

# Self Reported Baseline Mechanisms for Demand Response Programs

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**Abstract**—Demand Response (DR) is used to reduce the demand for electricity at times when the supply is scarce and expensive. Participating consumers are paid for reducing their energy consumption from an established baseline. Baselines are estimates of the counter-factual consumption against which the load reductions are estimated for deciding payments to consumers. This creates an incentive for consumers to inflate their baseline and increase the payments they receive. There are celebrated cases of consumers gaming this baseline to derive economic benefit. In the mechanism proposed here, each consumer forecasts her baseline consumption and reports her marginal utility to the aggregator who manages the DR program. The aggregator selects a set of consumers for each DR event to meet the load reduction requirement and pays them according to the observed reductions from their reported baseline. By a suitable reward and penalty for deviation from the reported baseline we show that reporting true baseline consumption and marginal utility is both incentive compatible and individually rational. Hence the mechanism establishes the correct baseline and the aggregator is able to meet any random load reduction requirement

can support balancing supply and demand to compensate for the variability of renewable generation. Adeptly managing flexible demand is a far better alternative to increased reserve generation, since it produces no emissions and consumes no resources. This is recognized and encouraged by the Federal Energy Regulatory Commission through its Order 745, which mandates that demand response be compensated on par with the conventional generation that supplies grid power. This paper, however, is concerned only with peak shaving DR applications.

There are three key components of any DR program that need to be designed: (a) a baseline against which demand reduction is measured, (b) the payment scheme to customers who reduce their consumption from this baseline, and (c) various terms and conditions such as limits on the frequency of DR events or penalties for nonconforming customers. Commonly used baselines include historical averages of consumption on similar days (by the consumer, or by a peer group of similar consumers). However, there are celebrated cases where the participants artificially inflated their baseline for extracting more payment [1]. Also, there is a problem of under payment and over payment because of the inaccurate baseline. This can result in inadequate consumer response.

**Our Contribution:** We approach baseline reporting as a mechanism design problem from the perspective of the demand response aggregator. Each consumer submits the values of her baseline consumption and marginal utility to the aggregator. The objective of the aggregator is to design an incentive mechanism such that (i) each consumer reports her true baseline and true marginal utility, (ii) meet any (random) load reduction requirement by effective scheduling and (iii) scheduling is efficient.

We propose a mechanism where reporting the true baseline consumption and marginal utility is a dominant strategy for each consumer. We also show that each consumer sticks to their baseline consumption when she is not called for DR and reduces maximum possible load when she is called for DR. Also, the aggregator can ensure adequate response to meet the load reduction requirement. The proposed mechanism is also nearly efficient since it selects consumers with the smallest marginal utilities where each selected consumer contributes the maximum possible load reduction. We also propose a second mechanism with a uniform payment scheme. In this second mechanism, under some assumptions, we show that truthful reporting of baseline consumption and marginal utility is a Nash equilibrium. Using simulations, we argue that such a

## I. INTRODUCTION

The core problem in power systems operations is to maintain fine balance of electricity supply and demand. This must be done economically while respecting reliability constraints. At certain times such as mid-afternoons on hot summer days, the supply of additional electric power is scarce and expensive. At these times, it is more cost-effective to reduce demand than to increase supply to maintain power balance.

Demand Response (DR) programs are used to reduce the demand for electricity. In these programs, aggregators recruit residential or industrial customers who are willing to reduce their electricity consumption at certain times. The aggregator serves as an intermediary and represents these flexible consumers to the system operator (SO). The aggregator typically receives a capacity payment from the SO for ability to reduce demand at short notice, and in turn, pays the customers it represents for their flexibility.

While most DR programs are limited to infrequent peak shaving applications, it is recognized that demand flexibility offers the potential to offer more lucrative ancillary services such as frequency regulation or load-following. These applications

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Nash equilibrium is an expected outcome when the number of consumers is very large.

**Related Work:** Traditional DR programs reward customers for load reduction during peak consumption periods. Consumers have an incentive to artificially inflate their baseline to increase their profits [2], [3], [4], [5]. These incentives persist even when the probability of occurrence of the DR event is low [3]. Alternative payment mechanisms that avoid resulting inefficiencies are offered in [3], but these do not explicitly address baseline inflation concerns. Adverse selection and double payment effects are two other issues that arise as a result of rewarding consumers based on load reduction from estimated baselines [2]. Addressing these gaming issues while ensuring fairness to participating consumers is essential to encourage and sustain wider use of DR programs.

Chen *et al.* [6], consider penalties when the consumer deviates from the baseline. The authors propose a two-stages game for DR. It is shown that the penalty (linear in deviations) induces users to report their true baseline assuming knowledge of consumer's utility function. The aggregator realizes its DR objective by tuning the retail price. In [7], assuming fixed costs for DR participation and linear costs for deviating from a known baseline, a centralized DR scheduling algorithm is proposed guaranteeing incentive compatibility in the case of two participants. A DR market assuming known baselines is proposed in [8], [9] where the objective is to maximize a social benefit function in the DR market. The approaches in [6], [7], [8] and [9] either assume knowledge of utility function or baselines. The remainder of the paper is organized as follows. We describe our problem formulation in Section II. Section III introduces the self-reported baseline mechanism. In Section IV we introduce a uniform payment mechanism. In section V we compare the proposed mechanism with that of the averaging schemes used by CAISO. We conclude in Section VI.

## II. MODEL AND PROBLEM FORMULATION

We consider a setting where the baseline-based DR program is managed by a single aggregator with  $N$  participating consumers. The aggregator's role is to manage the DR program for these consumers - enrolling them for the DR program, sending signals whenever a load reduction is required, and giving rewards for participating in the program.

**Consumer Model:** Let  $u_k(q_k)$  be the utility of consumer  $k$  derived by consuming  $q_k$  units of energy. We assume that  $u(\cdot)$  has the following form.

$$u_k(q_k) = \begin{cases} \pi_k q_k & \text{if } q_k < b_k \\ \pi_k b_k & \text{if } q_k \geq b_k \end{cases} \quad (1)$$

Here  $b_k$  is the maximum energy requirement of consumer  $k$ . Any additional consumption will not increase her utility. We call  $\pi_k$  the *true marginal utility* of consumer  $k$ . We assume that  $\pi_k \leq \pi_{max}, \forall k$ . Let  $\pi^e$  be the retail price for unit energy. Then the net utility  $U_k(\cdot)$  of consumer  $k$  is given by,

$$U_k(q_k) = \begin{cases} \pi_k q_k - \pi^e q_k & \text{if } q_k < b_k \\ \pi_k b_k - \pi^e q_k & \text{if } q_k \geq b_k \end{cases} \quad (2)$$

We assume that  $\pi_k > \pi^e, \forall k$ . The optimal consumption for consumer  $k$  which maximizes her net utility is clearly  $b_k$ . We call this the *true baseline consumption* of consumer  $k$ . We formulate baseline reporting and subsequent load reduction as a two stage mechanism.

