Physics 314 (Survey of Astronomy) Exam 2

Please show all significant steps clearly in all problems.
Please give clear, legible, and reasonably complete (although brief) responses to qualitative questions.
See the last page for values of constants.

1. Black holes

(a) (4) Calculate the escape velocity from the surface of the Earth, which has a radius of 6.37×10^{6} meters and a mass of 5.98×10^{24} kg. (The sum of the kinetic energy and gravitational potential energy is the same at the Earth's surface as at infinity, so you can first algebraically solve for the escape velocity in terms of the known quantities.)

As in Eq. (7.3) of the textbook,

$$v_{esc} = \sqrt{\frac{2GM}{R}} = 1.12\times10^{4} \text{ m/s.}$$

(b) (4) Again using nonrelativistic physics and Newtonian gravity, just as in part (a), calculate the Schwarzschild radius of a black hole with mass M. Comment on the validity of each of these two approximations for very strong gravitational fields, and on the validity of the final answer.

See Eq. (7.4), and the first paragraph on p. 135.

(c) (4) Calculate the numerical value of the Schwarzschild radius for a supermassive black hole with a mass of 3×10^{6} $M_{\odot}$. Give your answer in light hours. (1 hour = 3600 sec.)

Answer is 8.85×10^{9} m = 0.0082 light hour.

(d) (4) Is there evidence for a supermassive black hole like that in part (c)? Where is it, and what is the evidence?

Center of our Milky Way, as shown by orbits of stars around it (which establish its mass and an upper limit on its spatial extent), and various kinds of emissions (including radio and X-ray).

(e) (4) What are the identifying observable characteristics of a stellar-sized black hole in a binary system?

X-ray emission from matter in accretion disc spiraling in, absence of material surface (no impact with such a surface evidenced in emissions), mass great than maximum allowed for white dwarf ($1.4\ M_\odot$) or neutron star (about $2.6\ M_\odot$ even in theory, with observed neutron stars more like $1.4\, M_\odot$).

(f) (4) Explain the origin of Hawking radiation from a black hole. (You may give either a simple argument, based on the uncertainty principle and virtual particles, or a highly technical description, based on creation and destruction operators, but the former is preferred here, even though an occasional exposure to technical details is good for us!)

See p. 141.

(g) (4) Using the uncertainty principle for time and energy, and the masses given on the last page of this exam, estimate how long a virtual electron-positron pair will last.

See solution to problem 7.11. Answer is 6.43×10^{-22} s if $\hbar$ is used in uncertainty principle.

2. **The cosmic distance ladder**

(a) (4) How can distances to nearby stars be measured directly, using the fact that the Earth moves to a position on the opposite side of the Sun in half a year? (A picture might help.)

Parallax, pp. 5 and 32, for example.

(b) (4) Why does a Cepheid variable star have an instability that causes it to pulsate, whereas a normal star does not?

We discussed this in class, referring to problem 9.7, and adding the fact that that the opacity is mainly due to H^- ions in ordinary stars, but He^+ and H^+ in Cepheids.

(c) (4) What is a typical range of periods for a Cepheid, and for an RR Lyrae?

See bottom of right column of p. 173, and top of left column of p. 174.

(d) (4) How can these stars be used to measure distances – e.g., to nearby galaxies?

See p. 172.

(e) (4) What are the Tully-Fisher and Faber-Jackson relations, and how can they be used to measure even larger distances?

See Eq. (13.5) and discussion on p. 305.

3. Gravity versus nuclear reactions

The mass of a ^{4}He nucleus is 4.0026032 amu, and the mass of a ^{12}C nucleus is (by definition) 12.0000000. The mass of a mole in grams is numerically equal to the nuclear (or, more generally, molecular) mass in amu (also called u); e.g., the mass of a mole of ^{12}C nuclei is exactly 12 grams.

(a) (10) Suppose 3 moles of ^{4}He nuclei fall (from a distant companion star in a binary system) onto the surface of a neutron star with a radius of 15 km and a mass of 1.5 $M_\odot$, where $M_\odot = 1.99 \times 10^{30}$ kg. Calculate the gravitational potential energy released.

