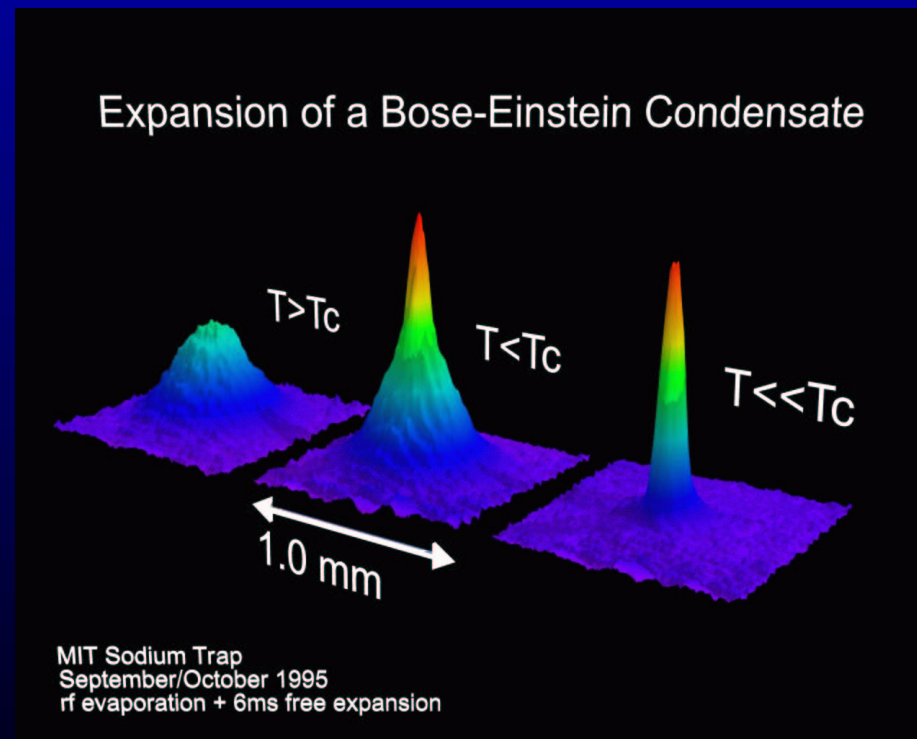


Bose-Einstein Condensate: A New state of matter

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Outline

- **Introductory Concepts**
 - ★ Bosons and Fermions
 - ★ Classical and Quantum Statistics
 - ★ Intuitive picture of Bose-Einstein Condensation (BEC)
 - ★ Statistical mechanics of BECs
 - ★ Experimental techniques to achieve BEC.
- **What can we do with Bose-Einstein condensates?**
 - ★ Coherence in the condensates
 - ★ Atom Lasers
 - ★ Generation of vortex lattices
 - ★ Other cool things that can be done with cold atoms

Bosons and Fermions

- The behavior of a gas of identical particles at low temperature depends on the spin of the particle.

Fermions

- Half-integral spin
- Obey Pauli's Exclusion principle
- No two of them can have the same quantum numbers

Bosons

- Integral spin
- Do not obey Pauli's Exclusion principle
- A large number of them can have the same quantum numbers

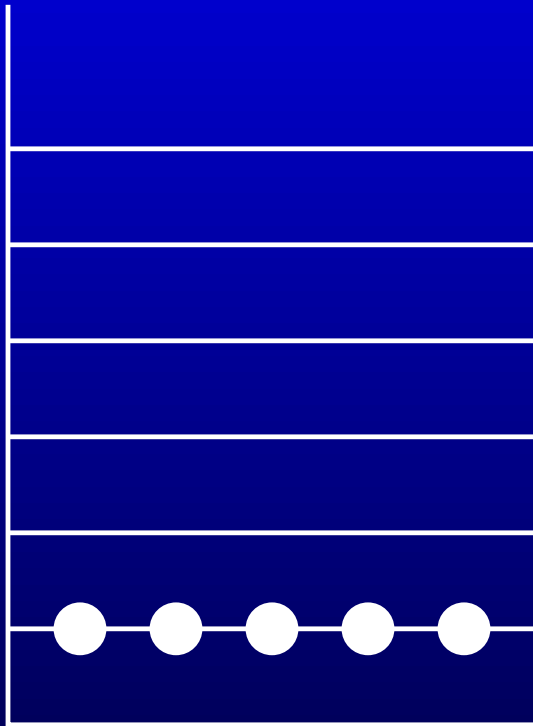
Simple example of bosonic and fermionic atoms

- Atoms consist of protons, neutrons and electrons which are all fermions.
- Atoms will be bosons if the total number of electrons, protons and neutrons add up to be an even number.
- Examples:
 - ★ Hydrogen atom: $1/2 \pm 1/2 = 0, 1$
 - ★ Helium isotope: ${}^4\text{He}$: 4 nucleons and 2 electrons : integer spin
 - ★ Helium isotope: ${}^3\text{He}$: 3 nucleons and 2 electrons : half-integer spin

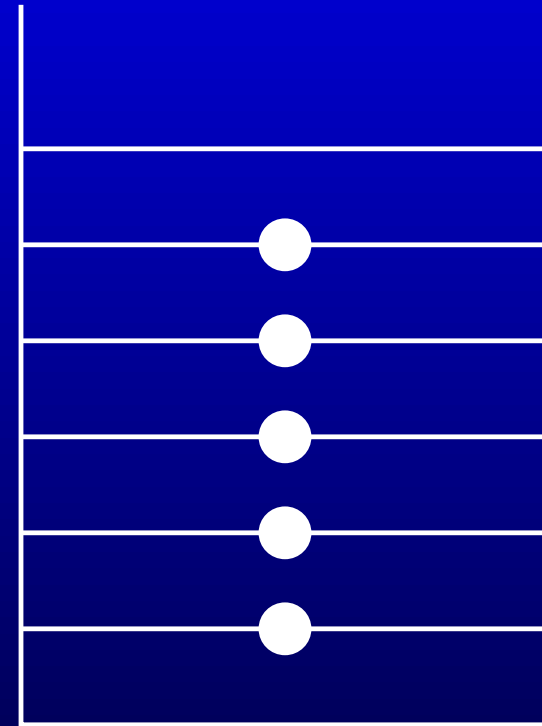
Classical and Quantum Statistics

- Boltzmann Statistics: $p(E_i) \propto \exp(-E_i/k_B T)$
 - ★ Applies at high temperature.
 - ★ Probability of occupation of any individual quantum level is small.
- As we lower the temperature, the atoms tend to occupy the lowest energy levels of the system.
- At low temperatures occupancy factor is no longer small.
- This is the regime where quantum effects become important.
- Spin of the atoms govern the statistics of the system.

