POLS 603 Spring 2010
Homework #4

(1) The following sample is drawn from a normal distribution with mean μ and standard deviation σ:

X=1.3, 2.1, 0.4, 1.3, 0.5, 0.2, 1.8, 2.5, 1.9, 3.2

Compute the mean, median, variance, and standard deviation of the sample.

(2) Using the data in the previous exercise, test the following hypotheses:

$$\mu \geq 2.0$$
$$\mu \leq 0.7$$
$$\sigma^2 = 0.5$$

Using a likelihood ratio test, test the following hypotheses:

$$\mu = 1.8$$
$$\sigma^2 = 0.8$$

(3) Suppose that the following sample is drawn from a normal distribution with mean μ and standard deviation σ

Y=3.1, - 0.1, 0.3, 1.4, 2.9, 0.3, 2.2, 1.5, 4.2, 0.4

Test the hypothesis that the mean of the distribution that produced these data is the same as the mean that produced the data in Exercise 1. Test the hypothesis assuming that the variances are the same. Test the hypothesis that the variances are the same using an F test and using a likelihood ratio test. (Do not assume that the means are the same).

(4) Using the data in Exercise 1, form confidence intervals for the mean and standard deviation.

(5) Based on a sample of 65 observations from a normal distribution, you obtain a median of 34 and a standard deviation of 13.3. Form a confidence interval for the mean. [HINT: Use the asymptotic distribution. See Example D24.] Compare your confidence interval with the one you would have obtained had the estimate of 34 been the sample mean instead of the sample median.

(6) In random sampling from the exponential distribution $f(x) = \frac{1}{\theta} e^{-\frac{x}{\theta}}$, $x \geq 0, \theta > 0$, find the maximum likelihood estimator of Θ and obtain the asymptotic distribution of this estimator.

(7) Suppose that in a sample of 500 observations from a normal distribution with mean μ and standard deviation σ, you are told that 35% of the observations are less than 2.1 and 55% of the observations are less than 3.6. Estimate μ and σ.

(8) Testing for normality. One method that has been suggested for testing whether the distribution underlying a sample is normal is to refer the statistic

$$L = n[\text{skewness}^2/6 + (\text{kurtosis} - 3)^2/24]$$

to the chi-squared distribution with two degrees of freedom. Using the data in Exercise 1, carry out the test.