Economics 475-500 | Economic Forecasting
Study Guide for Exam 2 | Spring 2024

The second exam will be administered on Thursday, March 28 and will cover material in lecture since the first exam. The format of the exam will be short answer, derivations, and application/interpretation problems.

Be able to describe conceptually how Holt-Winter exponential smoothing – with trend and seasonal effects – works.

Be able to "solve" and AR(1) of the form $y_t = \phi_1 y_{t-1} + \varepsilon_t$ into its MA(∞) representation by repeated substitution.

Know the statistical properties of White Noise.

Know the (general) statistical properties of a stationary time series variable.

Know the conceptual definitions of the Auto-Correlation Function (ACF) and Partial Auto-Correlation Function (PACF).

Be able to derive the forms of the ACF and PACF for a MA(1) model.

Given an example correlogram, be able to infer the nature of the model.

Given specific ARIMA(p,1,q) model output for a non-seasonal variable, be able to write out the forecasting equation for out of sample forecasts.

Given specific ARIMA(p,0,q)(P,1,Q) model output for a seasonal series, be able to write out forecasting equations for out of sample forecasts.

Example Problems

1. Suppose you fit an ARIMA(2,1,2) model to the U.S. – EU exchange rate (USEU_XR) to get the following estimated model:

ARIMA (2,1,2)			
ar1	ar2	ma1	ma2
0.5745	-0.0016	-0.2587	-0.1469

The last 6 months of actual data, fitted values, and estimated residual terms (e_t) are as follows:

Date	USEU_XR	Fitted	Residual
9/1/2020	1.1785	1.1933	-0.0148
10/1/2020	1.1768	1.1753	0.0015
11/1/2020	1.1826	1.1776	0.0050
12/1/2020	1.2168	1.1844	0.0324
1/1/2021	1.2178	1.2273	-0.0095
2/1/2021	1.2094	1.2160	-0.0066

Write out the forecasting equations for March and April 2021, ($Y_{T+1|T}$ *and* $Y_{T+2|T}$). (You do not need to calculate the forecasts.)

Solution:

First, write out the equation in conceptual lag operator form:

$$(1 - \phi_1 L - \phi_2 L^2)(1 - L)Y_t = (1 + \theta_1 L + \theta_2 L^2)\varepsilon_t$$

Expand and collect terms to get:

$$\left[1-\left(1+\phi_1\right)L+\left(\phi_1-\phi_2\right)L^2+\phi_2L^3\right]Y_t=\left(1+\theta_1L+\theta_2L^2\right)\varepsilon_t$$

Rearranging and applying the lag operator:

$$Y_{T+1|T}=\left(1+\phi_1\right)Y_T-\left(\phi_1-\phi_2\right)Y_{T-1}-\phi_2Y_{T-2}+\theta_1\varepsilon_T+\theta_2\varepsilon_{T-1}$$

In numbers:

$$Y_{T+1|T}=(1+0.5745)1.2094-(0.5745+0.0016)1.2178+(0.0016)(1.2168)+(0.2587)(0.0066)$$
$$+(0.1469)(0.0095)$$

For the T+2 forecast:

$$Y_{T+2|T}=\left(1+\phi_1\right)Y_{T+1|T}-\left(\phi_1-\phi_2\right)Y_T-\phi_2Y_{T-1}+\theta_1\varepsilon_{T+1}+\theta_2\varepsilon_T$$

And:

$$Y_{T+2|T}=(1+0.5745)Y_{T+1|T}-(0.5745+0.0016)1.2094+(0.0016)(1.2178)+(0.1469)(0.0066)$$

The $\theta_1\varepsilon_{T+1}$ goes to zero because the best estimate of the white noise error for T+1 is its mean, zero.

2. Suppose you fit an ARIMA(1,0,0)(1,1,1)[4] model to a Quarterly series for Candy production to get the following estimated model:

ARIMA(1,0,0)(1,1,1)[4]		
ar1	sar1	sma1
0.765	0.069	-0.642

The last 10 quarters of actual data, fitted values, and estimated residual terms (e_t) are as follows:

Year/Quarter	Actual	Fitted	Residual
2019-Q3	99.28	99.09	0.19
2019-Q4	111.33	110.01	1.31
2020-Q1	97.99	102.39	-4.41
2020-Q2	81.88	91.08	-9.20
2020-Q3	95.88	89.38	6.50
2020-Q4	112.90	108.06	4.84
2021-Q1	101.99	101.49	0.49
2021-Q2	91.84	90.22	1.62
2021-Q3	96.55	99.77	-3.22
2021-Q4	124.37	110.59	13.78

Write out the forecasting equations for 2022-Q1 and 2022-Q2, ($Y_{T+1|T}$ *and* $Y_{T+2|T}$). (You do not need to calculate the forecasts.)

Solution:

First, write out the model in conceptual lag operator form:

$$\left(1-\phi_1L\right)\left(1-\Phi_1L^4\right)\left(1-L^4\right)Y_t=\left(1+\Theta_1L^4\right)\varepsilon_t$$

Expand to get:

$$\left[1-\phi_1L-\left(1+\Phi_1\right)L^4+\phi_1\left(1+\Phi_1\right)L^5+\Phi_1L^8-\phi_1\Phi_1L^9\right]Y_t=\left(1+\Theta_1L^4\right)\varepsilon_t$$

Rearranging and applying the lag operator:

$$Y_{T+1|T}=\phi_1Y_T+\left(1+\Phi_1\right)Y_{T-3}-\phi_1\left(1+\Phi_1\right)Y_{T-4}-\Phi_1Y_{T-7}+\phi_1\Phi_1Y_{T-8}+\Theta_1\varepsilon_{T-3}+\varepsilon_{T+1}$$

Substitute the ar1 estimate for ϕ_1, the sar1 estimate for Φ_1, and the sma1 estimate for Θ_1 along with the appropriate lagged actual and residual terms. Any out-of-sample error terms go to zero.

For the $Y_{T+2|T}$ forecast:

$$Y_{T+2|T} = \phi_1 Y_{T+1|T} + (1 + \Phi_1)Y_{T-2} - \phi_1(1 + \Phi_1)Y_{T-3} - \Phi_1 Y_{T-6} + \phi_1 \Phi_1 Y_{T-7} + \Theta_1 \varepsilon_{T-2} + \varepsilon_{T+2}$$