Difference between the fermionic and bosonic systems as $T \rightarrow 0$.



Bose gas



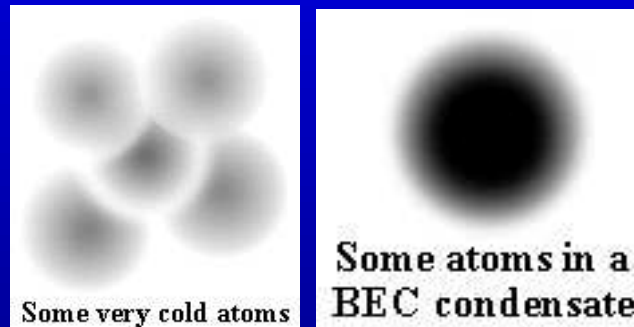
Fermi gas

Historical Introduction



- S. N. Bose in 1924 predicted a different statistics for the light quanta and derived Planck's radiation formula through very simple arguments. Einstein extended this idea to ordinary particles.
- He predicted that even a gas of completely non-interacting particles will undergo a phase transition at a sufficiently low temperatures.
- This phase transition has now come to be known as Bose-Einstein condensation.

Is Laser Cooling enough to obtain Bose-Einstein Condensates?



- The blob of atoms created at the focus of the laser beams through laser cooling techniques is still classical.
- It is governed by the Boltzmann distribution.
- One needs further cooling to achieve Bose condensation in atomic systems which are weakly interacting.
- This is very difficult to achieve.
- In strongly interacting systems like liquid helium Bose condensate is formed at relatively higher temperatures.
- Fritz London in 1938 interpreted superfluidity in liquid Helium discovered by W. H. Keesom in 1928 to be a Bose condensation phenomena.

The trick

- Here is the trick...



called evaporative cooling.

- It was proposed by Eric Cornell, Carl Wieman of JILA and Wolfgang Ketterle of MIT.



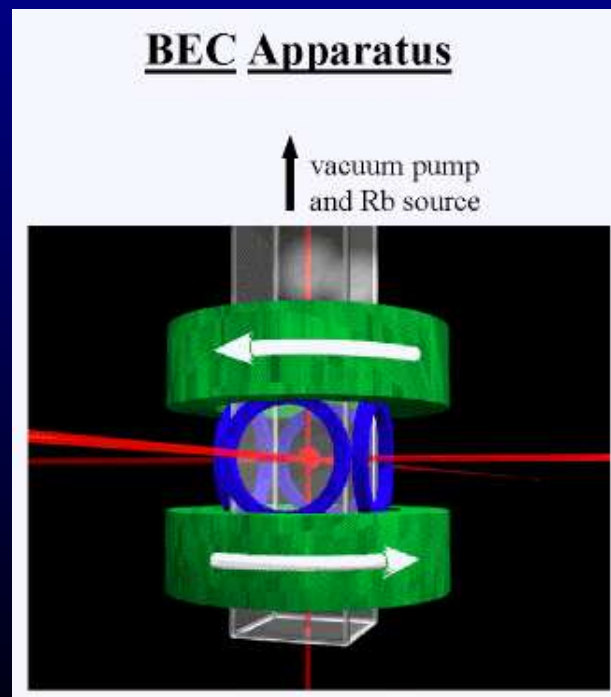
- They received the 2002 Nobel prize in Physics for their achievement.

Experimental Techniques for BEC

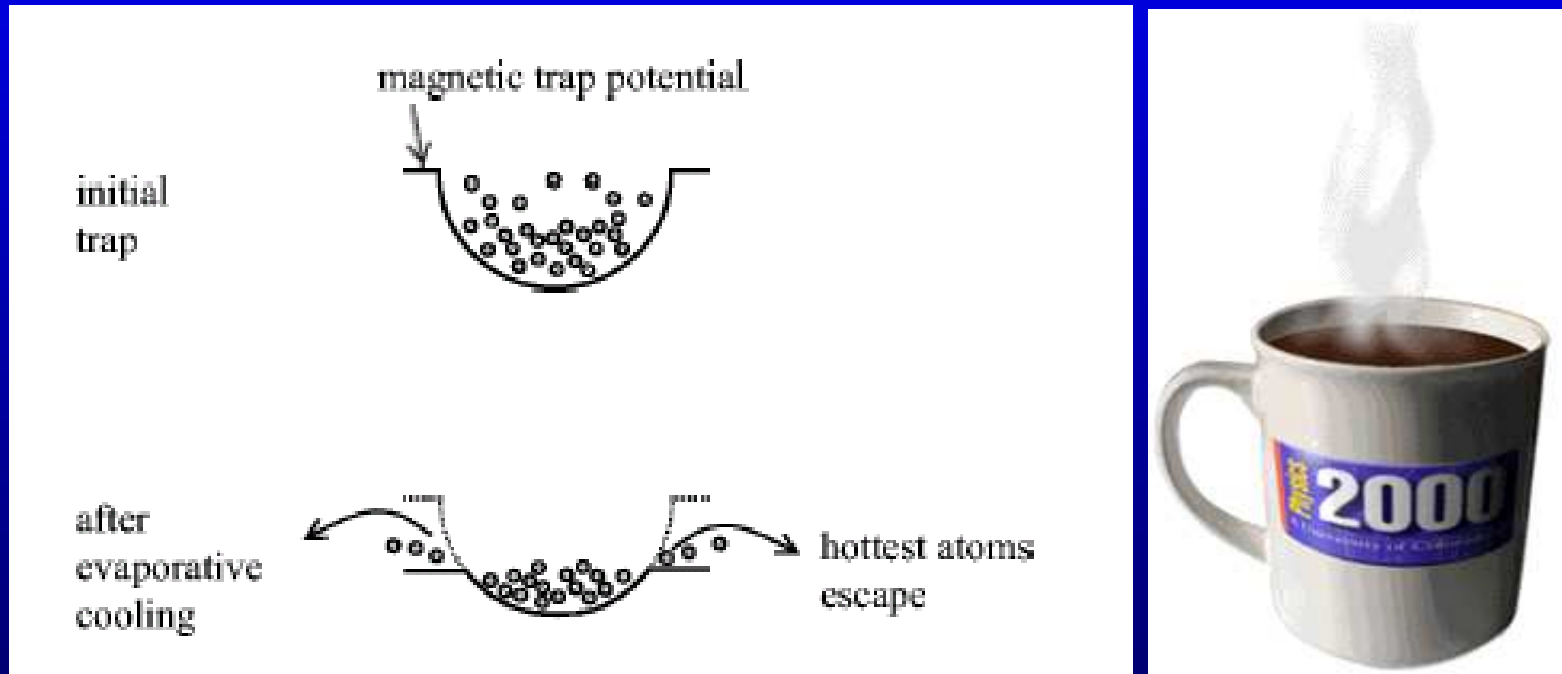
- The conditions required to achieve Bose-Einstein condensation in gaseous medium poses various technical challenges.
- To observe pure condensation effects with other effects such as liquefaction, the atoms has to be well apart from each other.
- This requires small particle density and in turn very small transition temperatures.
- For example, most of the successful experiments on gaseous systems have had condensation temperatures in the range between 500 nK and 2 μK , with particle densities of $10^{20} - 10^{21} \text{ m}^{-3}$.

