minimum diameter = 1.6×10^{-3} m

$$Y = \frac{F_\perp / A}{\Delta \ell / \ell_0} \quad , \quad \frac{\Delta \ell}{\ell_0} = \frac{0.20 \times 10^{-2}\ m}{2.00\ m} = 1.0 \times 10^{-3}$$

$$\Rightarrow A = \frac{1}{Y} \frac{F_\perp}{\Delta \ell / \ell_0}$$

$$= \frac{1}{2.0 \times 10^{11}\ N/m^2} \frac{400\ N}{1.0 \times 10^{-3}}$$

$$\Rightarrow \pi r^2 = 2.0 \times 10^{-6}\ m^2$$

$$\Rightarrow r = \sqrt{\frac{2.0 \times 10^{-6}}{\pi}}\ m = 8.0 \times 10^{-4}\ m$$

$$\Rightarrow \boxed{2r = 1.6 \times 10^{-3}\ m = 1.6\ mm}$$

[other version: 1.12 mm]

8. A wheel on a stationary bicycle at the Student Recreation Center is rotating at 2 rad/s when $t = 0$, where t is the time.

Former President George H. W. Bush is exercising on it. (I once was there at the same time as him, and he was indeed working out on a stationary bicycle, reading a book at the same time, all at age 80+.).

He exerts a torque that causes the angular acceleration to be constant and equal to 4 rad/s^2.

(a) (5) Calculate the angular velocity at $t = 3$ s.

angular velocity = $14 \ \frac{\text{rad}}{\text{s}}$

$$\boxed{\omega =} \ \omega_0 + \alpha t$$
$$= 2 \ \frac{\text{rad}}{\text{s}} + \left(4 \ \frac{\text{rad}}{\text{s}^2}\right)(3\text{s})$$
$$= \boxed{14 \ \frac{\text{rad}}{\text{s}}}$$

[other version: 20 rad/s]

(b) (5) Through what total angle $\Delta\theta$, in radians, has the wheel rotated between $t = 0$ and $t = 3$ s?

[In this, and all partial credit problems, please show all your work. Your grade is based entirely on your work as shown, and not the final answer.]

total angle $\Delta\theta$ = 24 rad

$$\boxed{\Delta\theta =} \ \omega_0 t + \frac{1}{2}\alpha t^2$$
$$= (2)(3) + \frac{1}{2}(4)(3)^2$$
$$= \boxed{24 \text{ rad}}$$

[other version: 33 rad]

9. You are watching an object with mass m that is attached to a spring with force constant k that is moving horizontally with simple harmonic motion. When the object is displaced 0.500 m to the right of its equilibrium position, it has a velocity of 2.00 m/s to the right, and an acceleration of 8.00 m/s^2 to the left.

(a) (4) Recalling Hooke's law for the force exerted by the spring on the mass, and Newton's second law relating this force to the mass and acceleration, calculate the ratio m/k.

$\frac{m}{k} =$ $0.0625\ s^2$

$$F = -kx, \quad F = ma \Rightarrow ma = -kx \Rightarrow \boxed{\frac{m}{k} =} -\frac{x}{a}$$

$$= -\frac{0.500\ m}{-8.00\ m/s^2}$$

$$= \boxed{0.0625\ s^2}$$

[other version: $0.125\ s^2$]

(b) (4) Calculate the period of oscillation for this mass and spring.

$$\boxed{T =} 2\pi\sqrt{\frac{m}{k}} = 2\pi\sqrt{0.0625\ s^2} = \boxed{1.57\ s}$$

[other version: $2.22\ s$]

(c) (4) Using conservation of energy, calculate the amplitude of oscillation A.

$A =$ $0.707\ m$

$$\cancel{\tfrac{1}{2}} kx^2 + \cancel{\tfrac{1}{2}} mv^2 = \cancel{\tfrac{1}{2}} kA^2$$

