Physics 314 Exam 3

Please show all significant steps clearly in all problems.

Boltzmann's constant = 1.38×10^{-23} J/K Coulomb's law constant = 8.99×10^{9} N m^2/C^2

h = Planck's constant = 6.63×10^{-34} J s $\hbar = h/2\pi$ $e = 1.60 \times 10^{-19}$ Coulomb

mass of electron = 9.11×10^{-31} kg mass of neutron ≈ mass of proton = 1.67×10^{-27} kg

1 eV = 1.60×10^{-19} Joule 1 MeV = 10^6 eV c = speed of light = 3.00×10^8 m/s

1 nm = 10^{-9} m 1 $\overset{o}{A} = 10^{-10}$ m 1 fermi = 10^{-5} $\overset{o}{A}$

1. Stars

(a) (7) Draw a rough sketch of the main-sequence line in the Hertzsprung-Russell diagram, which represents all stars when they are born. Show 3 points on this line which respectively correspond to **a very massive star, our Sun, and a red dwarf**. For each of these, use a tick mark to indicate the position on the horizontal axis, showing a roughly approximate temperature for each of the three. Also, indicate what quantity corresponds to the vertical axis. Finally, give a crude estimate, next to each point, of how long the star might be expected to live as a normal star.

(b) (3) What is the approximate range of possible sizes of stars, in terms of the solar mass M_{sun}?

(c) (6) Let L be the luminosity of a normal star, M be its mass, t_{life} be its lifetime, and T_e be its effective temperature. **Given** the roughly approximate relations $L \propto M^4$ and $R \propto M$, **obtain** the similarly rough relations (i) between t_{life} and M and (ii) between T_e and M.

(d) (4) Why does a large star move to the right and then up in the H-R diagram as its hydrogen fuel is consumed and converted to helium "ash"? Give simple qualitative arguments.

(e) (3) What will happen to our sun when it reaches its end as a normal star in a few billion years? (First it will emit a ___________, then it will become a ___________, and finally it will be a ___________ . State in a little more detail.)

(f) (3) What is a nova? I.e., what happens to produce a nova?

(g) (4) The opacity κ within a star varies with the density ρ and the temperature T, roughly as $\kappa \propto \rho/T^{3.5}$. (The reason is that H^- ions are largely responsible for the opacity, and they tend to be destroyed at higher temperature.) In a normal star, the temperature increases substantially with contraction under gravity. But in a Cepheid variable star, energy can be absorbed in creating helium ions rather than increasing the temperature. Give a simple qualitative argument why radial oscillations will be damped in normal stars but enhanced in Cepheids.

2. **Binary Systems**

(a) (15) Let ΔE_{grav} be the gravitational potential energy released when a mass m drops from an accretion disk onto a neutron star with

$$\frac{GM}{R} \approx 0.14\, c^2. \tag{1}$$

(This is what one gets for $R \approx 15$ km and $M \approx 1.4\, M_{sun}$.)

Let $\Delta E_{nuclear}$ be the nuclear binding energy released by ^{4}He nuclei reacting to ultimately produce ^{12}C.

For ^{4}He, the atomic mass is 4.0026032 u (atomic mass units), with 1 u = 1.67×10^{-27} kg. (The corresponding binding energy is 28295.673 keV, with 1 keV = 1000 eV).

For ^{12}C, the atomic mass is 12.0000000 u. (The corresponding binding energy is 92161.753 keV).

Calculate the approximate ratio $\Delta E_{grav}/\Delta E_{nuclear}$.

(b) (5) Using the result of Part (a), describe the two ways in which energy is emitted as helium is deposited onto a neutron star from an accretion disk, and how they differ in intensity of emission and in the total energy emitted over a long time.

(c) (10) Discuss the Roche lobes and Lagrangian points in a binary system, and the role they play in mass transfer from, e. g., a normal star to a compact object. Give as much detail as you can with only about three sentences and one drawing.

3. **Interstellar Space**

Here we consider the **Stromgren sphere** in a **H II** region.

(a) (5) What are H I and H II ?

(b) (5) In a steady state within the Stromgren sphere, the number of recombinations must equal the number of ionizations:

$$\mathcal{R}\frac{4}{3}\pi r^3 = N_{UV} \quad (2)$$

$$\mathcal{R} = \alpha n_e n_p = \alpha n_e^2. \quad (3)$$

Solve for r in terms of the other quantities.

(c) (5) What produces a H II region?

(d) (5) Explain the physical meaning of each of the following quantities in Part (b): r, N_{UV}, α, and n_e. Give a very crude estimate of n_e. E.g., is it 10^{23} per cm^3, 10^{16} per cm^3, 10^9 per cm^3, 10 per cm^3?

4. **Galaxies**

Here we consider **Oort's constants**, which are important for the rotation of our Galaxy (and others).

(a) (5) Let Ω be the mean angular velocity of disk stars at a distance r from the center of the Galaxy (measured, as always, in radians/sec). In particular, suppose our Sun is rotating around the galaxy every 240 000 000 years, and is about 25 000 light years from the center of the galaxy. What is Ω at the position of our Sun? (A year is 3.16×10^7 seconds.)

(b) (5) How fast are you now moving through space as you revolve around the Galaxy with the Sun? (The speed of light is 3.00×10^8 m/s.)

(c) (5) Oort's constants are defined by

$$A = -\frac{1}{2} r \frac{d\Omega}{dr} \quad (4)$$

$$B = -\frac{1}{2r} \frac{d}{dr} \left(r^2 \Omega \right) \quad (5)$$

Show that $\Omega = A - B$.

(d) (5) Show that $B = -\Omega$ for the special case of a uniformly rotating disk. Then show that this would require

$$M(r) \propto r^n \quad (6)$$

and determine the value of n. Here $M(r)$ is the mass contained within the radius r.