General cooling procedure

- Trap a gas of atoms and cool them using laser-cooling techniques.
- Turn the cooling laser off and compress the gas using a magnetic trap. This was achieved by JILA group by using static quadrupole and rotating dipole fields.
- Cool the gas again by evaporative cooling until condensation occurs.



Evaporative cooling mechanism



- The laser cooled atoms are first compressed in a magnetic trap.
- The trap potential is then reduced by decreasing the magnetic field strength, so that the hottest atoms can escape.
- This reduces the temperature, in the same way the evaporation cools a liquid

Observations of BEC

Atom	Isotope	Year of Observation
Rubidium	^{87}Rb	1995
Sodium	^{23}Na	1995
Lithium	^7Li	1997
Hydrogen	^1H	1998
Rubidium	^{85}Rb	2000
Helium	^4He	2001
Potassium	^{41}K	2001

Intuitive picture of Bose-Einstein condensation

- Consider a gas of identical non-interacting bosons of mass m at temperature T .
- The de Broglie wavelength λ_{dB} is determined by the free thermal motion:

$$\frac{p^2}{2m} = \frac{1}{2m} \left(\frac{h}{\lambda_{\text{dB}}} \right)^2 = \frac{3}{2} k_B T.$$

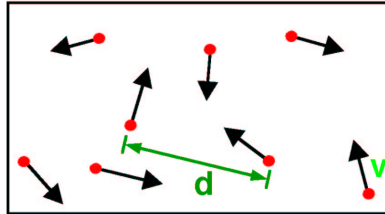
- This implies that:

$$\lambda_{\text{dB}} = \frac{h}{\sqrt{3mk_B T}}.$$

- The thermal de Broglie wavelength thus increases as T decreases.
- The quantum mechanical wave function of a free atom extends over a distance of $\approx \lambda_{\text{dB}}$.
- As we lower the temperature, eventually wave functions of neighboring atoms begin to overlap.
- The atoms will interact with each other and coalesce to form a “super atom” with a common wave function.
- This is the Bose-Einstein condensate.

Intuitive picture of Bose-Einstein condensation

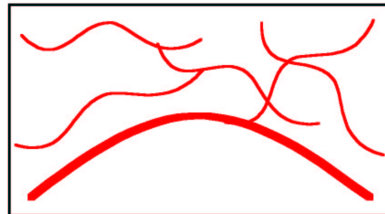
What is Bose-Einstein condensation (BEC)?



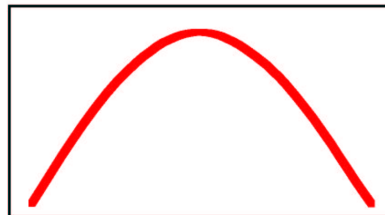
High Temperature T:
 thermal velocity v
 density d^{-3}
 "Billiard balls"



Low Temperature T:
 De Broglie wavelength
 $\lambda_{dB} = h/mv \propto T^{-1/2}$
 "Wave packets"