$$\Rightarrow A^2 = x^2 + \frac{m}{k}v^2$$

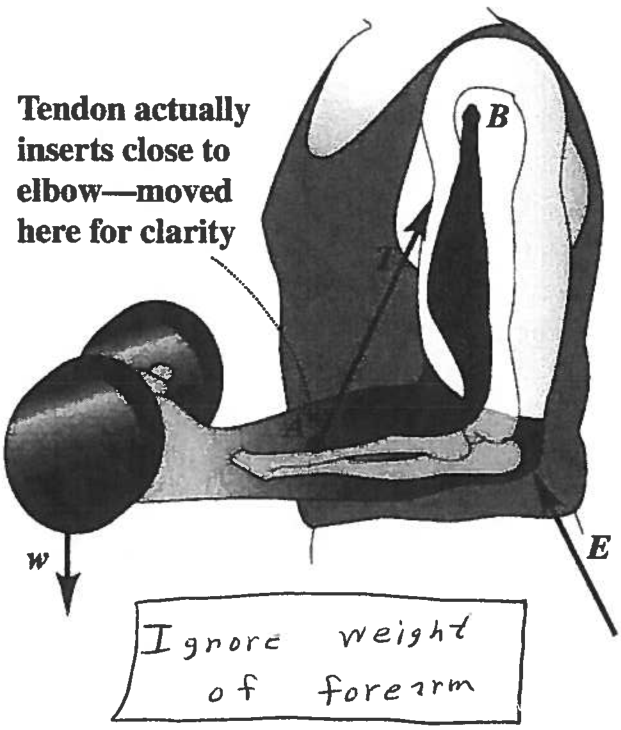
$$\Rightarrow \boxed{A =} \sqrt{x^2 + \frac{m}{k}v^2}$$

$$= \sqrt{(0.500)^2 + (0.0625)(2.00)^2}\ m$$

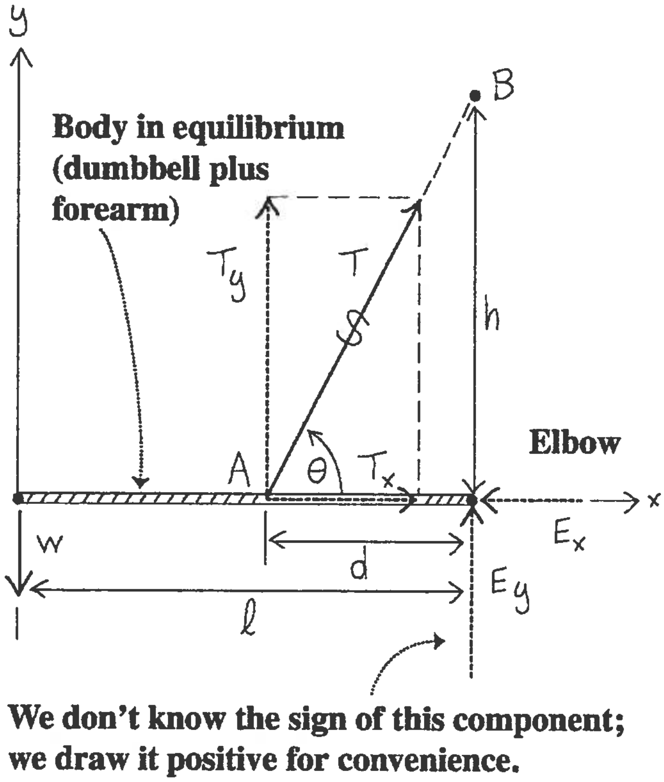
$$= \boxed{0.707\ m}$$

[other version: $0.866\ m$]

10. In the drawing below, neglect the weight of the arm, and
ℓ = length of arm = 0.6 m ,
d = distance from elbow joint to point where tendon is attached = 0.08 m ,
θ = angle of tendon with horizontal = 80° ,
w = weight of dumbbell = 40 N .



(a)



(b)

(a) (4) Calculate T_y, the y component of the tension in the tendon. elbow = center of rotation

T_y = 300 N

torques: $d\,T_y = \ell w \Rightarrow \boxed{T_y =} \dfrac{\ell w}{d}$

$= \dfrac{(0.6)(40)}{0.08}$

$= \boxed{300 \text{ N}}$

(b) (4) Calculate T_x, the x component of the tension in the tendon.

$T_x =$ 53 N

$$\frac{T_y}{T_x} = \tan\theta = \tan 80^\circ = 5.67$$

$$\Rightarrow \boxed{T_x =} \frac{T_y}{5.67} = \boxed{52.9\ N}$$

(c) (4) Calculate E_x, the x component of the force exerted on the forearm at the elbow.