With all quantities expressed in SI units (m, kg, s), but the units explicitly shown only in the final answer,

$$\frac{GMm}{R} = \frac{\left(6.67 \times 10^{-11}\right)(1.5)\left(1.99 \times 10^{30}\right)(3)(4.0026032)\left(10^{-3}\right)}{15000} = 1.59 \times 10^{14} \text{ J}.$$

(b) (10) Now suppose the 3 moles of ^{4}He reacts to form 1 mole of ^{12}C. Calculate the energy released in this nuclear reaction.

Again in SI units,

$$\left[(3)(4.0026032) \text{ -12.0000000}\right]\left(10^{-3}\right)\left(3.00 \times 10^{8}\right)^2 = 7..03 \times 10^{11} \text{ J}.$$

(c) (2) Calculate the ratio of these two energies: energy released in (a)/energy released in (b).

$$\frac{1.59 \times 10^{14}}{7..03 \times 10^{11}} = 226 \text{ which is } \sim 100$$

4. H-R Diagram

(a) (5) Draw a schematic picture of the H-R diagram, indicating what is plotted along the vertical and horizontal axes, and showing very roughly how the main sequence stars lie in this diagram.

See e.g. Fig. 9.7.

(b) (5) Consider a star with a mass of 30 $M_{\odot}$. Very roughly how long will this star live, how will it move in the H-R diagram as it gets older, what sequence of events will transpire in its interior, and what will be its final fate? Give a rough indication of some of the thermonuclear reactions. What is the "helium flash"?

Perhaps a million years – see e.g. Fig. 9.9, where its motion in the H-R diagram is also shown. The other questions are answered on pp. 149-155 – e.g. Fig. 8.13.

5. Virial theorem for harmonic oscillator model of atomic nucleus

Suppose that each nucleon (proton or neutron) in an atomic nucleus is effectively restrained by a harmonic oscillator force $F = -kr$, where r is the distance from the center of the nucleus. Let us also treat the nucleons as nonrelativistic classical particles (whose motion is obviously bounded).

(a) (5) Using $F = -dV/dr$ or $V = -\int F\,dr$ (with the integration constant set equal to zero), obtain the potential energy $V(r)$ of a nucleon in terms of k and r.

$$V = +k\int r\,dr = \frac{1}{2}kr^2$$

(b) (5) Write down the expression for the time average $\langle f \rangle$ of an arbitrary physical quantity $f(t)$, with the average taken from $t = 0$ to $t = T$.

$$\langle f \rangle = \frac{1}{T}\int_0^T f(t)\,dt$$

(c) (10) By considering the time average of the quantity

$$\frac{d(\vec{p}\cdot\vec{r})}{dt}$$

obtain a relation between the average potential energy and the average kinetic energy of a nucleon in our model. (Recall the physical meaning of $d\vec{p}/dt$ and $d\vec{r}/dt$, and recall that $\vec{A}\cdot\vec{B} = AB$ if the vectors $\vec{A}$ and $\vec{B}$ both point in the positive radial direction.) Your final result will have the form $\langle PE \rangle = \textit{something} \times \langle KE \rangle$, where you will determine the *something*.

$$\left\langle \frac{d(\vec{p}\cdot\vec{r})}{dt} \right\rangle = \frac{1}{T}\int_0^T \frac{d(\vec{p}\cdot\vec{r})}{dt}\,dt = \frac{1}{T}\left[\vec{p}\cdot\vec{r}\right]_0^T = 0$$ in the limit $T \to \infty$ since $\vec{p}\cdot\vec{r}$ is bounded

Also, $\left\langle \frac{d(\vec{p}\cdot\vec{r})}{dt} \right\rangle = \left\langle \frac{d\vec{p}}{dt}\cdot\vec{r} \right\rangle + \left\langle \vec{p}\cdot\frac{d\vec{r}}{dt} \right\rangle = \langle \vec{F}\cdot\vec{r} \rangle + \langle \vec{p}\cdot\vec{v} \rangle = \langle -kr^2 \rangle + \langle mv^2 \rangle$

So $\left\langle \frac{1}{2}kr^2 \right\rangle = \left\langle \frac{1}{2}mv^2 \right\rangle$ or $\langle PE \rangle = \langle KE \rangle$.