Physics 314 Final Exam

Please show all significant steps clearly in all problems.

Please also be clear, precise, and reasonably complete in answering qualitative questions.

Boltzmann's constant $= 1.38 \times 10^{-23}$ J/K Coulomb's law constant $= 8.99 \times 10^{9}$ N m^2/C^2
h = Planck's constant $= 6.63 \times 10^{-34}$ J s $\hbar = h/2\pi$ $e = 1.60 \times 10^{-19}$ Coulomb
mass of electron $= 9.11 \times 10^{-31}$ kg mass of neutron $\approx$ mass of proton $= 1.67 \times 10^{-27}$ kg
1 eV $= 1.60 \times 10^{-19}$ Joule 1 MeV $= 10^6$ eV c = speed of light $= 3.00 \times 10^8$ m/s
1 nm $= 10^{-9}$ m 1 $\overset{o}{A} = 10^{-10}$ m 1 fermi $= 10^{-5}$ $\overset{o}{A}$

1. Internal Heat of Planets and Moons

(a) (6) What are the three sources of internal heat that can lead to volcanic activity for a planet or moon? (One is relevant early in its history, and the other two might be relevant even after billions of years.) Please give an example and a few words of explanation in each case. (You may not remember the name of a specific satellite, but you can still say something about it.)

(i)

(ii)

(iii)

Now suppose that the initial radioactivity of the earth was spread uniformly throughout its interior, and that one cubic centimeter of material released about 8000 erg/year, or 1 m^3 released 800 J/yr. Suppose also that the rate of energy loss through the surface of the earth was about 50 erg/(cm^2 sec), or 1.6×10^6 J/(m^2 yr), and that the heat capacity was roughly equal to that of water: 1 calorie/(gram C°), or 4186 J/(kg C°).

Other relevant data:
radius of earth = 6.37×10^8 cm = 6.37×10^6 m
mass of earth = 5.98×10^{27} gm = 5.98×10^{24} kg

(b) (6) How much energy would be released by radioactivity within the earth during the first half billion years, or 5×10^8 years?

(c) (6) How much energy would escape from the earth during this time?

(d) (6) What would the approximate temperature of the earth have been after this first half billion years, in °C?

(e) (6) Radioactive decays are still occuring, of course, but suppose that they had suddenly turned off after the first half billion years. How long would it take for all the heat to escape from the earth?

(f) (6) Consider a moon which has the same basic composition as the earth but whose radius is only 1/100 that of the earth. With the same assumption as in part (e), how long would this moon take to lose all its initial heat? Derive and use a simple scaling argument (involving R_{earth} and R_{moon}) to get the answer.

2. **General Relativity**

(a) (9) In

$$ds^2 = R(t)^2 \left[\frac{dr^2}{1 - kr^2} + r^2 \left(d\theta^2 + \sin^2\theta \, d\phi^2 \right) \right] - c^2 dt^2 \tag{1}$$

explain what is meant by each of the following expressions (preferably with a bit of interesting commentary).

(i) ds (or, if you prefer, a finite Δs):

(ii) $R(t)$:

(iii) k:

(b) (5) What is the currently preferred value of k and what does this mean for cosmology?

3. **Origins**

(a) (8) What are the "four pillars" of the standard big-bang cosmology? I.e., what are the four observational tests for which theory and observation show convincing agreement?

[You can make up for a lack of detailed knowledge about one part by giving more detaills on another part.]

(i)

(ii)

(iii)

(iv)

(b) (9) Give a description, with a few well-chosen sentences, of how the Solar System came to be, and why it has its basic structure.

(c) (8) Give a brief description of how stars are born, with reference to the density-wave theory and other possible mechanisms for star formation.

4. **Resonances**

(a) (5) Let $T_{orbital}$ be the orbital period of Mercury as it revolves around the Sun. Let T_{rot} be the time for Mercury to make one complete rotation on its axis (as seen by a fixed observer). One Mercurian day is equal to two Mercurian years. I.e., an observer at a fixed point on the surface of Mercury sees two years go by from sunrise to sunrise. What is the ratio $T_{rot}/T_{orbital}$?

(b) (5) In this part, let $T_{orbital}$ be the orbital period of our Moon as it revolves around the Earth. Also, let T_{rot} be the time for the Moon to make one complete rotation on its axis (as seen by a fixed observer). For our Moon, what is the ratio $T_{rot}/T_{orbital}$?

