# Spectrum Sharing through Contracts for Cognitive Radios

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**Abstract**—Development of dynamic spectrum access and allocation techniques recently have made feasible the vision of cognitive radio systems. However, a fundamental question arises: Why would licensed primary users of a spectrum band allow secondary users to share the band and degrade performance for them? And how can we design incentive schemes to enable spectrum sharing using cooperative communication schemes? We consider a principal-agent framework, and propose a contracts-based approach. First, a single primary and a single secondary transmitter-receiver pair with a Gaussian interference channel between them are considered. The two users may contract to cooperate in doing successive-interference cancellation. Under full information, we give equilibrium contracts for various channel conditions. These equilibrium contracts yield Pareto-optimal rate allocations when physically possible. We then allow for time-sharing and observe that in equilibrium contracts there is no actual time-sharing. We show that the designed contracts can be made robust to deviation by either user post-contract. We also show how these can be extended to multiple secondary users. We show that under hidden information, when the primary user has a dominant role, neither user has an incentive to lie about their direct channel coefficients, or manipulate the cross channel measurements, and Pareto-optimal outcomes are achieved at equilibrium.

**Index Terms**—Cognitive radios, cooperative communications, spectrum sharing, game theory, contract design

## 1 Introduction

The scarcity of spectrum is becoming an impediment to the growth of more capable wireless networks. Several measures are sought to address this problem: Freeing up unused spectrum, sharing of spectrum through new paradigms such as cognitive radio sensing, as well as sophisticated information theoretic schemes and network coding methods. Nearly, all such methods presume perfect user cooperation. In particular, Multiuser communication theory has proved tremendously successful in developing almost capacity-achieving schemes [1]. Based on these, new hierarchical network architectures have been proposed [2] in which cluster of nodes act as multiantenna arrays and use MIMO coding techniques to improve network capacity scaling from $O(n/log\ n)$ to $O(n)$, where $n$ is the number of nodes [3]. This, however, is an unjustified assumption. And with noncooperative, selfish users who act strategically, network (sum-rate) capacity can be arbitrarily bad as shown for the single-hop Gaussian interference channel (GIC) in unlicensed bands [4].

For licensed bands, one of the key challenges when users have cognitive radio capability is, *why would primary users give up their ownership rights over their spectrum and share it with secondary users at the cost of performance degradation to themselves?* FCC mandates are not going to solve the problem as primary users can always transmit junk to keep channels busy and deter secondary users [5].

This incentive issue has been sought to be addressed by guaranteeing the primary user a payment in lieu of sharing his spectrum and suffering some performance degradation. This has been sought to be implemented in various ways: Dynamic competitive pricing [6], [7], [8], and spectrum auctions [9], [10], [11]. While competitive pricing is usually not incentive-compatible (IC) and not robust to manipulation by strategic users, carefully designed auctions can potentially be strategy proof and yield socially optimal outcomes. In many scenarios, we can even operate them as double-sided auctions or markets when there are both buyers and sellers. Unfortunately, for auctions to be practical, they must be operated by a neutral, disinterested party as an auctioneer. Otherwise, the auctioneer can manipulate the auctions to his advantage. This situation is unlikely to arise in most cognitive radio systems. The spectrum sharing and allocation must happen as a direct result of interaction between a primary user and one or more secondary users.

In information and communication theory, it is well known that when users use "cooperative communication" schemes, better performance (in terms of higher sum-rate) can be achieved. In fact, using such schemes (e.g., successive-interference cancellation (SIC)) can enable spectrum sharing between users without any performance degradation at all for the *dominant* user. Thus, spectrum sharing with naive coding, i.e., treating interference from other users as noise can lead to inefficient outcomes as Nash equilibria. Thus, a question arises whether it is possible to alleviate this inefficiency by introducing an incentive

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alignment mechanism. Our focus in this paper is a licensed band setting with cognitive radios, where there is a *primary user* who owns the spectrum band and a *secondary user* who wants to share the spectrum with the primary user. Spectrum sharing is desirable for two reasons. First, the primary user may not be using the channel all the time. When it is not being used, it can be used by the secondary user. Second, even when the primary user is using the channel, it can still share the spectrum with the secondary user with little or no performance degradation to itself. For example, when there are only two users sharing a GIC, if they both agree to cooperate in doing SIC, the *dominant user* suffers no performance degradation at all. His achievable rate is the same as if the other user were not present.

This, however, does not imply that if the primary user acts as the dominant user, it does not suffer any *externality cost* due to the presence of the other user. For one, the achievable rate region $\mathcal{R}_{SIC}$ is possible only asymptotically in the codeword length. So, with any practical SIC codes, there is going to be some performance loss. Second, SIC requires cooperation between the users in codebook design. Thus, coding and decoding at the primary transmitter and receiver is more complex than before again imposing *complexity externality*. Even if these issues are ignored, the primary user still may not have an incentive to share the spectrum unless he is compensated for it in some way. Furthermore, in some scenarios complexity considerations can entail that the primary acts as a nondominant user while the secondary acts as a dominant user. Thus, it is amply clear that there is a need to introduce incentive alignment schemes that will induce the primary user to share spectrum with the secondary user, and moreover cooperate in doing so by using advanced communication schemes such as SIC.