**Stage 1 (Reporting):** At the beginning of stage 1, aggregator announces a selection procedure and a reward function  $R(\cdot, \cdot, \cdot)$  for consumers who are selected to reduce load. Aggregator also announces a penalty function  $\Phi(\cdot, \cdot, \cdot)$  for consumers who deviate from their reported baseline. Depending on the selection procedure, the reward  $R(\cdot, \cdot, \cdot)$  and the penalty  $\Phi(\cdot, \cdot, \cdot)$ , each consumer  $k$  reports two parameters, the baseline consumption  $\hat{b}_k$  and marginal utility  $\hat{\pi}_k$ . Consumers can potentially give incorrect reports, i.e.,  $\hat{b}_k$  and  $\hat{\pi}_k$  need not be equal to  $b_k$  and  $\pi_k$  respectively.

**Stage 2 (Load reduction and Payment):** A DR event occurs where the aggregator has to collate a total load reduction of  $D$  units. We assume that  $D$  is random, and only the aggregator knows the value of  $D$ . This is reasonable considering the fact that a DR event and the extent of supply deficit is an exogenous event and is not observed by the consumer. Aggregator selects a set of consumers, depending on their reports  $(\hat{b}_k, \hat{\pi}_k)$ , to meet the total reduction  $D$  and sends a load reduction signal to each of these selected consumers. If consumer  $k$  is selected, it gets a reward  $R(q_k, \hat{b}_k, \hat{\pi}_k)$  when consuming  $q_k$ . Every consumer  $k$  gets a penalty  $\Phi(q_k, \hat{b}_k, \hat{\pi}_k)$  if the consumption  $q_k$  is different from the reported baseline  $\hat{b}_k$ .

We assume that the net load reduction requirement  $D$  is independent of the reports of the consumers. Even though, only the aggregator observes the actual realization of  $D$ , we assume that the distribution  $D$  is a common information, i.e., it is known to the aggregator and all consumers. In particular, each consumer knows the probability of a DR event, i.e.,  $\alpha = \mathbb{P}(D > 0)$ .

**Consumer's problem:** We assume that the consumers are non-cooperative. In particular, each consumer faces a two stage decision problem. In the first stage, it has to decide the value of the reports, i.e.  $\hat{b}_k$  and  $\hat{\pi}_k$ . In the second stage it has to decide on the energy consumption  $q_k$ . The value of  $q_k$  will depend on the consumer  $k$  being called for DR or not. The objective of each consumer is to maximize her expected benefit,  $\bar{J}_k$ . Consumer  $k$ 's two-stage optimization problem can be formulated as

$$(CP) \max_{\hat{b}_k, \hat{\pi}_k} \bar{J}_k(\hat{b}_k, \hat{\pi}_k), \quad \text{where,}$$

$$\bar{J}_k(\hat{b}_k, \hat{\pi}_k) = \mathbb{E}_{D, \hat{b}_{-k}, \hat{\pi}_{-k}} \left[ \max_{q_k} (U_k(q_k) + R(\cdot, \cdot, \cdot) - \Phi(\cdot, \cdot, \cdot)) \right] \quad (3)$$

Here  $\hat{b}_{-k} = (\hat{b}_1, \dots, \hat{b}_{k-1}, \hat{b}_{k+1}, \hat{b}_N)$  and  $\hat{\pi}_{-k} = (\hat{\pi}_1, \dots, \hat{\pi}_{k-1}, \hat{\pi}_{k+1}, \hat{\pi}_N)$ .

TABLE I  
NOTATIONS

$b_k$	Baseline consumption of consumer $k$
$q_k$	Energy consumption of consumer $k$
$\pi_k$	Marginal utility of consumer $k$
$u_k$	Utility of consumer $k$
$\pi^e$	Retail price of energy
$U_k$	Net utility of consumer $k$
$R(\cdot, \cdot, \cdot)$	Reward function for load reduction
$\Phi(\cdot, \cdot, \cdot)$	Penalty function for deviation from baseline
$\pi_k^r$	Reward/kWh awarded to consumer $k$
$\pi_k^p$	Penalty/kWh imposed on consumer $k$
$\hat{b}_k$	Baseline report of consumer $k$
$\hat{\pi}_k$	Marginal utility report of consumer $k$
$D$	Load reduction requirement

### III. SELF-REPORTED BASELINE MECHANISM

Here we introduce the Self-Reported Baseline Mechanism (SRBM). In the first stage, (each) consumer  $k$  reports her baseline consumption  $\hat{b}_k$  and marginal utility  $\hat{\pi}_k$  to the aggregator. Aggregator reorders the consumers in increasing order of their marginal utility. Formally, the aggregator forms the vectors  $\hat{B}$  and  $\hat{\Pi}$  such that

$$\hat{B} = (\hat{b}_1, \dots, \hat{b}_N), \hat{\Pi} = (\hat{\pi}_1, \dots, \hat{\pi}_N), \hat{\pi}_j \leq \hat{\pi}_{j+1}, \forall j. \quad (4)$$

In the second stage, aggregator observes the net load reduction requirement  $D$ . Then, the aggregator selects the first  $k^*(D)$  consumers (in the increasing order of their marginal utility) such that

$$\sum_{j=1}^{k^*(D)-1} \hat{b}_j < D \leq \sum_{j=1}^{k^*(D)} \hat{b}_j \quad (5)$$

If  $\hat{b}_j$  is the true baseline  $\forall j$  and consumers reduce their load by the maximum possible amount, this selection process would meet the load reduction requirement.

From now on we shorten the notation to  $k^*$  instead of  $k^*(D)$ . Also, let  $S(D) = \{1, \dots, k^*\}$  be the set of selected consumers. Aggregator then sends DR signal to every consumer in the set  $S(D)$ . Note that not all consumers are selected at each time. The number of selected consumers depends on  $D$ . Aggregator then observes the consumption  $q_k$  of all consumers.

Let  $S_{-k}(D)$  be the set of consumers who would be selected by the aggregator according to (5), if consumer  $k$  is not participating in the DR program. Let

$$\pi_k^r(D) = \max\{\hat{\pi}_j\} - \pi^e, j \in S_{-k}(D). \quad (6)$$

The reward function for consumer  $k$  is then defined as,

$$R(q_k, \hat{b}_k, \hat{\pi}_k) = \begin{cases} \pi_k^r(D) (\hat{b}_k - q_k)_+ & \text{if } k \text{ is selected} \\ 0 & \text{otherwise} \end{cases} \quad (7)$$

Clearly, consumer  $k$  gets a reward for load reduction only if it is selected. The consumer reward is proportional to the

measured load reduction  $(\hat{b}_k - q_k)_+$ . In the following, we denote  $\pi_k^r(D)$  simply as  $\pi_k^r$  making the dependence on  $D$  implicit.

The aggregator also imposes a penalty if the consumption  $q_k$  of consumer  $k$  is different from her reported baseline consumption  $\hat{b}_k$ . We define the penalty function as

$$\Phi(q_k, \hat{b}_k, \hat{\pi}_k) = \begin{cases} \pi_k^r (q_k - \hat{b}_k)_+ & \text{if } k \text{ is selected} \\ \hat{\pi}_k |(q_k - \hat{b}_k)| & \text{otherwise} \end{cases} \quad (8)$$

Clearly, no penalty for decreasing the load when the consumer is selected for DR. However, we impose a penalty for positive and negative deviation from the reported baseline when consumers are not selected for DR.