$E_x =$ 53 N

$$\boxed{E_x =} T_x = \boxed{53\ N}$$

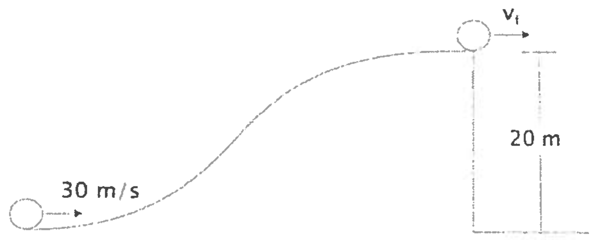
(d) (4) Calculate E_y, the y component of the force exerted on the forearm at the elbow.

$E_y =$ −260 N

$$E_y + T_y = w$$

$$\Rightarrow \boxed{E_y =} w - T_y = 40\ N - 300\ N = \boxed{-260\ N}$$

1 1. A ball begins rolling up a hill as shown, with no slipping or loss of mechanical energy. Its initial velocity in the horizontal flat region is 30 m/s. The moment of inertia of a solid sphere with mass M and radius R is given by $I = \frac{2}{5}MR^2$.



(a) (4) Calculate its velocity v_f just as it rolls horizontally off the edge of the cliff.

$v_f =$ 25 m/s

$$K_i = \frac{1}{2}Mv_{cm,i}^2 + \frac{1}{2}I\omega_i^2 \quad , \quad v_{cm} = v_{tan} = R\omega \Rightarrow \omega = \frac{v_{cm}}{R}$$

$$= \frac{1}{2}Mv_{cm,i}^2 + \frac{1}{2}\left(\frac{2}{5}M\cancel{R^2}\right)\left(\frac{v_{cm,i}}{\cancel{R}}\right)^2$$

$$= \left(\frac{1}{2} + \frac{1}{5}\right)Mv_{cm,i}^2$$

$$= \frac{7}{10}Mv_{cm,i}^2 \quad \text{and} \quad K_f = \frac{7}{10}Mv_{cm,f}^2$$

$$\therefore \frac{7}{10}\cancel{M}v_{cm,f}^2 + Mgh = \frac{7}{10}\cancel{M}v_{cm,i}^2$$

$$\Rightarrow v_{cm,f}^2 = v_{cm,i}^2 - \frac{10}{7}gh = (30)^2 - \frac{10}{7}(9.8)(20) = 620$$

$$\Rightarrow v_{cm,f} = 25\ \frac{m}{s}$$

(b) (4) Calculate the time t that it takes to hit the ground after falling off the cliff.

$t =$ 2.0 s

$$\underbrace{y - y_0}_{= -20\ m} = \underbrace{v_{0y}}_{=0}t + \frac{1}{2}(-g)t^2$$

$$\Rightarrow t^2 = \frac{40\ m}{9.8\ m/s^2} = 4.08\ s^2$$

$$\Rightarrow \boxed{t = 2.0\ s}$$

(c) (4) Calculate the distance from the base of the cliff to the point where the ball hits the ground.

distance from cliff = 50 m

$$\boxed{x - x_0 =} v_{0x} t + \frac{1}{2} \underbrace{a_x}_{=0} t^2$$

$$= (25 \tfrac{m}{s})(2.0 s)$$

$$= \boxed{50 \text{ m}}$$

(d) (4) Calculate the (center of mass) velocity that the ball has when it hits the ground.

velocity at ground = 32 m/s

$$v_y^2 = v_{0y}^2 + 2 a_y (y - y_0) \quad , \quad a_y = -9$$

$$= 0 + 2(-9.8)(-20)$$

$$= 392$$

$$\Rightarrow \boxed{v_y = 20 \tfrac{m}{s}}$$

$$\boxed{v =} \sqrt{v_x^2 + v_y^2} = \sqrt{25^2 + 20^2} = \boxed{32 \text{ m/s}}$$

(e) (4) Explain why the (center of mass) velocity of the ball is larger when it is at ground level on the right than it was at the same height on the left. Does this means that the ball somehow gained energy going up the hill?

less rotational kinetic energy

$\Rightarrow$ more translational kinetic energy

when back at same height

11. (5 points extra credit) Briefly describe how conservation of angular momentum plays a role in each of the following sports.

(i) (American) football

spiral pass provides stability,
so that ball does not tumble

(ii) bicycling

rotation of wheels provides stability
when bicycle is moving

(iii) diving (into a swimming pool)

curled up diver spins faster

(iv) figure skating on ice

figure skater spins faster with arms pulled in

(v) target practice with a rifle

rifling gives spin to bullet,
providing more stable trajectory