

The third exam will be administered on Tuesday, April 23rd. The format of the exam will be short answer and application – interpretation problems.

Given specific model output for a VAR system, be prepared to show how to construct point forecasts within the system.

Be prepared to explain conceptually what constitutes “co-integration” among two or more time-series variables and, again conceptually, how to test whether two variables are co-integrated.

Given specific output from an ARMA model and associated GARCH model of conditional variance, be prepared to show how to construct point forecasts of the conditional variance.

Example Problems

VAR

1. Presented below are the model results for a to equation VAR system for Housing Permits (“HPermits”) and Housing Sales. Data are monthly over the period Jan. 1991 – Jul. 2020, and the system is estimated based on the log-seasonal differences in the series. That is:

$$HSales_t = \ln(Sales_t) - \ln(Sales_{t-12})$$

$$HPermits_t = \ln(Permits_t) - \ln(Permits_{t-12})$$

The “.11” and “.12” below indication “t-1,” and “t-2,” respectively.

HPermits = HPermits.11 + HPermits.12 + HPermits.11 + HPermits.12 + const				
	Estimate	Std. Error	t value	Pr(> t)
HPermits.11	0.697	0.066	10.555	0.000
HPermits.12	-0.233	0.087	-2.678	0.008
HPermits.11	0.264	0.068	3.894	0.000
HPermits.12	0.049	0.087	0.566	0.572
const	0.009	0.006	1.528	0.127

HSales = HPermits.11 + HPermits.12 + HSales.11 + HSales.12 + const				
	Estimate	Std. Error	t value	Pr(> t)
HPermits.11	0.140	0.052	2.689	0.008
HSales.11	0.410	0.069	5.964	0.000
HPermits.12	0.088	0.054	1.640	0.102
HSales.12	0.001	0.069	0.011	0.991
const	0.017	0.005	3.673	0.000

- a. Write out the forecast equations for HSales for August (T+1) and September (T+2), 2020.

Solution:

$$HSales_{T+1} = 0.017 + 0.140 \times HPermits_T + 0.410 \times HSales_T + 0.088 \times HPermits_{T-1} + 0.001 \times HSales_{T-1}$$

$$HSales_{T+2} = 0.017 + 0.140 \times HPermits_{T+1} + 0.410 \times HSales_{T+1} + 0.088 \times HPermits_T + 0.001 \times HSales_T$$

- b. Given the data below, show how you would recover forecasts for the level of Sales, $Sales_{T+1}$ and $Sales_{T+2}$

Date	Permits	Sales	ln(Permits)	ln(Sales)
6/1/2019	9976	34101	9.208	10.437
7/1/2019	10652	36279	9.274	10.499
8/1/2019	11441	35477	9.345	10.477
9/1/2019	10206	29220	9.231	10.283
10/1/2019	11415	29473	9.343	10.291
11/1/2019	9199	25811	9.127	10.159
12/1/2019	9343	29918	9.142	10.306
1/1/2020	11212	21042	9.325	9.954
2/1/2020	11314	24881	9.334	10.122
3/1/2020	11984	30803	9.391	10.335
4/1/2020	9396	26111	9.148	10.170
5/1/2020	9713	27456	9.181	10.220
6/1/2020	12032	38666	9.395	10.563
7/1/2020	14216	43917	9.562	10.690

Solution:

Forecast of $HSales_{T+1}$ is the seasonal-log difference for August 2020 – need $\ln(Sales_{August\ 2019}) = 10.477$ to “undo” the seasonal difference

$$Sales_{T+1} = \exp(HSales_{T+1} + 10.477)$$

For September 2020:

$$Sales_{T+2} = \exp(HSales_{T+2} + 10.283)$$

2. For a model of the weekly return on the S&P 500 (measured as the log-difference in the weekly Index level) over the period 1/8/1990 – 4/10/2023, you specify an ARMA(1,0) for the weekly return series and a GARCH(1,1) process for the conditional variance of weekly returns to get the following results:

	Estimate	Std. Error	t value	Pr(> t)
mu	0.230	0.044	5.282	1.28E-07
ar1	-0.124	0.026	-4.689	2.75E-06
omega	0.267	0.060	4.418	9.96E-06
alpha1	0.195	0.024	7.973	1.55E-15
beta1	0.774	0.026	30.028	< 2e-16

Date	Return (%)	h_t	e_t	e_t^2
3/6/2023	1.648	4.654	1.371	1.880
3/13/2023	-4.876	4.234	-4.902	24.028
3/20/2023	2.454	8.222	1.620	2.624
3/27/2023	0.655	7.140	0.729	0.531
4/3/2023	3.629	5.895	3.480	12.107
4/10/2023	-0.374	7.185	-0.155	0.024

Write out the forecasting equations for the conditional variance of returns for the next two weeks: h_{T+1} and h_{T+2}

Solution (Note that is fine to use the coefficient names rather than the estimates).

The forecast of h_{T+1} uses h_T and e_T^2

$$h_{T+1} = \text{omega} + \text{alpha1} \times 0.024 + \text{beta1} \times 7.185$$

$$h_{T+2} = \text{omega} + (\text{alpha1} + \text{beta1}) \times h_{T+1}$$