Let  $J_k(q_k; \hat{b}_k, \hat{\pi}_k)$  be the ex-post benefit of consumer  $k$  given the first stage reports  $\hat{b}_k$  and  $\hat{\pi}_k$  and the second stage consumption  $q_k$ . Then using (7) and (8) the ex-post benefit, a consumer receives when selected for DR can be written as,

$$J_k(q_k; \hat{b}_k, \hat{\pi}_k | k \text{ is selected}) = \pi_k \min\{q_k, b_k\} - \pi^e q_k + \pi_k^r (\hat{b}_k - q_k). \quad (9)$$

The ex-post benefit when the consumer is not selected for DR can be written as,

$$J_k(q_k; \hat{b}_k, \hat{\pi}_k | k \text{ is not selected}) = \pi_k \min\{q_k, b_k\} - \pi^e q_k - \hat{\pi}_k |q_k - \hat{b}_k|. \quad (10)$$

We now specify the SRB mechanism formally, from the perspective of the aggregator.

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#### Algorithm 1 (Self-Reported Baseline Mechanism (SRBM))

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- 1) Receive reports  $\hat{b}_j$ s and  $\hat{\pi}_j$ s from all consumers
  - 2) Observe the net load reduction requirement  $D$
  - 3) Select  $S(D)$  consumers as specified by (5)
  - 4) Observe consumption  $q_j$  of every consumer  $j$
  - 5) Award the payment and impose the penalty as specified by (7)-(8)
- 

Before characterizing the equilibrium outcome under the SRB mechanism, we first formally define the following notions.

**Definition 1** (Dominant strategy). *Let  $\bar{J}_k((\hat{b}_k, \hat{\pi}_k); (\hat{b}_{-k}, \hat{\pi}_{-k}))$  be the expected net benefit of consumer  $k$  when she reports  $(\hat{b}_k, \hat{\pi}_k)$  and other consumers report  $(\hat{b}_{-k}, \hat{\pi}_{-k})$ . The pair  $(\hat{b}_k^*, \hat{\pi}_k^*)$  is a dominant strategy report for consumer  $k$  if  $(\hat{b}_k^*, \hat{\pi}_k^*) = \arg \max_{\hat{b}_k, \hat{\pi}_k} \bar{J}_k((\hat{b}_k, \hat{\pi}_k); (\hat{b}_{-k}, \hat{\pi}_{-k}))$ , for any report  $(\hat{b}_{-k}, \hat{\pi}_{-k})$  from other consumers.*

We make the following assumption on the probability a DR event.

**Assumption 1.** *Let  $\alpha = \mathbb{P}(D > 0)$  be the probability of a DR event. Then,  $\alpha \pi_{max} < (1 - \alpha) \pi^e$ .*

This is a mild assumption when the frequency of DR events is small, which is indeed the case in the current DR programs. We now give the main result which establishes that the

proposed mechanism achieves the desired properties.

**Theorem 1.** *In the SRB mechanism, under Assumption 1,*

- (i) *Reporting baseline and marginal utility truthfully is a dominant strategy i.e.  $(\hat{b}_k^*, \hat{\pi}_k^*) = (b_k, \pi_k)$*
- (ii) *When consumer  $k$  is not selected for DR, her optimal consumption is the same as the reported baseline consumption, i.e.  $q_k = \hat{b}_k$*
- (iii) *When consumer  $k$  is selected for DR, her optimal consumption is zero, i.e.  $q_k = 0$*

Proof is given in the appendix.

*Remark 1.* Note that under SRBM, each consumer reports its true baseline and sticks to its baseline consumption when it is not selected for DR. Hence the reduction that is measured from the reported baseline is indeed the true load reduction. Also, the selection process (5) meets the load reduction requirement because the consumers reduce their consumption to zero when selected. Note that, consumers reducing their consumption to zero is an artifact of the piece-wise linear utility function model.

*Remark 2.* From a social welfare point, neglecting the payment/penalty transactions, the selection process is also *nearly efficient* because it selects consumers with the smallest marginal utilities, where each selected consumer contributes the maximum possible load reduction. Hence the total disutility  $(-\sum_k u_k(\cdot))$  is minimized. However it is not clear if it is the best mechanism from the perspective of the aggregator: i.e., minimize the total payment to the consumers while (i) eliciting truthful reports from the consumers and (ii) achieving required load reduction.

*Remark 3.* Here we assumed that the true baseline consumption  $b_k$  is deterministic. However,  $b_k$  depends on (exogenous) random parameters like temperature. For example, a possible model can be  $b_k = \bar{b}_k + a_k|\theta - \theta_0|$  where  $\theta$  is the temperature and  $\theta_0$  is the nominal temperature. Here the consumers can report two parameters,  $\bar{b}_k$  and the temperature sensitivity  $a_k$ . Historical consumption data can be used to estimate these parameters. SRB mechanism can also be extended to such cases.

*Remark 4.* Clearly, the results depends on the form of the utility function; in particular, the deterministic and the piece-wise linear assumption on utility function  $u_k(\cdot)$ . This can be considered as the first step approximation, where such a self-reported baseline mechanism achieves desirable properties. The characterization of such a mechanism with a general utility function  $\tilde{u}(\cdot; \theta)$  which also depends on some exogenous random variable  $\theta$  is challenging and is an ongoing work.

#### IV. UNIFORM PAYMENT MECHANISMS

In the SRB mechanism the reward rate  $\pi_k^r$  is different for different consumers. In this section, we introduce a uniform payment mechanism for the baselining problem. We show that under certain conditions, this mechanism achieves the desired properties.

*Mechanism:* In the beginning of the first stage, consumer  $k$  submits her baseline consumption  $\hat{b}_k$  and marginal utility report  $\hat{\pi}_k$  to the aggregator. As in the previous mechanism, the aggregator reorders the consumers in increasing order of their *reported* marginal utility. Formally the aggregator forms the vectors  $\hat{B}$  and  $\hat{\Pi}$  where

$$\hat{B} = \{\hat{b}_1, \dots, \hat{b}_N\}, \hat{\Pi} = (\hat{\pi}_1, \dots, \hat{\pi}_N) \text{ such that } \hat{\pi}_j \leq \hat{\pi}_{j+1}, \forall j \quad (11)$$

In the second stage the aggregator receives the load reduction requirement  $D$ . The aggregator then selects the first  $k^* = k^*(D)$  consumers such that

$$\sum_{k=1}^{k^*(D)-1} \hat{b}_k < D \leq \sum_{k=1}^{k^*(D)} \hat{b}_k \quad (12)$$

The consumers who are selected are paid according to,

$$\pi^r(D) = \hat{\pi}_{(k^*(D)+1)} - \pi^e, \quad (13)$$

while others are paid zero. Then the reward function is given by,

$$R(q_k, \hat{b}_k, \hat{\pi}_k) = \begin{cases} \pi^r(D)(\hat{b}_k - q_k)_+ & \text{if } k \text{ is selected} \\ 0 & \text{otherwise} \end{cases} \quad (14)$$

The penalties are set as,

$$\Phi(q_k, \hat{b}_k, \hat{\pi}_k) = \begin{cases} \pi_{max}(q_k - \hat{b}_k)_+ & \text{if } k \text{ is selected} \\ \pi_{max}|(q_k - \hat{b}_k)| & \text{otherwise} \end{cases} \quad (15)$$

In the following theorem, we show that reporting marginal utility and baseline truthfully is a Nash equilibrium strategy. Also we show that the consumer provides maximum possible load reduction when signaled to reduce. We define the Nash equilibrium concept below.

