POLS 603 Spring 2010
Homework Exercises #3

(1) How many different five-card poker hands can be dealt from a deck of 52 cards?

(2) Compute the probability of being dealt four of a kind in a poker game.

(3) If x has a normal distribution with mean 1 and standard deviation 3, what are the following?
   (a) \( \text{Prob}[|x| > 2] \)
   (b) \( \text{Prob}[x > -1|x < 1.5] \)

(4) Approximately what is the probability that a random variable with chi-squared distribution with 264 degrees of freedom is less than 297? (HINT: Use B-37).

(5) Chebychev inequality. For the following two probability distributions, find the lower limit of the probability of the indicated event using the Chebychev inequality (B-18) and the exact probability using the appropriate table.
   (a) \( x \sim \text{normal}[0,3^2] \) and \(-4 < x < 4\)
   (b) \( x \sim \text{chi-squared}, 8 \text{ degrees of freedom}, 0 < x < 16\).

(6) Use the following joint probability distribution,

<table>
<thead>
<tr>
<th>Y</th>
<th>0</th>
<th>1</th>
<th>2</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0.05</td>
<td>0.1</td>
<td>0.03</td>
</tr>
<tr>
<td>1</td>
<td>0.21</td>
<td>0.11</td>
<td>0.19</td>
</tr>
<tr>
<td>2</td>
<td>0.08</td>
<td>0.15</td>
<td>0.08</td>
</tr>
</tbody>
</table>

   to compute the following.

   (a) compute the following probabilities:
      (1) \( \text{Prob}[Y < 2] \)
      (2) \( \text{Prob}[Y < 2, X > 0] \)
      (3) \( \text{Prob}[Y=1, X \geq 1] \)
   (b) Find the marginal distributions of X and Y.
   (c) Calculate \( E[X], E[Y], \var[X], \var[Y], \cov[X,Y], \text{and } E[X^2 Y^3] \)
   (d) Calculate \( \cov[Y, X^2] \)
   (e) What are the conditional distributions of Y given \( X=2 \) and of X given \( Y > 0 \)?
   (f) Find \( E[Y|X] \) and \( \var[Y|X] \). Obtain the two parts of the variance decomposition.

(7) If the probability density of \( y \) is \( \alpha y^2(1-y)^3 \) for \( y \) between 0 and 1, then what is \( \alpha \)?
   What is the probability that \( y \) is between 0.25 and 0.75?
(8) Interpolation in the chi-squared table. To find a percentage point in the chi-squared table that is between two values, we interpolate linearly between the reciprocals of the degrees of freedom. The chi-squared distribution is defined for noninteger values of the degrees of freedom parameter, but your table does not contain critical values for noninteger values. Using linear interpolation, find the 99% critical value for a chi-squared variable with degrees of freedom parameter 11.3.

(9) Suppose that $x_1$ and $x_2$ distributed as independent standard normal. What is the joint distribution of $y_1 = 2 + 3x_1 + 2x_2$ and $y_2 = 4 + 5x_1$. (Hint: see B.7.4).

(10) If $x$ has a normal distribution with mean $\mu$ and standard deviation $\sigma$, what is the probability distribution of $y = e^x$? (HINT: use the change of variables technique).