(c) (5) Explain why we have the kind of spin resonances that are illustrated in parts (a) and (b).

(d) (5) Give another example of a spin resonance in the Solar System, or alternatively give an example of an orbital resonance.

5. **Lagrangian Points** (20)

Consider two objects with masses M_1 and M_2, rotating around their common center of gravity with an angular velocity $\vec{\Omega}$ (measured in radians/sec, and with a direction given by the right hand rule). Also suppose that there is a third mass m at a distance $\vec{r}$ from the center of mass, with a velocity $\vec{v}$ in the rotating frame of reference, and with $m \ll M_1$, M_2. In the rotating frame, m feels the two real forces

$$\vec{F}_1 = -G\frac{M_1 m}{r_{m1}^2}\frac{\vec{r}_{m1}}{r_{m1}} \quad , \quad \vec{r}_{m1} = \vec{r} - \vec{r}_1 \tag{2}$$

$$\vec{F}_2 = -G\frac{M_2 m}{r_{m2}^2}\frac{\vec{r}_{m2}}{r_{m2}} \quad , \quad \vec{r}_{m2} = \vec{r} - \vec{r}_2. \tag{3}$$

It also feels two fictitious forces: the centrifugal force

$$\vec{F}_{centrifugal} = mr\Omega^2\frac{\vec{r}}{r} \tag{4}$$

and the Coriolis force

$$\vec{F}_{Coriolis} = -2m\vec{\Omega} \times \vec{v}. \tag{5}$$

(a) (5) Draw a picture representing M_1 and M_2 and show the positions of the 5 Lagrangian points.

(b) (15) Use the above real and fictitious forces to discuss the behavior of a small mass m if it is placed at or very near each point. Assume that $M_1 > 25M_2$.

(i) L_1

(ii) L_2

(iii) L_3

(iv) L_4

(v) L_5

6. Planets

(a) (10) Pick your favorite planet (other than the Earth) and describe five of its most interesting features.

(i)

(ii)

(iii)

(iv)

(v)

(b) (10) Now let us come back to the Earth. Near the poles the magnetic field lines converge, making the magnetic field $\vec{B}$ stronger.

For an incident cosmic ray proton with mass m, the kinetic energy $\left(p_{\|}^2 + p_{\perp}^2\right)/2m$ and the "adiabatic invariant" $p_{\perp}^2/B$ are both approximately conserved.

What does this imply about the behavior of protons (and other energetic charged particles) that approach earth along converging magnetic field lines? Give a convincing justification for your answer, using the facts given in the preceding paragraph.

7. The Cosmic Distance Ladder

(15) Define and explain each of the following, and tell where it fits into the requirements of astronomers for measuring the distances to stars and galaxies.

(a) parallax:

(b) Cepheid variables:

(c) Faber-Jackson relation for elliptical galaxies:

(d) Tully-Fisher relation for spiral galaxies:

(e) Type Ia supernovae:

8. Effective Potential

In Problem 5, let

$$\overrightarrow{F} = \overrightarrow{F}_1 + \overrightarrow{F}_2 + \overrightarrow{F}_{centrifugal} \tag{6}$$

and let the x-axis point from M_1 to M_2. In parts (a), (b), and (c) below, the small mass m is also regarded as lying somewhere along the x-axis.

(a) (5) Referring back to Problem 5, write down the expression for F_x (the x-component of $\overrightarrow{F}$) in terms of x, x_1, and x_2 (in an obvious notation). Assume that M_1 and M_2 make nearly circular orbits about the center of gravity, so that x_1 and x_2 can be regarded as constants.

(b) (5) Obtain the effective potential energy $V(x)$ which satisfies

$$F_x = -\frac{dV}{dx} \qquad \text{or} \qquad V(x) = -\int F_x \, dx. \tag{7}$$

(c) (5) Sketch the graph of the function $V(x)$ for the full range of relevant values of x, and mark the positions of L_1, L_2, and L_3 on this graph.

(d) (5) If you are really ambitious, get $V(\vec{r})$ in 3 dimensions, and try to sketch the contours of constant $V(\vec{r})$ (or equipotential surfaces) in the plane defined by M_1, M_2, and m.

(e) (5) What do you think is the role of the velocity-dependent $\vec{F}_{Coriolis}$, and what is meant by a stable position in the present context?

Happy Friday the 13th!

Also, Merry Christmas and a Happy Holiday Season!