**Definition 2** (Nash equilibrium). *Let  $\bar{J}_k((\hat{\pi}_k, \hat{b}_k), (\hat{\pi}_{-k}, \hat{b}_{-k}))$  be the expected benefit of consumer  $k$  when she reports  $(\hat{\pi}_k, \hat{b}_k)$  and other consumers report  $(\hat{\pi}_{-k}, \hat{b}_{-k})$ . Then  $(\hat{\pi}^*, \hat{b}^*) = ((\hat{\pi}_1^*, \hat{b}_1^*), \dots, (\hat{\pi}_N^*, \hat{b}_N^*))$  is a Nash equilibrium if  $\bar{J}_k((\hat{\pi}_k, \hat{b}_k), (\hat{\pi}_{-k}^*, \hat{b}_{-k}^*)) \leq \bar{J}_k((\hat{\pi}_k^*, \hat{b}_k^*), (\hat{\pi}_{-k}^*, \hat{b}_{-k}^*))$*

We need the following assumption. In the next subsection, we also argue that this assumption is not really necessary when the number of consumers is very large.

**Assumption 2.** *Let  $\{\pi_k, b_k\}$  be the marginal utility and baseline consumption of consumers. If  $\pi_j \leq \pi_k$ , then  $b_j \leq b_k$ ,  $\forall j, k$ .*

**Theorem 2.** *Under Assumption 2, with the condition that  $\pi_{max} < (1 - \alpha) \min\{\pi_k - \pi^e, \pi^e\}/\alpha \forall k$ , reporting the baseline and marginal utility truthfully is a Nash equilibrium of mechanism SRBM-UP. Under this Nash-Equilibrium strategy,*

- (i) *When consumer  $k$  is not selected for DR, consumer  $k$  consumes the reported baseline i.e.  $q_k = \hat{b}_k$*
- (ii) *When consumer  $k$  is selected for DR, consumer reduces load consumption to zero i.e.  $q_k = 0$*

Proof and a detailed explanation is given in [10].

## A. Simulations

In Section IV we proved that reporting truthfully is a Nash equilibrium under Assumption 2. However this assumption may not be necessary when the number of consumers are very large, which we argue by simulation. More specifically, simulation experiments suggest that the fraction of consumers who give incorrect reports approaches zero as the total number of consumers increases.

Given the number of consumers  $N$ , we generate each consumer's baseline consumption  $b_k$  and marginal utility  $\pi_k$  uniformly at random. We assume  $b_k \sim U[0, 1]$  and  $\pi_k \sim U[0, 1]$ . Let  $Q = (b_1, \dots, b_N)$  and  $\Pi = (\pi_1, \dots, \pi_N)$ . Also, we assume that the net load reduction requirement  $\mathbb{P}(D|D > 0) \sim U[0, 50]$ . Simulation procedure is as follows:

- (i) For a given  $N$  generate  $Q$  and  $\Pi$  as specified above.
- (ii) In [10] we show that reporting baseline truthfully is a dominant strategy. Hence, we set  $\hat{b}_k = b_k$  as the optimal baseline report of consumer  $k$ . Then we compute the optimal report  $\hat{\pi}_k$  by numerically solving the consumer problem (3) assuming that other consumers report truthfully. While solving, we assume that consumer  $k$  knows both  $Q$  and  $\Pi$  perfectly. So, the deviation  $\delta_k = \hat{\pi}_k - \pi_k$  is in a way the worst case deviation because  $\hat{\pi}_k$  is computed with perfect information, which is not available to consumer  $k$  in the original setting.
- (iii) Compute the cdf of the vector of deviations  $\Delta = (\delta_1, \dots, \delta_N)$ .
- (iv) Repeat this for different realizations of  $Q$  and  $\Pi$ . Plot the averaged cdf.

We repeat the above procedure for different values of  $N$ . Figure 1 shows that as  $N$  increases, the cdf approaches a step function. So, the fraction of consumers who deviate from truthful reporting decreases to zero as  $N$  increases. This extends the scope of uniform payment mechanisms to more general conditions provided the number of consumers is large.

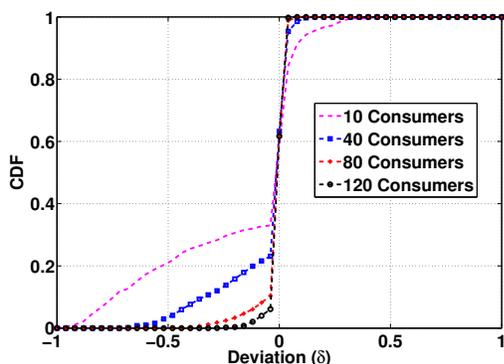


Fig. 1. CDF of deviations from the truthful reporting ( $\delta$ )

## V. COMPARISON WITH CAISO BASELINE METHOD

In the CAISO method of payment for the consumers, CAISO estimates the baseline by computing the mean of the consump-

tion of the most recent  $m$  similar but non-event days and then multiplies this mean by an adjustment factor that accounts for any difference in the consumption pattern from the past. The adjustment factor is calculated based on the consumption profile of the consumers in the hours prior to the DR event on the DR event day [11].

*Baseline Calculation:* After the DR event the baseline is calculated in the following way. Denote the consumption of the most recent  $m$  similar but non-event days collectively by  $\mathbf{b}_m$  and the calculated adjustment factor by  $C_b$ . Then the estimated baseline is

$$b_k^c = \bar{b}_k C_b$$

Where  $\bar{b}_k = (\text{sum}\{\mathbf{b}_m\}/m)$ . Denote the consumption in the hours prior to the DR event by  $q_k^-$  and the consumption during the hours prior to the hour that corresponds to the DR event hour on the DR event day, of the most recent  $m$  similar but non event days collectively as,  $\mathbf{b}_m^-$ . Let  $\bar{b}_k^- = (\text{sum}\{\mathbf{b}_m^-\}/m)$ . Then the adjustment factor is given by  $C_b = q_k^-/\bar{b}_k^-$ . This completes baseline calculation. Let the price for unit reduction be  $\pi$ . So the payment for reduction in the CAISO method is given by  $R(q_k, b_k^c, \pi) = \pi(b_k^c - q_k)$ . Below we provide the theorem that characterizes CAISO mechanism

**Theorem 3.** *The following holds for the CAISO mechanism,*

- (i) *When  $\pi < \pi_{max} - \pi^e$  CAISO mechanism cannot guarantee the required load reduction*
- (ii) *When  $\pi \geq \pi_{max} - \pi^e$ , CAISO mechanism payment is larger than the payment in SRBM*
- (iii) *When consumers are informed of the DR event several hours prior to the DR event then  $b_k^c \rightarrow \infty$  when  $\pi > \pi^e$ , i.e. the baseline estimate is inflated*

## VI. CONCLUSION

In this paper we addressed the baseline problem that is central to incentive-based DR programs. We proposed a mechanism where the consumers self-report their baseline and their marginal utility. In the proposed mechanism the consumers reveal their true baseline and also provide the maximum load reduction when signaled for DR. Also they stick to their baseline consumption when they are not signaled. So, the aggregator is able to meet any random load reduction requirement reliably by selecting consumers whose measured reductions adds up to the total reduction. Finally we show that in the current CAISO method either (i) one cannot guarantee reliable load reduction (ii) Or the payments are larger (iii) And in some scenarios the baseline estimate is inflated.

