Physics 314 (Survey of Astronomy) Exam 1

Please show all significant steps clearly in all problems.
Please give clear and legible responses to qualitative questions.
See the last page for values of constants.

1. Nuclear reactions in stars

(a) (3) Write down the proton-proton chain of reactions that converts H to He, being careful to show all the particles in each reaction.

See Eqs. (6.10) – (6.12).

(b) (3) Write down the triple alpha set of reactions that converts He to C. What qualitative statement can you make about the lifetime of ^{8}Be and its significance?

See Box 6.2. Since the lifetime of these nuclei is very short, the reaction requires that they be continually produced at high temperature and density.

(c) (3) Why does fusion produce energy for light elements, and fission for heavy elements?

See Fig. 6.7 and p. 116..

(d) (3) What is the heaviest element that can be produced in normal fusion reactions in stars, and why?

Iron 56, see p. 116.

(e) (3) How are heavier elements produced, with larger atomic numbers Z? E.g., how were the selenium ($Z = 34$) and iodine ($Z = 53$) in your body produced?

Supernovae, see p. 119.

2. Some simple ideas

(a) (2) Why does a solar eclipse not occur whenever there is a new moon, and a lunar eclipse whenever there is a full moon?

See `http://www.astronomycafe.net/qadir/ask/a11517.html` for example. We gave a more detailed discussion with a picture in class.

(b) (3) By considering a rotating frame of reference, with a (fictitious) centrifugal force, explain how the Sun produces tides. (The same principle applies with the more important influence of the Moon, but let us consider only the Sun here for simplicity.) It may help to draw a picture.

Our simplistic picture involves a gravitational force which is proportional to $1/r^2$ and a centrifugal force (in the rotating frame of reference) which is proportional to r, with the Earth turning under the resulting bulges toward and away from the Sun. See also Fig. 1.7.

3. (10) Average photon energy at temperature T

For a gas of photons, the average value of the energy pc is given by an expression that is similar to that for the average kinetic energy of an ordinary molecule in a gas:

$$\langle pc \rangle = \frac{\int_0^\infty e^{-pc/kT} pc \cdot 4\pi p^2 dp}{\int_0^\infty e^{-pc/kT} 4\pi p^2 dp}.$$

Using the transformation of variables $x = pc / kT$, and then integrating by parts in the numerator (with $u = x^3$ and $v = e^{-x}$), obtain the average value $\langle pc \rangle$ of the photon energy in a photon gas at temperature T.

The answer for these extremely relativistic particles is $3kT$, in contrast to the answer $3kT / 2$ for nonrelativistic particles that you got in the similar Problem 4.3, for which you have a solution.

4. Identification of a white dwarf

(a) (10) A star is seen with a blackbody radiation spectrum whose peak (i.e., maximum intensity as a function of wavelength) is at a wavelength of 500 nm (with 1 nm $= 10^{-9}$ meter). Calculate the surface temperature of this star.

The Wien displacement law gives 5800 K.

(b) (10) From observations of its motion, and other observational data, this star is known to lie within a globular cluster that is 7.2×10^3 light years away. When its luminosity L is compared with that of a star at the same distance which is essentially identical to our Sun, with a radius $R_\odot$, luminosity $L_\odot$, and surface temperature of 5800 K, it is found that $L = 10^{-4} L_\odot$. Calculate the radius R of this star in terms of $R_\odot$. (I.e., obtain $R / R_\odot$.)

With T the same, the Stefan-Boltzmann law implies that the surface area is 10^{-4} times that of the Sun, so the radius is $0.01\, R_\odot$, or about the size of the Earth.

5. Formation of a neutron star

(a) (10) Using the Heisenberg uncertainty principle, estimate the energy of an electron confined to the typical size of a nucleus or nucleon. This is then roughly the Fermi energy that an electron would have in nuclear matter – i.e., the maximum kinetic energy of an electron in this ultradense matter.

