Physics 314 Exam 2

Please show all significant steps clearly in all problems.

Boltzmann's constant $= 1.38 \times 10^{-23}$ J/K Coulomb's law constant $= 8.99 \times 10^9$ N m^2/C^2
h = Planck's constant $= 6.63 \times 10^{-34}$ J s $\hbar = h/2\pi$ $e = 1.60 \times 10^{-19}$ Coulomb
mass of electron $= 9.11 \times 10^{-31}$ kg mass of neutron $\approx$ mass of proton $= 1.67 \times 10^{-27}$ kg
1 eV $= 1.60 \times 10^{-19}$ Joule 1 MeV $= 10^6$ eV c = speed of light $= 3.00 \times 10^8$ m/s
1 nm $= 10^{-9}$ m 1 $\overset{o}{A}= 10^{-10}$ m 1 fermi $= 10^{-5}$ $\overset{o}{A}$

1. Black Holes: Time Dilation and Gravitational Red Shift

(a) (4) Using Planck's formula for the energy E of a photon in terms of its frequency f, together with $E = mc^2$, obtain the inertial mass m of the photon as a function of f.

(b) (4) Now let us make the leap of using Newtonian gravity, with the m of part (a) interpreted as the gravitational mass in this picture. What is the gravitational potential energy of the photon when it is outside a spherically symmetric mass M, and a distance r from its center?

(c) (4) Assuming that the sum of the "kinetic energy" of part (a) and the "potential energy" of part (b) is conserved, show that

$$f(r)\,F(r) = \text{constant} \tag{1}$$

where $f(r)$ is the frequency of the photon (when it is at the position represented by r) and

$$F(r) = \left(1 - \frac{GM/c^2}{r}\right). \tag{2}$$

(d) (8) Now for some fun with calculus, using the fact that

$$d \log u = du/u \tag{3}$$

where u is any function: Differentiate the expression of part (c) and then integrate to find the function $f(r)$.

(e) (5) Show that your result in part (c) agrees to first order with the exact result obtained with Einstein gravity (i.e. general relativity):

$$\frac{\lambda}{\lambda_0} = \left(1 - \frac{R_S}{r}\right)^{-1/2} \tag{4}$$

where R_S is the Schwarzschild radius. [If you are feeling very ambitious, use the back of this page to also show that it disagrees in second order, indicating that Einstein's theory is needed in strong gravitational fields. This part is not required, however.]

2. Nuclear Forces and Reactions.

(a) (3) In the vacuum, or in certain unstable nuclei, the neutron decays into a proton plus other particles. (This is, of course, called beta decay, and it results from the weak nuclear interaction.) Fill in the particles for this reaction below:

$$\mathrm{n}^0 \to \mathrm{p}^+ + \_\_\_ + \_\_\_ \,.$$

(b) (3) In the modern view of this reaction, it is mediated by a W particle (or W boson). Draw the Feynman diagram for the reaction above involving n^0 and p^+. Show all the particles, and indicate the charge of the W boson – i.e., whether it is a W^+ or a W^-.

(c) (3) The latest Nobel Prize was for x-ray and neutrino astronomy. One of the recipients was Ray Davis, who has operated a neutrino observatory in the Homestake Gold Mine of South Dakota for about 30 years. In his experiment, a neutrino is produced in the sun and then detected by the reaction

$$^{37}_{17}Cl + \nu \to ^{37}_{18} Cl + e^-. \tag{5}$$

This reaction also involves the weak interaction, and is mediated by a W particle. Draw the Feynman diagram for the reaction above, with a n^0 and a ν at the bottom, and a p^+ and an e^- at the top. (Again, indicate the charge of the W boson. In each Feynman diagram, show all the particles.)

(d) (3) Now redo the Feynman diagram of part (d) by writing it in terms of **quarks**, a ν, and an e^-, as well a W^+ or W^-.

(e) (3) In the proposed grand unified theories (GUTs), proton decay can occur, with

$$p^+ \to e^+ + \pi^0 \tag{6}$$

mediated by a superheavy X boson. Draw the Feynman diagram for this process, showing the **quarks** that are involved, as well as e^+ and X.

(f) (3) What is the charge of the X boson?

Which of the following quantities is conserved during this reaction: B, L, $(B+L)$ or $(B-L)$, where B is the baryon number and L is the lepton number?

(g) (3) The rest mass energy of the X boson is roughly 10^{13} TeV, where 1 TeV $= 10^{12}$ eV. What is the approximate range of the "GUT force" mediated by the X boson, in fermis, where 1 fermi $= 10^{-15}$ m? [You may find it convenient to use $\lambda_{Compton} = 1240$ nm if $m_0c^2 = 1$ eV, where $\lambda_{Compton} = h/m_0c = hc/\ m_0c^2$.]

What does this imply about the lifetime of the proton?

(h) (4) Fill in the blanks: ${}^3_2\mathrm{He} + {}^1_1\mathrm{H} \to {}^4_2\mathrm{He} +$ ___ $+$ ___

3. Neutron Stars

(a) (5) What supports a neutron star against gravity? What is is the approximate upper limit on the mass of a neutron star, in solar masses?

(b) (5) In a few sentences, preferably with some drawings, describe the standard picture of how a neutron star emits the electromagnetic radiation that makes it appear to us as a pulsar.

(c) (5) Why do neutron stars exist? I.e., why do the protons and electrons in normal matter combine to form neutrons?

4. Relativity (10) The fundamental relation between momentum and energy in Einstein's special theory of relativity is

$$E^2 = p^2c^2 + m_0^2c^4. \tag{7}$$

Suppose that this relation is known to hold, and that the mass is also known to increase with velocity according to

$$m = \frac{m_0}{\sqrt{1 - (v/c)^2}} \tag{8}$$

with

$$p = mv. \tag{9}$$

Show that Einstein's famous relation $E = mc^2$ follows from these three equations.

5. White Dwarfs

(a) (5) What supports a white dwarf against gravity? What is the approximate upper limit on the mass of a white dwarf, in solar masses?

(b) (15) The average spacing r between ions is $n_+^{-1/3}$, where n_+ is the number density of ions. Thus, the electrostatic interaction energy between ions is of order $K\,(Ze)^2\,/r = KZ^2e^2n_+^{1/3}$ per ion. On the other hand, the average thermal energy per ion is kT (actually $3kT/2$). Calculate the critical temperature for which kT equals $KZ^2e^2n_+^{1/3}$ for a helium white dwarf of mean density $\rho = Am_pn_+ = 10^9$ kg/m^3.

(c) (5) What is the physical significance of this critical temperature (in order of magnitude)?