Our work assumed that the true baseline consumption is deterministic and that the utility function is piece-wise linear. In the presence of uncertainty, designing of an incentive scheme has to take in to account the fact that the consumers themselves are uncertain about their baseline consumption. The characterization of the DR mechanism for a general utility function  $u(q; \theta)$  where  $\theta$  is an exogenous random variable is an ongoing work. Also we have not addressed the optimality of the DR program from the point of view of the aggregator.

We will also incorporate the recruitment problem - recruiting the minimum possible consumers to achieve any random load reduction reliably when the recruitment cost is high and consumption is random.

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## VII. APPENDIX

We first prove the following two propositions. Let

$$\hat{b}_k^*(\hat{\pi}_k) = \arg \max_{\hat{b}_k} \bar{J}_k(\hat{b}_k, \hat{\pi}_k).$$

**Proposition 1.** *In SRB mechanism, under Assumption 1,  $\hat{b}_k^*(\hat{\pi}_k) = b_k$  and is unique.*

*Proof.* Each consumer's decision can be formulated as a two stage optimization problem. In the first stage, consumer  $k$  reports  $\hat{b}_k$  and  $\hat{\pi}_k$ . In the second stage, the consumption  $q_k$  depends on whether consumer  $k$  is selected or not. We first consider the second stage optimization problem.

Second Stage optimization: Let,

$$J_k(q_k; \hat{b}_k, \hat{\pi}_k) = U_k(q_k) + R(q_k, \hat{b}_k, \hat{\pi}_k) - \Phi(q_k, \hat{b}_k, \hat{\pi}_k) \quad (16)$$

which is the net benefit of consumer  $k$  in the second stage as a function of  $q_k$ , given the first stage reports  $(\hat{b}_k, \hat{\pi}_k)$ . Note that the exact form of  $J_k$  depends on whether consumer  $k$  is selected for DR or not. So, we separately consider these two cases.

*Case:* Consumer  $k$  is selected for DR.

Let  $q_k^{dr}$  be the optimal consumption of consumer  $k$  when she is selected for DR. Formally,

$$\begin{aligned} q_k^{dr} &= \arg \max_{q_k} J_k(q_k; \hat{b}_k, \hat{\pi}_k | k \text{ is selected}), \text{ where,} \\ J_k(q_k; \hat{b}_k, \hat{\pi}_k | k \text{ is selected}) &= \pi_k \min\{q_k, b_k\} - \pi^e q_k \\ &\quad + \pi_k^r (\hat{b}_k - q_k). \end{aligned} \quad (17)$$

Of course,  $q_k^{dr}$  will depend on the first stage reports  $(\hat{b}_k, \hat{\pi}_k)$ . In particular, there are two possible values for  $q_k^{dr}$ .

*Sub-Case*  $\pi_k^r \geq \pi_k - \pi^e$ : It is straightforward to show that  $q_k^{dr} = 0$ . Then, substituting for  $q_k^{dr}$  back in (17) we get,

$$J_k(q_k^{dr}; \hat{b}_k, \hat{\pi}_k | k \text{ is selected}) = \pi_k^r \hat{b}_k \quad (18)$$

*Sub-Case:*  $\pi_k^r < \pi_k - \pi^e$

It is again straightforward to show that  $q_k^{dr} = b_k$ . Then we get,

$$J_k(q_k^{dr}; \hat{b}_k, \hat{\pi}_k | k \text{ is selected}) = (\pi_k - \pi^e) b_k + \pi_k^r (\hat{b}_k - b_k) \quad (19)$$

*Case* Consumer  $k$  is not selected for DR: Let  $q_k^{ndr}$  be the optimal consumption of consumer  $k$  when she is not selected for DR. Formally,

$$\begin{aligned} q_k^{ndr} &= \arg \max_{q_k} J_k(q_k; \hat{b}_k, \hat{\pi}_k | k \text{ is not selected}), \text{ where,} \\ J_k(q_k; \hat{b}_k, \hat{\pi}_k | k \text{ is not selected}) &= \pi_k \min\{q_k, b_k\} \\ &\quad - \pi^e q_k - \hat{\pi}_k |q_k - \hat{b}_k|. \end{aligned} \quad (20)$$

We make the following observation: From the assumption  $\pi_k - \pi^e > 0$  it follows that  $\hat{\pi}_k > \pi^e$ . Similar to Case 1, we consider different sub-cases on the first stage reports  $(\hat{b}_k, \hat{\pi}_k)$ .

*Sub-case*  $\hat{\pi}_k \geq \pi_k - \pi^e$ : Using  $\hat{\pi}_k > \pi^e$ , we get  $q_k^{ndr} = \hat{b}_k$  and

$$J_k(q_k^{ndr}; \hat{b}_k, \hat{\pi}_k | k \text{ is selected}) = \pi_k \min\{\hat{b}_k, b_k\} - \pi^e \hat{b}_k \quad (21)$$

*Sub-case*  $\hat{\pi}_k < \pi_k - \pi^e$  and  $\hat{b}_k \leq b_k$ : we get  $q_k^{ndr} = b_k$  and

$$J_k(q_k^{ndr}; \hat{b}_k, \hat{\pi}_k | k \text{ is selected}) = (\pi_k - \pi^e) b_k + \hat{\pi}_k (\hat{b}_k - b_k) \quad (22)$$

*Sub-case*  $\hat{\pi}_k < \pi_k - \pi^e$  and  $\hat{b}_k > b_k$ : Using  $\hat{\pi}_k > \pi^e$ , we get  $q_k^{ndr} = \hat{b}_k$  and

$$J_k(q_k^{ndr}; \hat{b}_k, \hat{\pi}_k | k \text{ is selected}) = \pi_k b_k - \pi^e \hat{b}_k \quad (23)$$

This completes the characterization of the second-stage decisions  $q_k^{dr}$  and  $q_k^{ndr}$  of consumer  $k$ .

First stage optimization: Let  $\mathbb{I}_k(D)$  denote the indicator function of selection of consumer  $k$ . Then,

$$\mathbb{I}_k(D) = \begin{cases} 0 & \text{if } k \text{ is not selected} \\ 1 & \text{if } k \text{ is selected} \end{cases} \quad (24)$$

By the selection process (5), consumer  $k$  will not be selected up to a  $D_k$  where  $D_k = \sum_{m=1}^{k-1} \hat{b}_m$  and will always be selected for  $D > D_k$  i.e.,

$$\mathbb{I}_k(D) = \begin{cases} 0 & \text{if } D \leq D_k \\ 1 & \text{if } D > D_k \end{cases} \quad (25)$$

And it follows that,

$$\frac{\partial D_k}{\partial \hat{b}_k} = 0 \quad (26)$$

The expected benefit consumer  $k$  receives in the first stage is given by,

$$\begin{aligned} \bar{J}_k(\hat{b}_k, \hat{\pi}_k) &= \mathbb{E}[\mathbb{I}\{k \text{ is selected}\}J_k(\cdot|k \text{ is selected})] \\ &+ \mathbb{E}[\mathbb{I}\{k \text{ is not selected}\}J_k(\cdot|k \text{ is not selected})] \end{aligned} \quad (27)$$

