HW 12.1. A small perturbation λx^4 is added to the Hamiltonian for the harmonic oscillator. (This represents the anharmonic effects in a real system.)

(a) Calculate the first-order correction to the energy eigenvalues, ΔE_n.

Give your answer in terms of n, λ, $\hbar$, the mass m, and the force constant k.

HW 12.2. For the same problem as above, calculate the leading correction to the ground-state wavefunction.

This involves calculating the matrix element $\langle m|\lambda x^4|0\rangle$ for the lowest intermediate state m which gives a nonvanishing correction in lowest-order perturbation theory.

Give your answer as a coefficient (involving the same parameters as in problem 1 above) times the unperturbed wavefunction $\psi_m(x)$, which is mixed into $\psi_0(x)$ by the perturbation (with m specified, of course). You do not need to normalize the corrected wavefunction to one.

HW 12.3. A hydrogen atom is placed in a uniform static electric field ε that points along the z axis. The (relatively small) perturbation in the Hamiltonian is $-e\varepsilon z$, where e is the fundamental charge. It removes the degeneracy of some of the states, and this phenomenon is called the Stark effect.

Using degenerate perturbation theory, calculate the lowest-order shifts in energy for the four $n=2$ states in hydrogen. Give the answer in terms of e, ε, and the Bohr radius a_0.

You may use the fact that $\langle 2,\ell=0,m=0|z|2,\ell=1,m=0\rangle = -3a_0$.