You are graded on your work, with partial credit. See the last pages of the exam for formula sheets. **Please be clear and well-organized, so that we can easily follow each step of your work.**

1. (a) (15) A metal airplane with a wingspan of 10 m moves through the earth's magnetic field, whose magnitude we will approximate as 0.5 T. If the airplane has a velocity of 300 km/hour perpendicular to the magnetic field, what is the induced emf across its wing?

(b) (5) Suppose the airplane were made of insulating plastic rather than conducting metal. Would there still be an induced emf across its wing? Explain in a few words.

2. Two point charges are moving as shown in the figure.

At a given moment in time, the charge of $q = +3.00 \ \mu\text{C}$ is located on the *Y* axis at a distance of 0.20 m from the origin, and is moving to the right with a velocity $v = 2.00 \times 10^6 \text{ m/s}.$

At the same time, the charge of $q' = -2.00 \ \mu C$ is located on the x axis at a distance of 0.10 m from the origin, and is moving upward with a velocity $v = 1.00 \times 10^6$ m/s.



(a) (6) Determine the magnitude of the magnetic field at the origin due to the first charge q.

(b) (2) What is the direction of this field? [up, down, to left, to right, into paper, out of paper?]

(c) (6) Determine the magnitude of the magnetic field at the origin due to the second charge q'.

(d) (2) What is the direction of this field?

(e) (6) Calculate the magnitude of the total magnetic field at the origin.

(f) (3) What is the direction of the total magnetic field? [up, down, to left, to right, into paper, out of paper?]

3. In the figure below, the long straight wire carries a current which is increasing at a rate di / dt.



(a) (5) At a given instant when the current is i, what is the magnitude of the magnetic field \vec{B} at a distance r to the right of the wire (in terms of r, i, and constants)?

(b) (5) Calculate the flux $d\Phi_B$ through the narrow strip of thickness dr and length L at a distance r from the long straight wire (in terms of dr, L, r, and i).

(c) (5) Integrate to find the total flux through the rectangular loop (in terms of i, a, b, and L).

(d) (5) Calculate the induced emf around the loop (in terms of di / dt, a, b, and L).

(e) (5) Calculate the numerical value of the induced emf if a = 10.0 cm, b = 24.0 cm, L = 20.0 cm, and di / dt = 5.00 A/s.

4. (a) (6) For the circuit shown in the figure, use Kirchhoff's loop rule to write an equation which involves the charge q on the capacitor with capacitance C, the current i through the resistor with resistance R, the rate of the change di / dt of the current through the inductor with inductance L, and the battery which has emf \mathcal{E} (and negligible internal resistance).



(b) (4) Rewrite this as a second-order differential equation in the time-dependent charge q(t).

(c) (4) Now consider the case where the battery and resistance are removed, so that $\varepsilon = 0$ and R = 0. Write down the differential equation for q(t) in this case.

(d) (4) Show that the differential equation in Part (c) is satisfied by $q(t) = Q\cos(\omega t + \phi)$, and at the same time **determine** the value of ω (in terms of L and C.

(e) (4) Determine the value of ϕ if q(t) has its maximum value at time t = 0.

(f) (4) Calculate the maximum value of the electric field energy in the capacitor if $Q = 2.00 \times 10^{-3} \text{ C}_{\text{and}} C = 4.00 \times 10^{-6} \text{ F}_{\text{.}}$

(g) (4) Calculate the maximum value of the magnetic field energy in the inductor if $i_{\text{max}} = \omega Q = 2.00 \times 10^{-3} \text{ A}_{\perp}$

5. (5 points extra credit) Suppose that the laws of Nature suddenly changed, so that the displacement current term involving $d\Phi_E / dt$ were suddenly eliminated from Ampere's law, giving only

$$\oint \vec{B} \cdot d\vec{\ell} = \mu_0 i_c$$

(instead of the full version which includes the displacement current).

What sudden dramatic change would transpire in the world experienced by all human beings (even those who have never come close to a radio or cellphone or any other device employing a capacitor)?