Recall that  $\mathbb{P}(D > 0)$  is the probability that a DR event occurs. From (25) we get

$$\begin{aligned} &\mathbb{E}[\mathbb{I}\{k \text{ is selected}\}J_k(\cdot|k \text{ is not selected})] \\ &= \mathbb{E}[\mathbb{I}_k(D)J_k(\cdot|k \text{ is selected})] \\ &= \mathbb{P}(D > 0)\mathbb{E}[\mathbb{I}_k(D)J_k(\cdot|k \text{ is selected})|D > 0] \end{aligned} \quad (28)$$

And similarly,

$$\begin{aligned} &\mathbb{E}[\mathbb{I}\{k \text{ is not selected}\}J_k(\cdot|k \text{ is not selected})] \\ &= \mathbb{P}(D > 0)\mathbb{E}[(1 - \mathbb{I}_k(D))J_k(\cdot|k \text{ is not selected})|D > 0] \\ &+ (1 - \mathbb{P}(D > 0))J_k(\cdot|k \text{ is not selected}) \end{aligned} \quad (29)$$

Using (29) and (28) we can express  $\bar{J}_k(\hat{b}_k, \hat{\pi}_k)$  as,

$$\begin{aligned} \bar{J}_k(\hat{b}_k, \hat{\pi}_k) &= \mathbb{P}(D > 0)\mathbb{E}[\mathbb{I}_k(D)J_k(\cdot|k \text{ is selected})|D > 0] \\ &+ \mathbb{P}(D > 0)\mathbb{E}[(1 - \mathbb{I}_k(D))J_k(\cdot|k \text{ is not selected})|D > 0] \\ &+ (1 - \mathbb{P}(D > 0))J_k(\cdot|k \text{ is not selected}) \end{aligned} \quad (30)$$

Also let,  $\mathbb{P}(D \leq D'|D > 0) = \int_{0+}^{D'} f(D)$ . Then using (25) and  $\alpha = \mathbb{P}(D > 0)$  we get,

$$\begin{aligned} \bar{J}_k(\hat{b}_k, \hat{\pi}_k) &= \alpha \int_{D_k}^{\bar{D}} J_k(q_k^{dr}; \hat{b}_k, \hat{\pi}_k | k \text{ is selected}) f(D) \\ &+ \alpha \int_{0+}^{D_k} J_k(q_k^{ndr}; \hat{b}_k, \hat{\pi}_k | k \text{ is not selected}) f(D) \\ &+ (1 - \alpha) J_k(q_k^{ndr}; \hat{b}_k, \hat{\pi}_k | k \text{ is not selected}) \end{aligned} \quad (31)$$

Define,

$$\hat{b}_k^*(\hat{\pi}_k) = \arg \max_{\hat{b}_k} \bar{J}_k(\hat{b}_k, \hat{\pi}_k) \quad (32)$$

Where,  $\hat{b}_k^*(\hat{\pi}_k)$  is the optimal report of baseline consumption that maximizes the expected net utility of consumer  $k$ . Next we solve this optimization problem to derive consumer's optimal report  $\hat{b}_k^*(\hat{\pi}_k)$  corresponding to the marginal utility report  $\hat{\pi}_k$ . Differentiating  $\bar{J}_k(\hat{b}_k, \hat{\pi}_k)$  w.r.t  $\hat{b}_k$  and using (26) we get,

$$\begin{aligned} \frac{\partial \bar{J}_k(\hat{b}_k, \hat{\pi}_k)}{\partial \hat{b}_k} &= \alpha \int_{D_k}^{\bar{D}} \frac{\partial J_k(\cdot|k \text{ is selected})}{\partial \hat{b}_k} f(D) \\ &+ \alpha \int_{0+}^{D_k} \frac{\partial J_k(\cdot|k \text{ is not selected})}{\partial \hat{b}_k} f(D) \\ &+ (1 - \alpha) \frac{\partial J_k(\cdot|k \text{ is not selected})}{\partial \hat{b}_k} \end{aligned} \quad (33)$$

Using the fact that  $\pi_k^r$  is independent of consumer  $k$ 's reports, it follows from (18) and (19) that  $\frac{\partial J_k(\cdot|k \text{ is selected})}{\partial \hat{b}_k} = \pi_k^r$ .

Substituting for  $\frac{\partial J_k(\cdot|k \text{ is selected})}{\partial \hat{b}_k}$  in (33) we get,

$$\begin{aligned} \frac{\partial \bar{J}_k(\hat{b}_k, \hat{\pi}_k)}{\partial \hat{b}_k} &= \alpha \int_{D_k}^{\bar{D}} \pi_k^r f(D) \\ &+ \alpha \int_{0+}^{D_k} \frac{\partial J_k(\cdot|k \text{ is not selected})}{\partial \hat{b}_k} f(D) \\ &+ (1 - \alpha) \frac{\partial J_k(\cdot|k \text{ is not selected})}{\partial \hat{b}_k} \end{aligned} \quad (34)$$

To complete the analysis we consider the following two cases, (i) when  $\hat{\pi}_k \geq \pi_k - \pi^e$  (ii) when  $\hat{\pi}_k < \pi_k - \pi^e$ . In each case we show that  $\frac{\partial \bar{J}_k}{\partial \hat{b}_k} > 0$  when  $\hat{b}_k \leq b_k$  and  $\frac{\partial \bar{J}_k}{\partial \hat{b}_k} < 0$  when  $\hat{b}_k > b_k$  which establishes that  $\hat{b}_k^*(\hat{\pi}_k) = b_k$  is the unique maximizer.

*Case  $\hat{\pi}_k \geq \pi_k - \pi^e$ :* We consider the following two sub-cases (i)  $\hat{b}_k \leq b_k$  and (ii)  $\hat{b}_k > b_k$ .

*Sub-case Consumer reports a baseline that is less than her true baseline i.e.  $\hat{b}_k \leq b_k$ :* Using (21) in (34) we get,

$$\begin{aligned} \frac{\partial \bar{J}_k(\hat{b}_k, \hat{\pi}_k)}{\partial \hat{b}_k} &= \alpha \int_{D_k}^{\bar{D}} \pi_k^r f(D) + \alpha \int_{0+}^{D_k} (\pi_k - \pi^e) f(D) \\ &+ (1 - \alpha)(\pi_k - \pi^e) \end{aligned} \quad (35)$$

From (6) we get  $\pi_k^r \geq \hat{\pi}_k - \pi^e$ . Since  $\hat{\pi}_k > \pi^e$  it follows that  $\pi_k^r \geq \hat{\pi}_k - \pi^e > 0$ . This implies  $\frac{\partial \bar{J}_k(\hat{b}_k, \hat{\pi}_k)}{\partial \hat{b}_k} > 0$  when  $\hat{b}_k \leq b_k$ .

