Astrophysics (Physics 489) Final Exam

1. A star emits radiation with a characteristic wavelength $\lambda_{max} = 100$ nm . (λ_{max} is the wavelength at which the Planck distribution reaches its maximum.) The apparent (bolometric) magnitude of the star is $m = 12.26$.

(a) (5) What is the energy in eV of a photon with wavelength $\lambda_{max} = 100$ nm ?

(b) (5) Is this (100 nm, or 1000 $\overset{\circ}{A}$) photon visible, or does it lie in some other part of the electromagnetic spectrum? (Please specify.)

(c) (5) What is the surface temperature of this star? (This is the effective temperature of the star, treated as a blackbody.)

(d) (5) We found the relation between apparent magnitude and absolute magnitude to be
$$m = M_{Sun} - 2.5\log_{10}\left(F / F_{10,\odot}\right) \quad , \quad M_{Sun} = 4.76 \quad , \quad F_{10,\odot} = 3.196 \times 10^{-10} \ (\text{J/s})/\text{m}^2 .$$
Calculate the radiant flux of this star at the position of the Earth.

(e) (5) Suppose that the luminosity of this star is known to be 50 000 $L_\odot$, where $L_\odot = 3.826 \times 10^{26}$ J/s. What is the distance from this star to the Earth?

(f) (5) Using the Stefan-Boltzmann law, calculate the radius R of this star. Give your final answer in terms of the radius of the Sun, $R_\odot = 6.96 \times 10^5$ km.

(g) (5) In a very crude sketch, indicate a dot showing where this star would go in the Herzsprung-Russell diagram. Indicate, along the horizontal and vertical axes, what the relevant values are for the quantities plotted in the H-R diagram in the case of this star.

2. A quasar is moving directly away from us, and its H_α spectral line is found to be redshifted to 7385 $\overset{\circ}{A}$ from 6565 $\overset{\circ}{A}$ (where this second value is the unshifted reference value on Earth). You may assume the formulas

$$t_0 - t = \frac{z}{H_0} - \left(1 + \frac{1}{2}q_0\right)\frac{z^2}{H_0} + \dots$$

$$d_L = \frac{cz}{H_0}\left[1 + \frac{1}{2}(1 - q_0)z + \dots\right]$$

where z is the redshift, t is the time at which the radiation is emitted by the star, t_0 is the present time, d_L is the luminosity distance, $H_0 \approx 72 \ (\text{km/sec})/\text{Mpc}$ is the present value of the Hubble constant, and q_0 is the present value of the deceleration parameter (which is now known to be negative, but which is not needed here).

In this problem let us neglect the terms of order z^2 and higher, and also assume that the luminosity distance can be treated as the actual distance from this star to the Earth.

(a) (5) Calculate the redshift z .

(b) (5) How long ago did the quasar emit the radiation that we see now?

(c) (5) What is the distance from this quasar to the Earth?

(d) (5) With the effects of both motion and time dilation included, the formula for the Doppler effect associated with the full velocity $\vec{u}$ of an object emitting electromagnetic radiation is

$$v = v_0 \frac{\sqrt{1-(u/c)^2}}{1+v_r/c}$$

where v is the shifted frequency and v_r is the radial component of the velocity (i.e., the component in the direction away from the observer). If the tangential velocity is zero, and $u/c = v_r/c \ll 1$, obtain a very simple approximate formula for z in terms of u and c.

(e) (5) Calculate the approximate value of the recessional velocity u for our quasar.

3. Give a qualitative, but reasonably substantial, discussion of the typical nuclear reactions for each of the following. Please write down a representative nuclear reaction wherever you can.

(a) (4) A star with mass $M \approx 1 M_{\odot}$, or $M < 1 M_{\odot}$, burning H to He.

(b) (4) A star with mass $M > M_{\odot}$, burning H to He.

(c) (4) Later stages in the evolution of a star with $M > 8 M_{\odot}$.

(d) (4) Final stages in the evolution of a star with $M > 8 M_{\odot}$.

(e) (4) Remnant of a Type II supernova in the days and weeks after the explosion of the core.

4. Consider a binary system in which the two stars (or sun and planet, or planet and moon, or supermassive black hole and star, or pair of galaxies, etc.) have masses m_1 and m_2, with

$$M \equiv m_1 + m_2 \quad , \quad \frac{1}{\mu} \equiv \frac{1}{m_1} + \frac{1}{m_2}.$$

In parts (a) and (b), you may assume the results that we obtained in class for the angular momentum $\vec{L}$ and gravitational potential energy U in terms of the total mass M and reduced mass μ.

(a) (5) What is the expression for $\vec{L}$ in terms of μ, the relative coordinate $\vec{r}$, and the relative velocity $\vec{v} \equiv d\vec{r}/dt$?

(b) (5) What is the expression for U in terms of μ, r, and M?

(c) (5) In class, we derived the following results for the values of r and v at "perihelion" (closest approach) and "aphelion" (farthest apart):

$$r_p = a(1-e) \quad , \quad r_a = a(1+e) \quad , \quad v_p^{\,2} = \frac{1+e}{1-e}\frac{GM}{a} \quad , \quad v_a^{\,2} = \frac{1-e}{1+e}\frac{GM}{a}.$$

Here a is the semimajor axis and e is the eccentricity. Show that these results are consistent with angular momentum conservation, in the sense that the magnitude of L has the same value at perihelion and aphelion.

(d) (5) Is the direction of $\vec{L}$ the same at these two points? Draw and label a picture, and give a clear argument why it is or is not.

(e) (5) Evaluate the total energy at these two points (perihelion and aphelion). Is it the same?

5. (10) Consider a white dwarf, in which matter originally in a star with $\sim 10^6$ times the mass of the Earth is squeezed down to about the size of the Earth. Then the volume per electron in the white dwarf is $\sim 10^{-6}$ times the volume per electron in the atoms that make up the Earth, so the linear size per electron is roughly

$$\Delta x \sim 10^{-2} \, \overset{\circ}{A} = 10^{-12} \, \text{m} .$$

Use the Heisenberg uncertainty principle to estimate the speed of electrons in a white dwarf, and to determine whether relativistic effects are important.

6. The Stefan-Boltzmann equation relates the luminosity of a star to its temperature T and surface area $4\pi R^2$.

(a) (10) Linearize this equation to show that
$$\frac{\delta L}{L_0} = 2\frac{\delta R}{R_0} + 4\frac{\delta T}{T_0}$$

(b) (10) Linearize the relation for an adiabatic process,
$$TV^{\gamma-1} = \text{constant}$$
and so find a relation between $\delta L / L_0$ and $\delta R / R_0$ for a spherical blackbody model of a pulsating star composed of an ideal monatomic gas.

7. (15) A CN (cyanogen) molecule has a single rotational ground state (with $\ell = 0$, $m_\ell = 0$), and three rotational first excited states (with $\ell = 1$, $m_\ell = -1, 0, +1$). These excited states are degenerate, and lie 4.8×10^{-4} eV above the ground state. There are 27 molecules in the excited states for every 100 molecules in the ground state (so that $N_{excited} / N_{ground} = 0.27$). Assume that these molecules are in thermal equilibrium with the cosmic microwave background radiation, and use the Boltzmann factor $e^{-E/kT}$ to calculate the temperature of the CMB radiation.

Whatever your activities may be, have a fun summer!