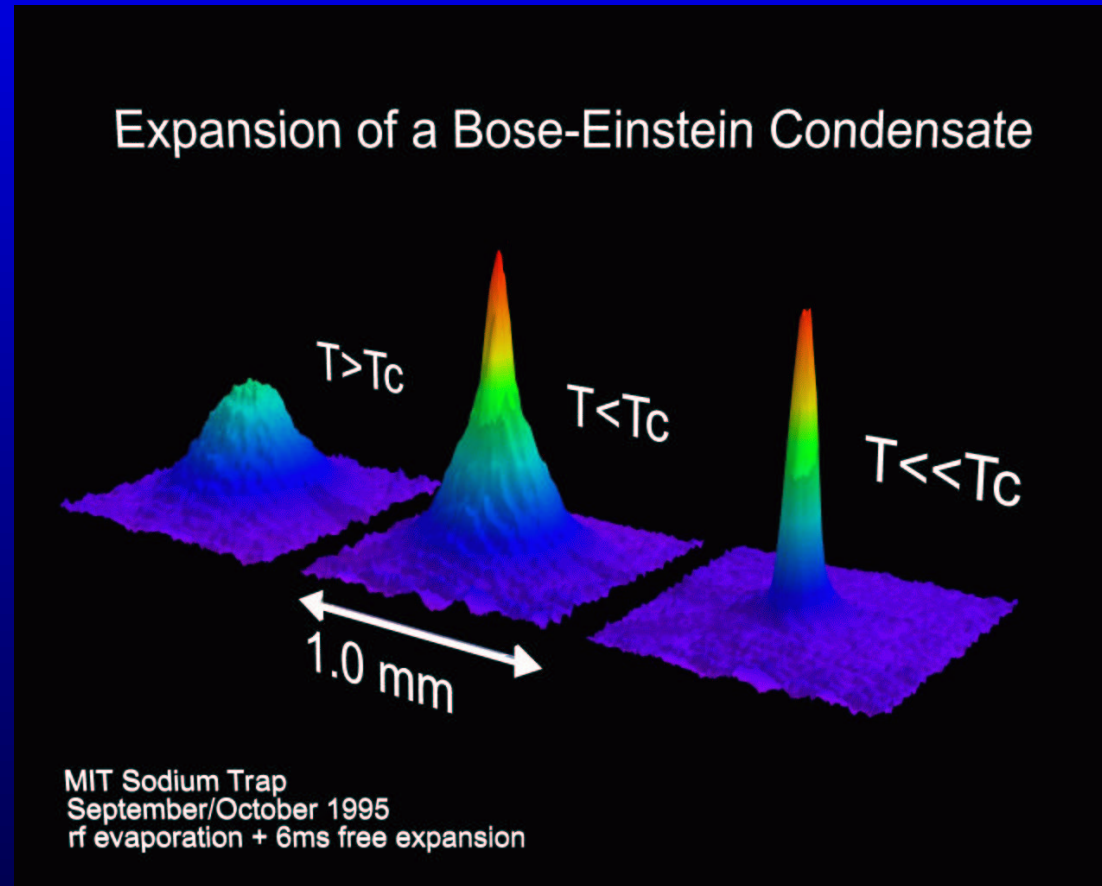


$T = T_{crit}$:
 Bose-Einstein
 Condensation
 $\lambda_{dB} \approx d$
 "Matter wave overlap"



$T = 0$:
 Pure Bose
 condensate
 "Giant matter wave"

Intuitive picture of Bose-Einstein condensation



Another one of Prof W. Ketterle's viewgraphs

Simple derivation of the critical temperature

- The condition for wavefunction overlap:
“The reciprocal of the effective particle volume determined by the de Broglie wavelength should be equal to the particle density.”
- If we have N particles in volume V then: $N/V \approx 1/\lambda_{dB}^3$
- Thus using earlier relation for λ_{dB} we obtain for T :

$$T_c \approx \frac{1}{3} \frac{h^2}{mk_B} \left(\frac{N}{V} \right)^{2/3}$$

- Despite the simplicity of the argument this formula is very close to the one obtained from rigorous statical mechanical arguments apart from a numerical factor.
- Low density systems such as gases will have a very low transition temperatures. This explains why it has been so difficult to observe condensation in gases until recently.

Statistical Mechanics of Bose-Einstein condensation

- Consider a gas of N non-interacting bosons of mass m at temperature T in a volume V .
- The occupation of the quantum state with energy E is given by

$$n_{\text{BE}}(E) = \left[\exp\left(\frac{E - \mu}{k_B T}\right) - 1 \right]^{-1}$$

where, μ is the chemical potential.

- Chemical potential is determined by:

$$\frac{N}{V} = \int_0^{\infty} n_{\text{BE}}(E) g(E) dE$$

$g(E)$ is the density of states per unit volume.

- If lowest energy state is $E = 0$ then the maximum value for μ is zero.

More on the distribution function

$$n_{\text{BE}}(E) = \left[\exp\left(\frac{E-\mu}{k_B T}\right) - 1 \right]^{-1}$$

- If $(E - \mu) \gg k_B T$, then we get Boltzmann form :

$$n(E) \approx \exp(-E/k_B T).$$

Thus classical statistics applies when μ takes large negative value.

- We notice, from N/V expression, that μ is negative for small densities and large T .
- Thus, we must have large particle density and low temperature in order to achieve Bose-Einstein condensation.

Statistical mechanical considerations continued...

- For non-interacting particles possessing only kinetic energy:

$$g(E)dE = 2\pi(2S + 1) \left(\frac{2m}{h^2}\right)^{3/2} \sqrt{E}dE$$

where $(2S + 1)$ is the spin multiplicity. This accounts for the fact that there are $(2S + 1)$ degenerate spin states for each momentum state. We choose $S = 0$.

- We fix the particle density N/V and vary the temperature. The condition to determine the chemical potential is:

$$\frac{N}{V} = 2\pi \left(\frac{2m}{h^2}\right)^{3/2} \int_0^\infty \frac{1}{\exp(E - \mu)/k_B T - 1} \sqrt{E} dE$$

- At high T , μ will have a large negative value. As we cool the gas while keeping N/V constant, μ must increase to compensate for the decrease in $k_B T$. This process continues, until μ reaches its maximum value of zero.

Critical temperature

- The critical temperature T_c at which this occurs is given by:

$$\begin{aligned}
 \frac{N}{V} &= 2\pi \left(\frac{2m}{h^2} \right)^{3/2} \int_0^\infty \frac{1}{\exp(E/k_B T) - 1} \sqrt{E} \, dE \\
 &= 2\pi \left(\frac{2mk_B T_c}{h^2} \right)^{3/2} \int_0^\infty \frac{x^{1/2}}{e^x - 1} \, dx \\
 &= 2\pi \left(\frac{2mk_B T_c}{h^2} \right)^{3/2} \times \Gamma(3/2)\zeta(3/2) = 2\pi \left(\frac{2mk_B T_c}{h^2} \right)^{3/2} \times 2.315
 \end{aligned}$$

where $x = E/k_B T$.