As we argued before, auction mechanisms [10], [9] are not the right framework for this problem because there is no independent, impartial entity that can coordinate the auction and act as an auctioneer. Here, the primary user is an interested party with incentives to manipulate the auction. We, thus, consider this as a *principal-agent model* [12], [13], where one user (possibly the primary) acts as a principal, and offers several contracts to the agent(s) (possibly the secondary user(s)). The agent(s) then picks one of the possible contracts or may reject all of them. We specify the class of contracts that a primary user can offer such that both the primary and the secondary users are able to maximize their individual utilities while still achieving a social welfare objective. The principal-agent model has been used in solving some problems in communication networks such as network formation [14] and wireless multihop routing [15]. The principal-agent model and the contractual mechanism approach to spectrum sharing are new.

We focus on a two-user setting and assume that their radios are sophisticated enough to employ SIC techniques [16]. One receiver takes a dominant role and decodes his signal the last after decoding signals of all the other users. Typically, this would be the primary user but there can be scenarios where the secondary user radio is more sophisticated and acts as a dominant user. Again, either the primary or the secondary user can be the principal and offer contracts to the other user. Our findings are the following:

1. Under the full information case, i.e., when all the exact channel coefficients is common knowledge, in general, it is *impossible* to design *(first-best)* contracts that are Pareto-optimal at equilibrium. Nevertheless, we can specify channel conditions, and contract formats that are Pareto-optimal at equilibrium under them.
2. Even if we allow for time-sharing of a "dominant" role as part of the contractual negotiation, it is still *impossible* to design contracts that are Pareto-optimal at equilibrium, though as earlier, we can specify various contract formats that do result in Pareto-optimal outcomes under various channel conditions.
3. Deviation from agreed contract is a serious concern given lack of policing. Nevertheless, we show that simple incentive schemes can be devised that make the contracts robust to any deviation postcontract.
4. The contract design methodology can be extended to multiple secondary users.
5. We also show that under hidden information, when the primary user has a dominant role, neither user has an incentive to lie about their direct channel coefficients, or manipulate the cross channel measurements.

This is not the case when the secondary user has a dominant role. Thus, the lesson we learn is that when the information about channel coefficients is hidden, it is better for the primary user to be the dominant user. This yields *(second-best)* contracts with *(first-best)* Pareto-optimal outcomes at equilibrium.

A related work is [19] wherein a spectrum sharing contract scheme for resource exchange in cognitive radio networks is proposed. In their model, the primary and secondary users transmit in turn. Moreover, the primary user in return for sharing spectrum (via a TDMA scheme) receives help from the secondary users in transmitting to the primary receiver by use of space-time codes (which can be difficult to implement). The exact level of power used by the secondary users in transmitting the primary user's data, and the fraction of time secondary users get for their own transmission is determined via a contract. Our focus, however, is on spectrum sharing wherein both users transmit at the same time, and in the same frequency band. But they "cooperate" in using multiuser communication schemes such as the SIC.

The paper is organized as follows: In Section 2, we describe the physical layer channel model. In Section 3, we introduce the principal-agent framework and the contract design problem in a cognitive radio setting. In Section 4, we propose IC contracts under full information. We then consider the incomplete information setting in Section 5. We end with a discussion of future work.

## 2 The Physical Model and Related Work

We consider a situation where there is a primary user 0 who owns the spectrum license, and $M-1$ secondary cognitive radio users who want to share the spectrum with the primary user. We will model it as a GIC of bandwidth $W$ (assumed unity, for simplicity) with $M$ transmitter-receiver pairs. We consider a discrete-time channel model:

$$y_i[n] = \Sigma_{j=0}^{M-1} h_{j,i} x_j[n] + z_i[n], \quad i = 0, \cdots M-1, \tag{1}$$

where $x_j$ is the signal from the transmitter $j$, $y_i$ is the signal received at the receiver $i$, $h_{j,i}$ is the channel attenuation coefficient from transmitter $j$ to receiver $i$, and the noise process $\{z_i\}$ is i.i.d. over time with distribution $\mathcal{N}(0, N_0)$. We assume a flat fading channel.

Each user could treat the signal from other users as interference. This is reasonable when interference cancellation and alignment schemes cannot be used due to limitations on decoder complexity, delay constraints, or uncertainty in channel estimation. We assume that users use random Gaussian codebooks for transmission. Then, the maximum rate that the system can achieve is given by

$$R_i = \log\left(1 + \frac{c_{i,i} P_i}{N_0 + \Sigma_{j \neq i} c_{j,i} P_j}\right), \quad i = 0, \cdots M-1, \tag{2}$$

where $P_i$ is the transmitted power of user $i$, and $c_{i,j} = |h_{i,j}|^2$. Each transmitter has power constraints. Thus, $P_i$ must satisfy $P_i \leq \bar{P}_i$ for each $i$.