*Sub-case The consumer reports a baseline that is greater than her true baseline i.e.  $\hat{b}_k > b_k$ :* Using (21) in (34) we get,

$$\begin{aligned} \frac{\partial \bar{J}_k(\hat{b}_k, \hat{\pi}_k)}{\partial \hat{b}_k} &= \alpha \int_{D_k}^{\bar{D}} \pi_k^r f(D) + \alpha \int_{0+}^{D_k} -\pi^e f(D) - (1 - \alpha)\pi^e \\ &= \int_{D_k}^{\bar{D}} (\alpha\pi_k^r - (1 - \alpha)\pi^e) f(D) - \int_{0+}^{D_k} \pi^e f(D) \end{aligned} \quad (36)$$

Note that consumers cannot report a value more than  $\pi_{max}$  which implies  $\pi_k^r \leq \pi_{max} - \pi^e$ . Then from assumption 1 it follows that  $\alpha\pi_k^r - (1 - \alpha)\pi^e < 0$ . And from (36) it follows that  $\frac{\partial \bar{J}_k(\hat{b}_k, \hat{\pi}_k)}{\partial \hat{b}_k} < 0$  when  $\hat{b}_k > b_k$ .

*Case  $\hat{\pi}_k < \pi_k - \pi^e$ :* As before we have two sub-cases.

*Sub-case Consumer reports a baseline that is less than her true baseline i.e.  $\hat{b}_k \leq b_k$ :* Using (22) in (34) we get,

$$\frac{\partial \bar{J}_k(\hat{b}_k, \hat{\pi}_k)}{\partial \hat{b}_k} = \alpha \int_{D_k}^{\bar{D}} \pi_k^r f(D) + \alpha \int_{0+}^{D_k} \hat{\pi}_k f(D) + (1 - \alpha)\hat{\pi}_k \quad (37)$$

Since  $\hat{\pi}_k > \pi^e > 0$  and  $\pi_k^r \geq \hat{\pi}_k - \pi^e > 0$  it follows that  $\frac{\partial \bar{J}_k(\hat{b}_k, \hat{\pi}_k)}{\partial \hat{b}_k} > 0$  when  $\hat{b}_k \leq b_k$ .

*Sub-case The consumer reports a baseline that is greater than her true baseline i.e.  $\hat{b}_k > b_k$ :* Using (23) in (34) we get,

$$\begin{aligned} \frac{\partial \bar{J}_k(\hat{b}_k, \hat{\pi}_k)}{\partial \hat{b}_k} &= \alpha \int_{D_k}^{\bar{D}} \pi_k^r f(D) + \alpha \int_{0+}^{D_k} -\pi^e f(D) \\ &- (1 - \alpha)\pi^e \\ &= \int_{D_k}^{\bar{D}} (\alpha\pi_k^r - (1 - \alpha)\pi^e) f(D) - \int_{0+}^{D_k} \pi^e f(D) \end{aligned} \quad (38)$$

This expression is exactly the same as (36) and it follows that  $\frac{\partial \bar{J}_k(\hat{b}_k, \hat{\pi}_k)}{\partial \hat{b}_k} < 0$ .

The two cases together imply  $\hat{b}_k^*(\hat{\pi}_k) = b_k$  is the unique maximizer for any given  $\hat{\pi}_k$ .  $\square$

Let

$$\hat{\pi}_k^*(\hat{b}_k) = \arg \max_{\hat{\pi}_k} \bar{J}_k(\hat{b}_k, \hat{\pi}_k)$$

**Proposition 2.** *In the SRB mechanism, under Assumption 1,  $\pi_k \in \{\pi | \bar{J}_k(\hat{b}_k = b_k, \pi) = \bar{J}_k(\hat{b}_k = b_k, \hat{\pi}_k^*(\hat{b}_k = b_k))\}$ .*

*Proof.* Here we show that reporting marginal utility truthfully maximizes consumer's utility when  $\hat{b}_k = b_k$ . Let  $J_k(q_k; \hat{\pi}_k, \hat{b}_k = b_k | D)$  be the ex-post benefit of consumer  $k$  when it reports  $(\hat{\pi}_k, \hat{b}_k)$  and when the realized value of load reduction requirement is  $D$ . Define,  $\hat{S}(D; \hat{\pi}_k)$  to be the set of consumers who would be selected (according to (5)) when consumer  $k$  reports  $\hat{\pi}_k$  and  $\hat{S}_{-k}(D)$  to be the set of consumers who would be selected when consumer  $k$  is excluded. From (6) the payment for unit reduction when consumer  $k$  gets selected is then given by,

$$\pi_k^r(D; \hat{\pi}_k) = \max\{\hat{\pi}_j\} - \pi^e, \quad j \in \hat{S}_{-k}(D) \quad (39)$$

To show that reporting marginal utility truthfully maximizes consumer's utility when  $\hat{b}_k = b_k$ , we consider the following two cases; (i) consumer  $k$  gets selected on reporting truthfully (ii) consumer  $k$  does not get selected on reporting truthfully. And show that in either of the scenarios consumer  $k$  does not gain in terms of the net benefit by deviating from reporting her marginal utility truthfully.

*Case 1: consumer  $k \notin \hat{S}(D; \pi_k)$ :*

Here, on reporting truthfully consumer  $k$  will not be selected. Hence by (21) consumer  $k$  consumes  $q_k = b_k$  and her ex-post benefit is given by  $J_k(q_k = b_k; \pi_k, \hat{b}_k = b_k | D) = (\pi_k - \pi^e)b_k$ . We then compare the net benefit of consumer  $k$  on over-reporting and under-reporting her marginal utility from that of reporting truthfully.

Over Reporting,  $\hat{\pi}_k > \pi_k$ :

By the selection process (5) consumer  $k$  will not be selected on over reporting her marginal utility. Hence by (21)  $J_k(q_k = b_k; \hat{\pi}_k, \hat{b}_k = b_k | D) = (\pi_k - \pi^e)b_k$  and consumer  $k$  is indifferent to over reporting in this case.

Under Reporting,  $\hat{\pi}_k < \pi_k$ :

Let consumer  $k$  under report such that consumer  $k$  gets selected. Because  $k \notin \hat{S}(D; \pi_k)$ ,  $\hat{S}(D; \pi_k) = \hat{S}_{-k}(D; \pi_k)$ . Note that  $\hat{S}_{-k}$  is not dependent on the report  $\hat{\pi}_k$ . Hence  $\hat{S}(D; \pi_k) = \hat{S}_{-k}(D; \hat{\pi}_k)$ . And because  $k \notin \hat{S}(D; \pi_k)$ ,  $\pi_k > \max\{\hat{\pi}_j | j \in \hat{S}(D; \pi_k)\}$  i.e.  $\pi_k > \max\{\hat{\pi}_j | j \in \hat{S}_{-k}(D; \hat{\pi}_k)\}$ . Then from (39) it follows that,  $\pi_k^r(D; \hat{\pi}_k) < \pi_k - \pi^e$ . By (19) consumer  $k$  consumes  $q_k = b_k$  and her ex-post benefit is given by  $J_k(q_k = b_k; \hat{\pi}_k, \hat{b}_k = b_k | D) = (\pi_k - \pi^e)b_k$ . Hence consumer  $k$  is indifferent to under reporting in this case.

For this case we have established that consumer  $k$  is indifferent to deviating from reporting her marginal utility truthfully.

*Case 2: consumer  $k \in \hat{S}(D; \pi_k)$ :*

Similarly for this case we show that consumer  $k$  does not gain by over-reporting or under-reporting her marginal utility.