- Thus,

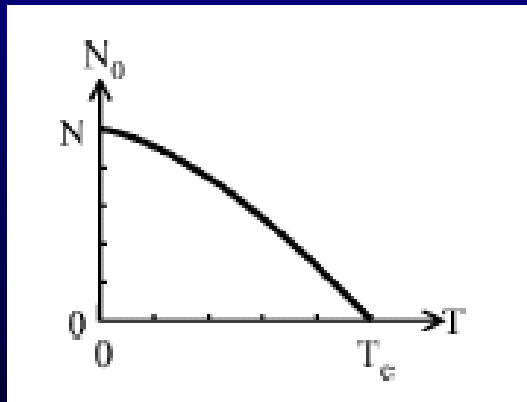
$$T_c = \frac{h^2}{2mk_B} \left(\frac{N}{V} \right)^{2/3} \frac{1}{[2\pi\Gamma(3/2)\zeta(3/2)]^{2/3}} = 0.0839 \frac{h^2}{mk_B} \left(\frac{N}{V} \right)^{2/3}$$

Number of particles in the condensate

- At T_c a macroscopic fraction of the total number of particles condense into the state with $E = 0$. This is because $n_{\text{BE}}(E)$ has a singularity at $E = 0$ and $\mu = 0$.
- The remainder of particles continue to distribute thermally between the rest of the levels.
- The number of particles with $E = 0$ is given by

$$N_0(T) = (N - N_{\text{ex}}) = N \left[1 - \left(\frac{T}{T_c} \right)^{3/2} \right],$$

where N is the total number of particles.

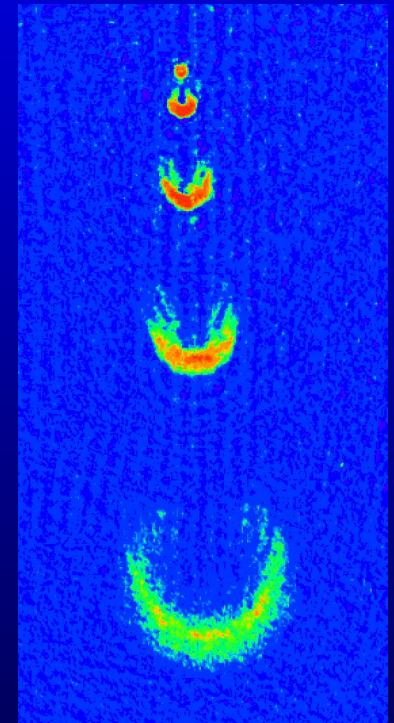
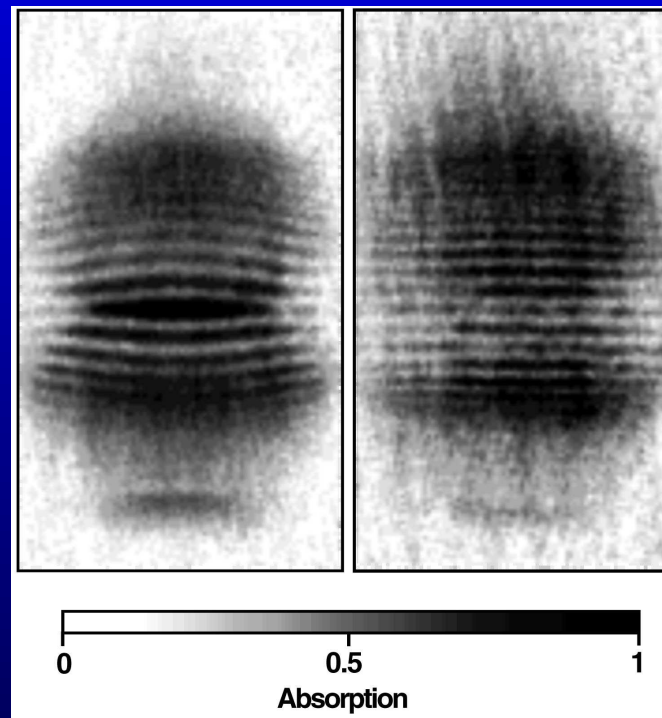
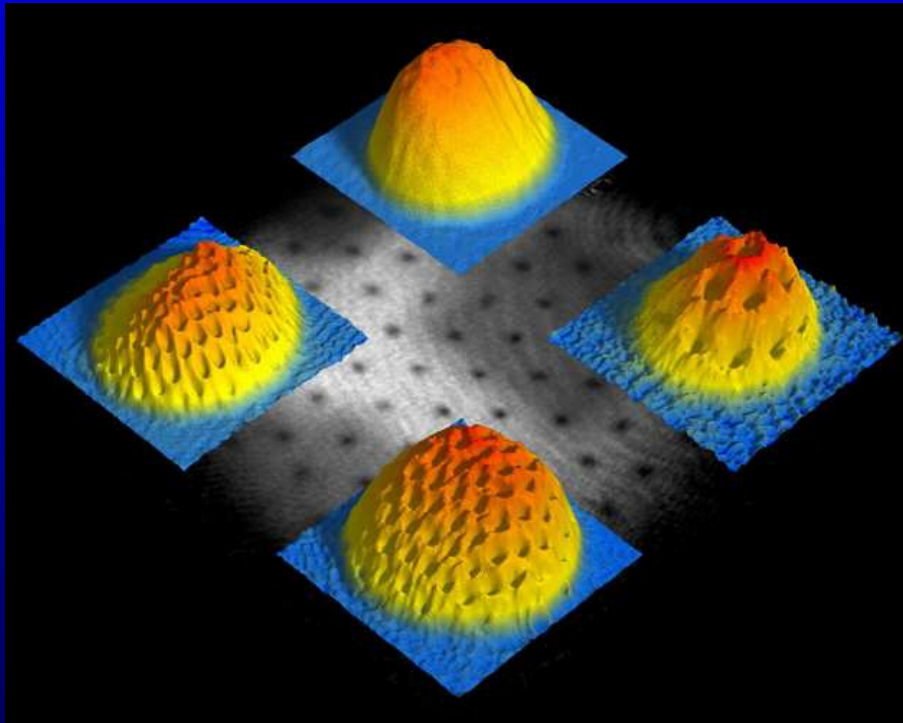


Thus, at T_c a phase transition occurs and a substantial fraction of the total number of particles condense in the state with $E = 0$.

