Physics 314 Exam 1

Please show all significant steps clearly in all problems.

Boltzmann's constant = 1.38×10^{-23} J/K   Coulomb's law constant = 8.99×10^{9} N m^2/C^2

h = Planck's constant = 6.63×10^{-34} J s $\hbar = h/2\pi$ $e = 1.60 \times 10^{-19}$ Coulomb

mass of electron = 9.11×10^{-31} kg   mass of neutron ≈ mass of proton = 1.67×10^{-27} kg

1 eV = 1.60×10^{-19} Joule   1 MeV = 10^6 eV c = speed of light = 3.00×10^8 m/s

1 nm = 10^{-9} m   1 $\mathring{A} = 10^{-10}$ m   1 fermi = 10^{-5} $\mathring{A}$

1. (20) **Our Sun**. Briefly describe 10 interesting facts about the Sun, with as much hard information as possible.

When appropriate, give semiquantitative information about temperatures, distances, time scales, etc.

Also, discuss a variety of different aspects and features.

A good description will leave the reader with a good overall understanding, in some detail, of what the Sun is like and how it works.

Fact 1:

Fact 2:

Fact 3:

Fact 4:

Fact 5:

Fact 6:

Fact 7:

Fact 8:

Fact 9:

Fact 10:

2. Energies of Atoms, Nuclei, Photons, and Ions.

Parts (a) and (b): According to Heisenberg's uncertainty principle, $\Delta p \Delta x \geq \hbar/2$. For an estimate of ground-state properties one can often make the rough approximations $\Delta p \Delta x \sim \hbar$ and $p \sim \Delta p$, where p is the momentum of a particle and Δx is the size of the region to which it is confined. We can then use these approximations to estimate the energies of particles in various situations. In parts (a) and (b) of this problem, assume that the usual nonrelativistic expression for the kinetic energy gives a reasonable order-of-magnitude estimate for the total energy. Give your answers in eV.

(a) (5) Estimate the energy of an electron in an atom with diameter 1 $\mathring{A}$. Is your answer reasonable?

(b) (5) Estimate the energy of a nucleon in an atomic nucleus with diameter 5 fermi. Is your answer reasonable?

(c) (5) Now consider a visible photon with wavelength 500 nm. What is the energy of this photon, in eV? Can photons with about this much energy excite atoms like that of part (a)?

(d) (5) Suppose your answer to (b) gives a rough estimate of the spacing between energy levels in a nucleus. Use this spacing to obtain a rough estimate of the the characteristic wavelengths for the gamma ray photons emitted by an excited nucleus when it decays.

(e) (5) Estimate the thermal energy of an ion in the center of the Sun.

(f) (5) Suppose that two protons must "touch" in order to fuse together, and that the radius of a proton is 1 fermi. What energy must they have in order to approach each other this closely? If this differs from the average thermal energy of part (e), explain why there is no discrepancy.

3. **Bohr Theory for a Nucleus with a Harmonic Oscillator Potential**

Let us solve a crude model for nucleons in a nucleus. The mass of each nucleon is m, and it is taken to move in a circular orbit and in a harmonic oscillator potential

$$V(r) = \frac{1}{2}kr^2 \tag{1}$$

where r is the distance from the center of the nucleus. An excitation of the nucleus is (simplistically) interpreted as the excitation of a single nucleon, just as for an electron in an atom.

(a) (5) Write down Bohr's assumption about the quantization of angular momentum (or equivalently, the assumption that an integral number of deBroglie wavelengths fit into an orbit) to obtain a relation between the velocity v of the particle and the radius r of its orbit.

(b) (5) Calculate the centripetal force

$$F = -\frac{dV(r)}{dr} \tag{2}$$

corresponding to the above potential.

(c) (5) Solve for the allowed radii r_n in terms of the quantum number n and the constants m, k, and $\hbar$.

(d) (5) Solve for the allowed energies in terms of n, $\hbar$, and $\omega_0 = \sqrt{k/m}$.

(e) (5) If the effective force constant k is 100 N/fermi, calculate the energy of the photon emitted when a nucleon falls from the first excited state to the ground state.

4. (a) (10) Consider a distribution of dark matter in a galaxy which is in a steady state. By taking the average of the time derivative of $\vec{r} \cdot \vec{p}$ for a given set of the dark matter particles, obtain a relationship between the average kinetic energy and average gravitational potential energy for these particles.

(b) (10) In the observations of luminous matter orbiting a galaxy, it is found that the velocity v is approximately constant with respect to the distance r from the center of the galaxy, even at large r where there is essentially no more luminous matter.

Suppose that $\rho(r)$ is the density of the dark matter (and thus of all the matter) at large r. Using the fact that

$$dM(r) = \rho(r) \cdot 4\pi r^2 dr \tag{3}$$

calculate the approximate dark matter distribution $\rho(r)$, using the fact that $v(r)$ is constant for orbiting hydrogen atoms. I.e., show that $\rho(r)$ is proportional to some simple function of r.

5. (5) Using the fact that

$$P = \frac{1}{3} n \langle \overrightarrow{v} \cdot \overrightarrow{p} \rangle \tag{4}$$

for particles in a gas, show that

$$P_{rad} = \frac{1}{3} n_{rad} \langle E \rangle \tag{5}$$

for a blackbody radiation field. What is the temperature dependence of P_{rad} – i.e., if P_{rad} is proportional to T^n, what is n?