Since  $k \in \hat{S}(D; \pi_k)$ ,  $\pi_k^r(D; \pi_k) \geq \pi_k - \pi^e$ . By (18), consumer  $k$  consumes  $q_k = 0$ . Her ex-post benefit is given by,  $J_k(q_k = 0; \pi_k, \hat{b}_k = b_k | D) = \pi_k^r(D; \pi_k)b_k$

Over Reporting,  $\hat{\pi}_k > \pi_k$ :

On over reporting if the consumer gets selected then  $\pi_k^r(D; \hat{\pi}_k) = \pi_k^r(D; \pi_k) \geq \pi_k - \pi^e$ . By (18) consumption  $q_k = 0$  and  $J_k(q_k = 0; \hat{\pi}_k, \hat{b}_k = b_k | D) = \pi_k^r(D; \hat{\pi}_k)b_k = \pi_k^r(D; \pi_k)b_k = J_k(q_k = 0; \pi_k, \hat{b}_k = b_k | D)$ . Hence consumer  $k$ 's ex-post benefit remains same if it gets selected on over reporting. On the other hand if the consumer does not get selected, then by (21)  $J_k(q_k; \hat{\pi}_k, \hat{b}_k = b_k | D) = (\pi_k - \pi^e)b_k \leq \pi_k^r(D; \pi_k)b_k = J_k(q_k = 0; \pi_k, \hat{b}_k = b_k | D)$ . And the consumer can strictly loose but never gain in terms of ex-post benefit. Hence over-reporting is a loss to the consumer in this case.

Under Reporting,  $\hat{\pi}_k < \pi_k$ :

Here, on under reporting consumer  $k$  will always be selected and so  $\pi_k^r(D; \hat{\pi}_k) = \pi_k^r(D; \pi_k) \geq \pi_k - \pi^e$ . By (18) consumption  $q_k = 0$  and  $J_k(q_k; \hat{\pi}_k, \hat{b}_k = b_k | D) = \pi_k^r(D; \hat{\pi}_k)b_k = \pi_k^r(D; \pi_k)b_k = J_k(q_k; \pi_k, \hat{b}_k = b_k | D)$  and so it does not gain by under-reporting.

Both cases together imply that reporting marginal utility maximizes consumer's utility when  $\hat{b}_k = b_k$ . And it follows from the proof that this maximizer may not be unique. Hence  $\pi_k \in \{\pi | \bar{J}_k(\hat{b}_k = b_k, \pi) = \bar{J}_k(\hat{b}_k = b_k, \hat{\pi}_k^*(\hat{b}_k))\}$   $\square$

We now complete the proof of Theorem 1

*Proof.* For any given  $\hat{b}_k$  and  $\hat{\pi}_k$ ,  $\bar{J}_k(\hat{b}_k, \hat{\pi}_k) \leq \bar{J}_k(\hat{b}_k, \hat{\pi}_k^*(\hat{b}_k))$ . From proposition 1 it follows that,  $\bar{J}_k(\hat{b}_k, \hat{\pi}_k^*(\hat{b}_k)) \leq \bar{J}_k(b_k, \hat{\pi}_k^*(b_k))$ . And from proposition 2 it follows that  $\bar{J}_k(b_k, \hat{\pi}_k^*(b_k)) \leq \bar{J}_k(b_k, \pi_k)$ . Hence  $\bar{J}_k(\hat{b}_k, \hat{\pi}_k) \leq \bar{J}_k(b_k, \pi_k)$ . This implies that reporting the baseline and marginal utility truthfully is a dominant strategy, i.e.  $(\hat{b}_k^*, \hat{\pi}_k^*) = (b_k, \pi_k)$ . When consumer is not selected, it follows from  $\hat{\pi}_k = \pi_k$ ,  $\hat{b}_k = b_k$  and (20) that  $q_k^{adr} = b_k$ , which gives assertion (i). When consumer  $k$  is selected,  $\hat{\pi}_k = \pi_k$  implies that  $\pi_k^r > \pi_k - \pi^e$ . Then from (17) it follows that  $q_k^{dr} = 0$ , which gives assertion (iii)  $\square$

### A. Proof of Theorem 3

We give a rough outline of the proof here.

(i) For all  $\pi < \pi_{max} - \pi^e$ , we can construct a set of consumers such that when a particular consumer  $j$  is selected  $\pi < \pi_j - \pi^e$ . So when the reward per unit reduction  $\pi$  is such that  $\pi < \pi_{max} - \pi^e$  one cannot guarantee that all selected consumers would reduce

(ii) For a particular load reduction requirement  $D$ , even if CAISO's selection is as efficient as in SRBM, each selected consumer is paid  $\pi$  per unit of reduction where  $\pi \geq \pi_{max} - \pi^e \geq \pi_k - \pi^e$  for all  $k$ . Hence the total payment made by CAISO would be greater than that made in SRBM.

(iii) *When the consumers are informed of the DR event several hours prior to the DR event:* This is done in scenarios where the participating consumers need sufficient time to prepare for the DR event. We model the average benefit of consumer  $k$

on the DR event day as the mean of the utility during the DR event  $J_k^{dr}$  and the utility during the hours prior to the DR event  $J_k^{-1}$ . Then,

$$\begin{aligned} J_k(\cdot | k \text{ is selected}) &= (J_k^{-1} + J_k^{dr}) / 2 \\ &= (u_k(q_k^-) - \pi^e q_k^- + u_k(q_k) - \pi^e q_k + R(q_k, b_k^c, \pi)) / 2 \\ &= (u_k(q_k^-) - \pi^e q_k^- + u_k(q_k) - \pi^e q_k + \pi(b_k^c - q_k)) / 2 \end{aligned} \quad (40)$$

Here we compute the consumption decisions of the consumer  $q_k^-$  prior to the DR event hour on the DR event day and use it to derive a lower bound on the customer baseline. On the DR event day,  $b_m$  and  $b_m^-$  are constants so differentiating the average utility  $J_k(\cdot)$  in (40) w.r.t  $q_k^-$  gives,

$$\begin{aligned} \frac{\partial J_k(\cdot)}{\partial q_k^-} &= \left( \frac{\partial u_k(q_k^-)}{\partial q_k^-} - \pi^e + \pi \frac{\partial b_k^c}{\partial q_k^-} \right) / 2 \\ &= \begin{cases} \pi_k - \pi^e + \pi \frac{\bar{b}_k}{b_k} & \text{If } q_k^- \leq b_k \\ -\pi^e + \pi \frac{\bar{b}_k}{b_k} & q_k^- > b_k \end{cases} \\ &\geq \frac{\partial J_k(\cdot)}{\partial q_k^-} \geq \begin{cases} \pi_k - \pi^e + \pi & \text{If } q_k^- \leq b_k \\ -\pi^e + \pi & q_k^- > b_k \end{cases} \quad \text{since } \bar{b}_k > \bar{b}_k^- \end{aligned} \quad (41)$$

Note that  $\frac{\partial J_k(\cdot)}{\partial q_k^-} > 0$  for all values of  $q_k^-$  when  $\pi > \pi^e$ . This implies  $q_k^- \rightarrow \infty$  when  $\pi > \pi^e$ . This implies that

$$b_k^c = \bar{b}_k C_b = \bar{b}_k q_k^- / \bar{b}_k^- > q_k^- \rightarrow \infty \text{ when } \pi > \pi^e \quad (42)$$

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