More on the theory of Bose-Einstein condensation

- So far we have seen a very simple treatment for BEC within so called grand canonical ensemble of statistical mechanics.
- More complete description of BECs has to be within microcanonical ensemble.
- The correct theory should also take into account the interaction among the bosons.
- There is so called the mean-field approach to study BECs:
 - ★ Since BECs behave as macroscopic objects, one ascribes to them a macroscopic wavefunction.
 - ★ One then writes a Schrödinger type nonlinear wave equation for this wavefunction.
 - ★ This is so called non-linear Schrödinger or Gross-Pitaevskii (GP) equation.
 - ★ GP equation is useful to study most of the observed properties of BECs.

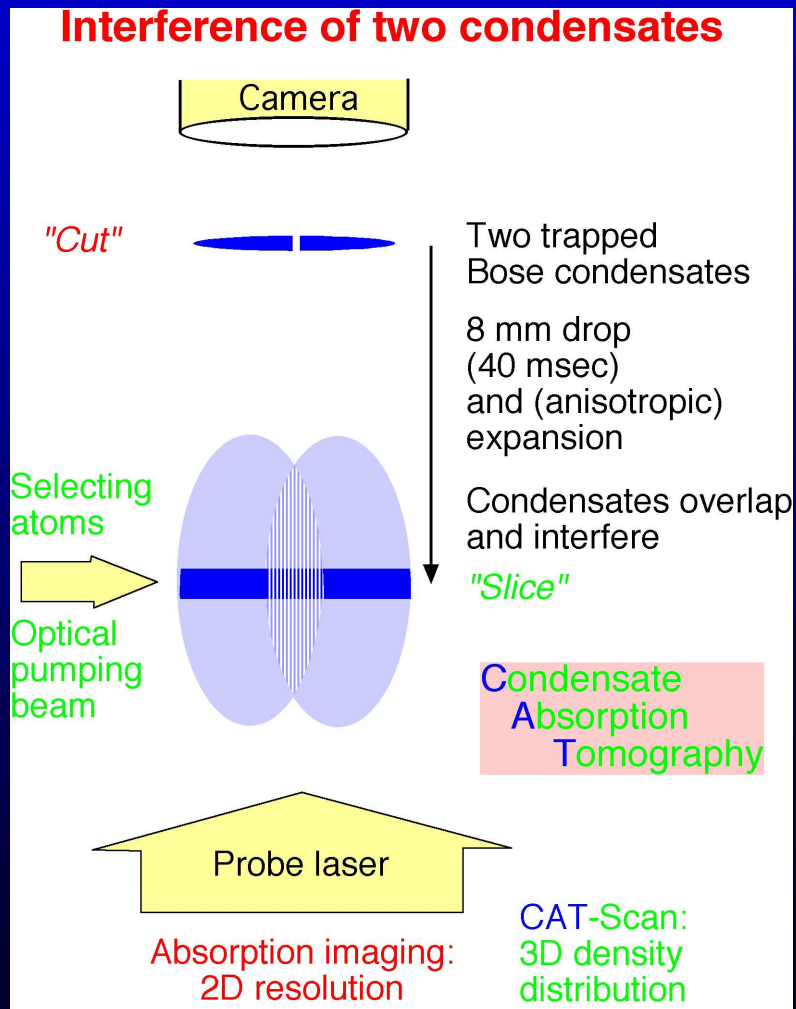
What can we do with the cold atoms in BEC ?



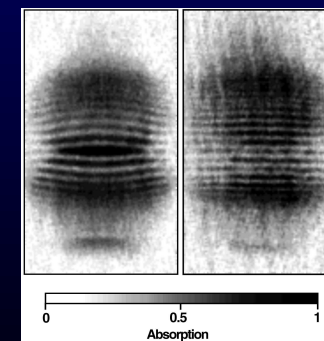
Coherence in the condensates

- Perhaps the most striking feature of quantum mechanical theory is the fact that it predicts that matter can behave like waves.
- This means that coherent matter waves can form interference patterns.
- The demonstration of interfering pathways in a neutron interferometer was perhaps the strongest affirmation of quantum theory and the wavelike nature of matter.
- Atom interferometer experiments extended this demonstration to the size of an entire atom.
- Now, Bose-Einstein condensation has allowed us to make another advance, making it possible to interfere two completely independent clouds of atoms with each other. This was demonstrated for the first time in the month of November, 1996 at MIT by the group of Ketterle.

Interference of two condensates



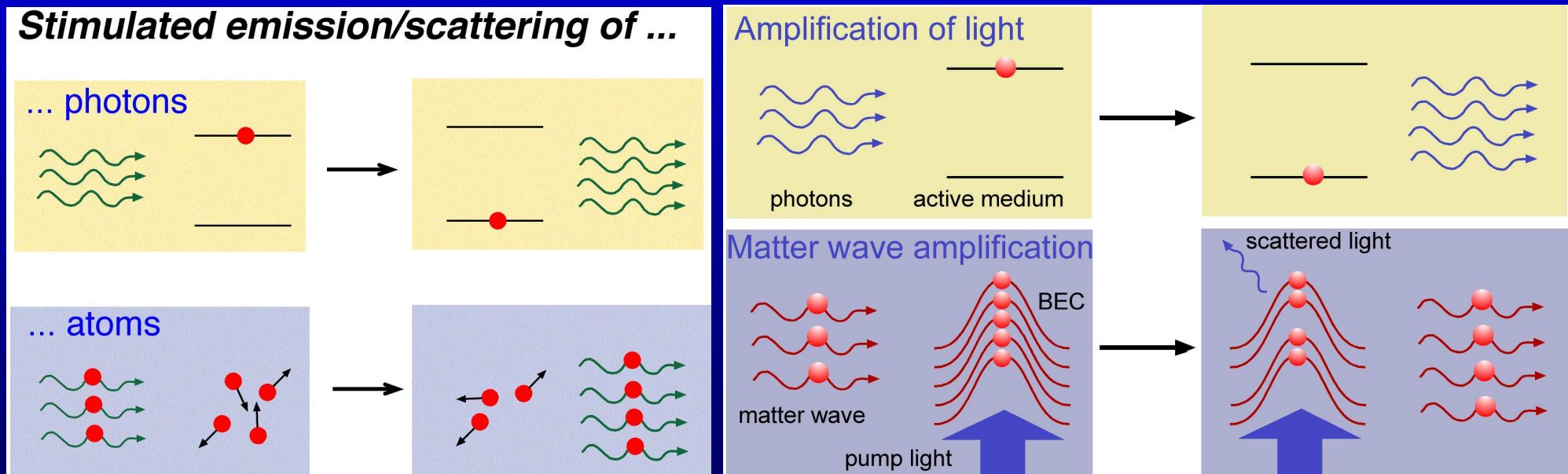
- This is done by cutting the atom trap in half with an argon-ion laser beam.
- The sodium atoms are then cooled in the two halves of the trap to form two independent Bose condensates.
- At this point, the laser and the magnetic field are quickly turned off, allowing the atoms to fall and expand freely.
- As the two condensates began to overlap with one another, interference fringes form.



