Problem 3a.   A hydrogen atom is in its ground state at $t = -\infty$. An electric field, taken to be in the z direction, and with magnitude

$$\varepsilon\, e^{-t^2/\tau^2} \;,$$

is applied until $t = \infty$.

(a) Calculate the matrix elements $\langle 2\,\ell m | z | 100 \rangle$.

[Answer: $\dfrac{2^{15/2}}{3^5} a_0$ for the relevant one, and prove that the others are zero]

(b) Calculate, in first order, the probability that the atom ends up in any of the $n = 2$ states.

[Answer: $\left(\dfrac{e\varepsilon}{\hbar} a_0 \tau \right)^2 \dfrac{2^{15}}{3^{10}} \pi\, e^{-\omega_{21}{}^2 \tau^2/2}$ where $\omega_{21} \equiv \dfrac{\mathcal{E}_2 - \mathcal{E}_1}{\hbar}$]