Physics 634 Exam 2

Please be clear in all steps.

1. With the use of Feynman diagram rules and Feynman parameters, we obtained

$$\overline{u}\left(p+q\right)\,\delta\Gamma^{\mu}_{rel}\left(p+q,p\right)\,u\left(p\right)=$$
$$2ie^2\int\frac{d^4k}{\left(2\pi\right)^4}\int_0^1 dx\,dy\,dz\,\delta\left(x+y+z-1\right)\,\frac{2}{D^3}\,\overline{u}\left(p+q\right)\,i\sigma^{\mu\nu}q_\nu\,m\,z\left(1-z\right)\,u\left(p\right) \quad (1)$$

where

$$D=k^2-\Delta+i\epsilon \quad (2)$$

and

$$\Delta=-xyq^2+\left(1-z\right)^2m^2\,. \quad (3)$$

Also,

$$\delta\Gamma^{\mu}_{rel}\left(p+q,p\right)\;=\;\frac{i\sigma^{\mu\nu}q_\nu}{2m}F_2\left(q^2\right) \quad (4)$$

is the part of the vertex correction which contains $\sigma^{\mu\nu}$, and is therefore relevant in the calculation of the anomalous contribution to the magnetic dipole moment of the electron, as described by

$$a_e=\left(g-2\right)/2=F_2\left(0\right). \quad (5)$$

(a) (12) Using a Wick rotation, and the fact that

$$d\Omega_4=\sin^2\omega\,\sin\theta\,d\phi\,d\theta\,d\omega\,, \quad (6)$$

show that

$$\int d^4k\frac{1}{D^3}\;=-2\pi^2 i\,I \quad (7)$$

where

$$I=\int_0^\infty dk_E\,\frac{k_E^3}{\left(k_E^2+\Delta\right)^3}. \quad (8)$$

(b) (12) Now using integration by parts, show that

$$I=\frac{1}{4\Delta}. \quad (9)$$

(c) (12) Perform the integration over Feynman parameters to obtain a simple expression for $\left(g-2\right)/2$ in terms of the fine structure constant α. This is the result which was first obtained by Schwinger.

2. (a) (12) Write down the Feynman diagram for the vacuum polarization of quantum electrodynamics (in lowest order).

(b) (12) Translate this diagram into the mathematical expression for $i\Pi_2^{\mu\nu}(q)$ (involving, e.g., an integral over the internal momentum k).

(c) (12) We finally obtained

$$\alpha_{eff}\left(q^2\right) = \frac{\alpha}{1-(\alpha/3\pi)\log\left(-q^2/Am^2\right)} \quad \text{with } A = e^{5/3}. \tag{10}$$

In words, briefly explain the significance of this result, and also give a simple physical picture of how it arises.

3. (a) (14) By considering the infinite set of Feynman diagrams, show that the exact electron propagator S can be written in the form

$$S = \frac{i}{\not{p} - m_0 - \Sigma(p)}. \tag{11}$$

At the same time, define $\Sigma(p)$.

(b) (14) Near the single-particle pole, show that S has the form

$$S = \frac{iZ_2}{\not{p} - m}. \tag{12}$$

At the same time, define the physical mass m and the field renormalization constant Z_2.