Atom laser and the conventional laser

- An atom laser is analogous to an optical laser, but it emits matter waves instead of electromagnetic waves.
- A laser requires a cavity, an active medium and an output coupler.
- In a typical atom laser, the "resonator" is a magnetic trap in which the atoms are confined by "magnetic mirrors". The active medium is a thermal cloud of ultra-cold atoms, and the output coupler is an rf pulse which controls the "reflectivity" of the magnetic mirrors. The gain process in an atom laser.
- The analogy to spontaneous emission in the optical laser is elastic scattering of atoms (collisions similar to those between billiard balls).
- In a laser, stimulated emission of photons causes the radiation field to build up in a single mode. In an atom laser, the presence of a Bose-Einstein condensate causes stimulated scattering by atoms into that mode.

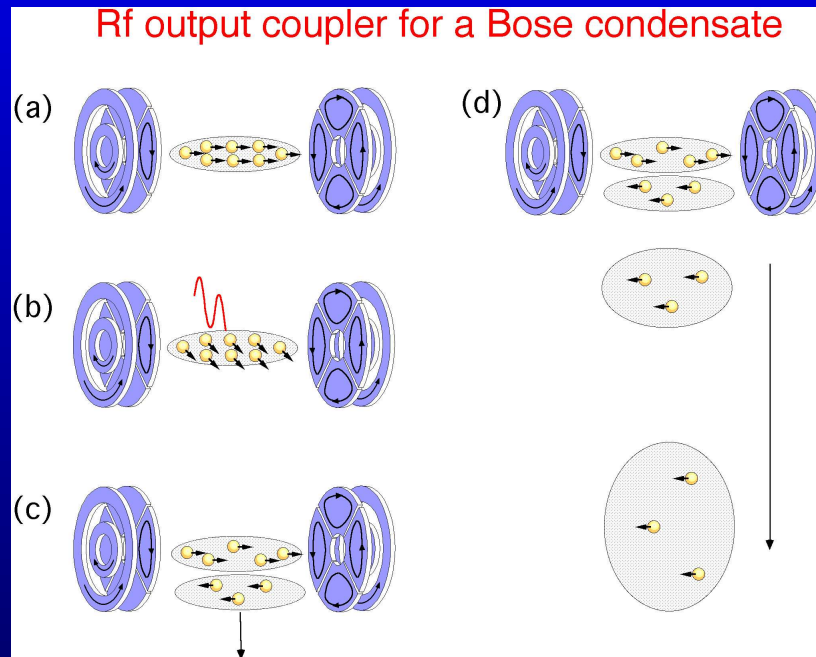
- Unlike optical lasers, which sometimes radiate in several modes (i.e. at several nearby frequencies) the matter wave laser always operates in a single mode.



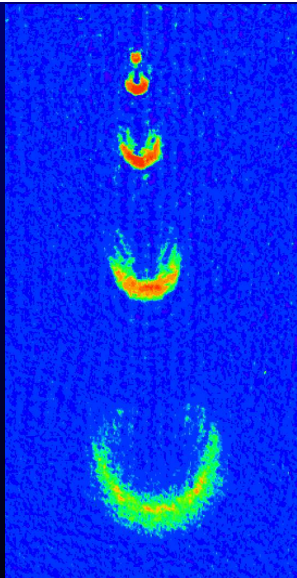
Differences between an atom laser and an optical laser

- Photons can be created, but not atoms. The number of atoms in an atom laser is not amplified. What is amplified is the number of atoms in the ground state, while the number of atoms in other states decreases.
- Atoms interact with each other - that creates additional spreading of the output beam. Unlike light, a matter wave cannot travel far through air.
- Atoms are massive particles. They are therefore accelerated by gravity.
- Bose condensates occupies the lowest mode (ground state) of the system, whereas lasers usually operate on very high modes of the laser resonator.
- A Bose condensed system is in thermal equilibrium and characterized by extremely low temperature. In contrast, the optical laser operates in a non-equilibrium situation which can be characterized by a negative temperature (which means "hotter" than infinite temperature!). There is never any population inversion in evaporative cooling or Bose condensation.

Atom Laser Output Coupler

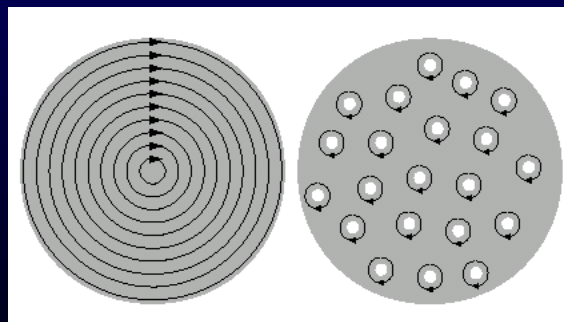


- Figure (a) shows a Bose condensate trapped in a magnetic trap. All the atoms have their (electron) spin up, i.e. parallel to the magnetic field.
- (b) A short pulse of rf radiation tilts the spins of the atoms.
- (c) Quantum-mechanically, a tilted spin is a superposition of spin up and down. Since the spin-down component experiences a repulsive magnetic force, the cloud is split into a trapped cloud and an out-coupled cloud.
- (d) Several output pulses can be extracted, which spread out and are accelerated by gravity.