With a position uncertainty of about 1 fermi, and the momentum taken to roughly equal its uncertainty, the order-of-magnitude estimate is roughly 10 – 100 GeV, with 1 GeV = 1000 MeV – i. e., very high.

(b) Now use the above result to estimate the energy released if this electron reacts with a proton to give a neutron and a neutrino. The energy released is the Fermi energy of this electron + the rest mass energy of an electron + the rest mass energy of a proton – the rest mass energy of a neutron. (The neutrino has a mass, but it is so small that the rest mass energy of the neutrino can be neglected.)

The rest mass energies of proton and neutron are roughly 1 GeV, so they can be neglected in a crude order-of-magnitude estimate, and the answer is again roughly 10 – 100 GeV.

6. (20) Spinning neutron star

In this problem you will need to remember some basic astronomy facts. Namely, you will need very approximate values for the size of the Sun relative to the Earth, the period of rotation of the Sun, and the size of a neutron star (in each case, to within about an order of magnitude). If you do not remember, you will still get partial credit. You are given that the radius of the earth is about 6400 km, and that the moment of inertia of a sphere with radius R and mass M is $I = (2/5)MR^2$.

A star with a mass $M \approx 10M_{\odot}$ and a radius $R \approx 10R_{\odot}$ uses up its nuclear fuel, collapses, and explodes in a supernova. A neutron star is left behind, with a mass $M_n \approx 2M_{\odot}$ and only about 1% of the angular momentum $I\omega$ of the original star, where ω is the angular velocity in rad/sec.

Using the above information, obtain a rough order-of-magnitude estimate of the period of rotation of the neutron star. Is your answer within one or two orders of magnitude of the measured periods for pulsars like those in the Crab and Vela nebulas?

The Sun is roughly 100 times as large in diameter as the earth, and it takes about a month to rotate once. A neutron star is roughly 10 km in radius. The calculated period then turns out to be roughly 0.01 second, or 10 msec, which is comparable to the periods of these pulsars – e.g. 1/30 sec for the Crab.

7. Bohr theory for an ion with atomic number Z

Consider an atom with an atomic number Z which has had all but one of its electrons stripped away.

(a) (2) Which of the following ions or atoms qualify for this description? Circle all the ones that do qualify. (The first 3 elements in the periodic table are H, He, Li.)

He I, He II, Li I, Li II, Li III, Fe I, Fe II, Fe III

Answers are He II and Li III, which each have one electron left.

(b) (3) As usual, we adopt the picture that the electron (with mass m) moves in a circular orbit with radius r and velocity v. For a general Z, write down the equation which equates the centripetal force on the electron to the electrostatic force. This gives one equation involving r and v.

In this problem you reproduce what we did in class, and repeated in homework problems 3.5 and 3.6, except that ke^2 is replaced by Zke^2, with $Z = 2$ rather than 1.

(c) (3) Write down Bohr's postulate for the quantization of angular momentum. (Alternatively, assume that $n\times$ the electron's de Broglie wavelength fits into the circumference of its orbit.) For a given value of the quantum number n=1, 2, 3, …., this gives a second equation involving r and v.

(d) (4) Using the relations in parts (b) and (c) above (two equations in the two unknowns r and v), solve for the allowed values of r in terms of n, Z, and the other constants.

(e) (4) Using the expressions for the kinetic energy and electrostatic potential energy of the electron, obtain its total energy in terms of n, Z, and the other constants.

(f) (4) Now consider He, for which $Z = 2$. Calculate the energy of a photon that is emitted when the electron falls from the $n = 2$ state to the $n = 1$ state in this singly ionized He atom. Give your final answer in eV. How does this compare with the energy when the electron in an H atom undergoes the same transition, from first excited state to ground state?

The energies are proportional to $\left(Zke^2\right)^2$, or Z^2. For hydrogen we would get 10.2 eV, and for helium we get a result that is 4 times larger, or about 41 eV.