Generation of vortex lattices in Bose-Einstein condensates

- Quantum mechanics and the wave nature of matter have subtle manifestations when particles have angular momentum, or more generally, when quantum systems are rotating.
- When a quantum-mechanical particle moves in a circle the circumference of the orbit has to be an integer multiple of the de Broglie wavelength.
- This quantization rule leads to the Bohr model and the discrete energy levels of the hydrogen atom.
- For a rotating superfluid, it leads to quantized vortices.

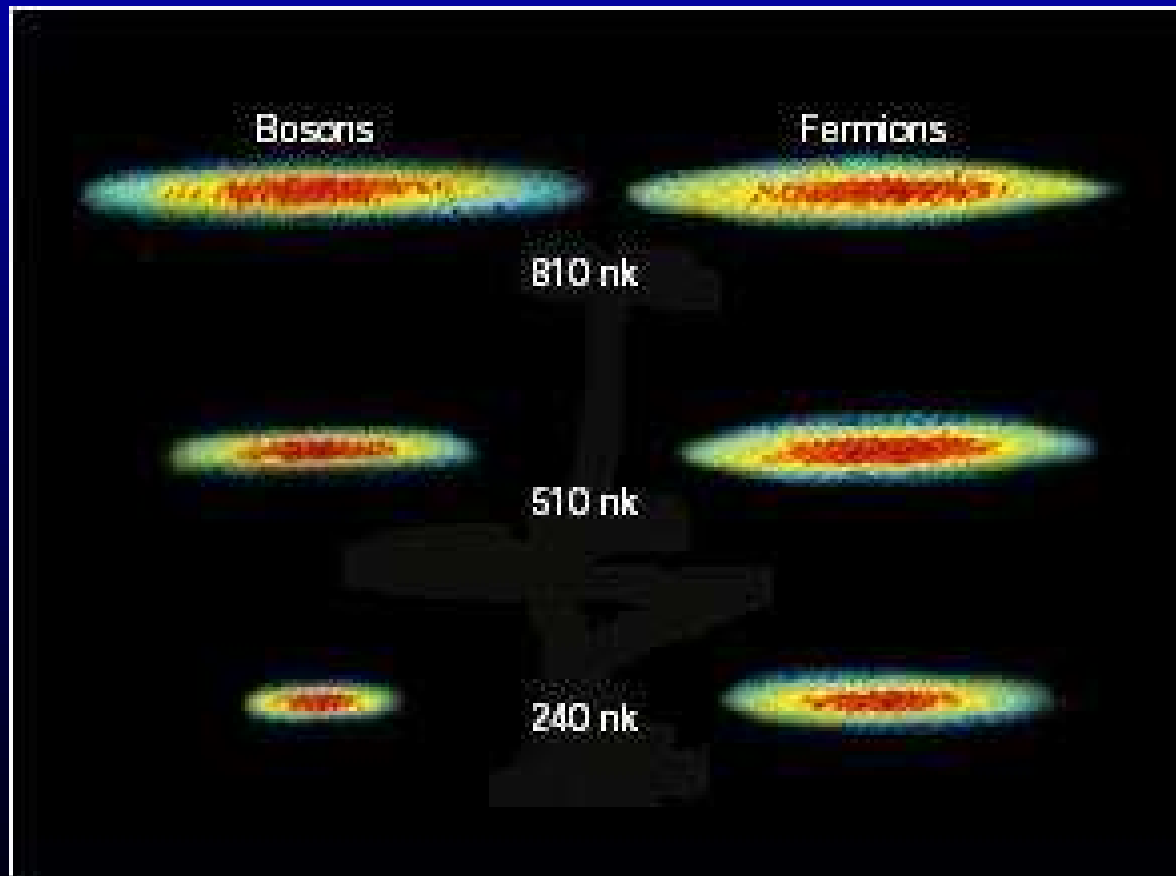


- If one spins a normal liquid in a bucket, the fluid will rotate, after an initial transient, as a rigid body where the velocity smoothly increases from the center to the edge.
- However, this smooth variation is impossible for particles in a single quantum state.
- To fulfill the above-mentioned quantization rule, the flow field has to develop singular regions where the number of de Broglie wavelengths on a closed path jumps up by one.
- Energetically, the most favorable configuration for this is an array of vortices, which are whirlpools similar to tornados or the flow of water in a flushing toilet.
- However, the whirlpools in a Bose-Einstein condensate are quantized when an atom goes around the vortex core, its quantum mechanical phase changes by exactly 2π .

Other cool things that can be done with cold atoms

Fermionic condensate

- Fermions can be cooled to form Fermi-degenerate systems. These are as interesting as the BECs. See Nature 398, 218-220 (1999)



More cool things that can be done with cold atoms

Nonlinear Atom Optics

- All the nonlinear optics effects possible with light waves can be achieved for matter waves as well. Here is an example.



Few more cool things that can be done with cold atoms

- All the known particles in the universe can be classified as either fermionic or bosonic.
- Thus, BECs and FDSs allow to test the properties of vast number of system on a tabletop in a laboratory.
- Laboratory achievement of white dwarfs (in Fermi Systems) and supernova explosion (for attractive interaction in Bose systems) are a very good example of the kinds of systems that can be studied with degenerate gases.
- Degenerate gases are useful tools to study complex condensed matter systems and many body effects.
- Multi-component and molecular condensates could become very helpful in studying chemistry in a controllable way.
- You will hear more about various application of Bose-Einstein Condensates to various areas of physics in a future talk by Prof. Allen