The spectrum sharing problem is to determine a set of power allocations $P = (P_1, \ldots, P_M)$ that maximize a given global utility function (such as the achievable sum-rate $\sum_i R_i$) while satisfying the power constraints. However, users are selfish and may not cooperate, with each wanting to maximize their own rate. Thus, they pick their power allocations $P$ each wanting to maximize their own rate and leading to a spectrum sharing game between them. To predict the outcome of such a game, we look at its' *Nash equilibrium* (NE) $P^*$ such that given the power allocations of all the other users $P^*_{-i}$, user $i$'s rate is maximized at $P^*_i$, i.e.,

$$R_i(P^*_i, P^*_{-i}) \geq R_i(P_i, P^*_{-i}), \quad \forall P_i \leq \bar{P}_i. \tag{3}$$

In [4], it was shown that in a flat fading GIC, a NE exists, all NE are pure strategy equilibria, and under certain conditions, *full-spread* power allocation is a NE.[1] Moreover, it was shown that under certain conditions, full-spread is the unique NE. In most cases, however, the set of rate vectors that result from the full-spread NE are not *Pareto-optimal*. So, there may be a significant performance loss if the $M$ users operate at any such point due to lack of cooperation. In fact, in many cases this inefficient outcome is the only possible outcome of the game. For general parallel GIC, existence of NE was proved in [17].

The above discussion assumed that users do not use cooperative communication schemes. In licensed settings, however, it can be possible to enable spectrum sharing using cooperative communication schemes [1]. A particular scheme of relevance is *SIC* which works as follows: Suppose user $M-1$ decodes his own signal by treating interference from all other users as noise, then he can achieve a rate $R_{M-1} = \log(1 + \frac{c_{M-1,M-1} P_{M-1}}{N_0 + \Sigma_{j<M-1} c_{j,M-1} P_j})$. Now, user $i$ can do the same and decode signal of all users $j > i$ first as above. Then, he can subtract these signals from his received signal, and decode his own signal by treating all other users as noise and achieve a rate

$$R_i = \log\left(1 + \frac{c_{i,i} P_i}{N_0 + \Sigma_{j<i} c_{j,i} P_j}\right), \tag{4}$$

which is greater than the rate in (2) if he had treated all other users' signals as noise. Proceeding in this way, user 0 then achieves a rate $R_0 = \log(1 + \frac{c_{0,0} P_0}{N_0})$. We will call user 0 as a *dominant user* (the user who decodes everyone else' signal first), and user $i$ as a $i$th *nondominant user* (in case of only two users, this user is just referred to as the *nondominant user*). From the achievable rate region (Fig. 1), it is clear that all users could potentially gain if we could find a way for them to cooperate.

1. The framework of [4] where the users do power allocation over a spectral band. Full spread power allocation then refers to spreading power uniformly over the whole band.

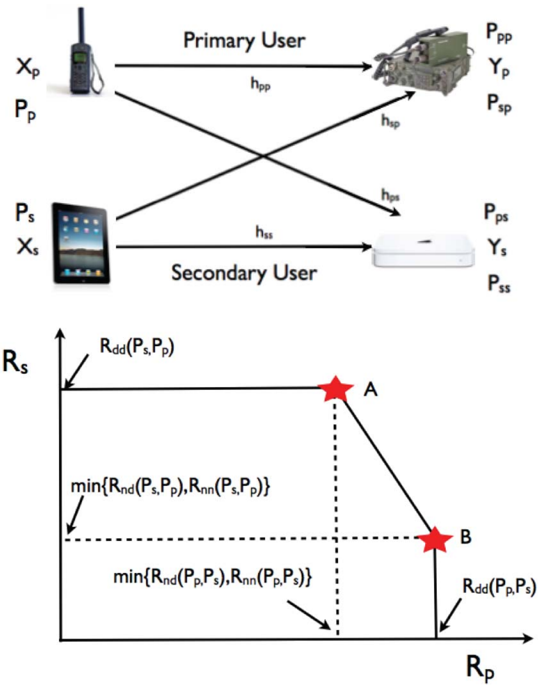


Fig. 1. The GIC, and it's achievable region with SIC with assigned dominant/nondominant roles.

**Remarks.**

1. A pertinent question in the context of cognitive radio systems is who will act as the dominant user? As we will see, our analysis indicates that the primary user should act as the dominant user, if he can. However, if he cannot, then the secondary user may act as the dominant user. This will depend on the scenario. Thus, in this paper, we consider both cases.
2. We note that a way to interpret rate expressions (2) and (4) are as long-term average rates for stationary, ergodic channels. If they are nonstationary, as is more realistically the case, the channel estimation is done, not in every time slot but every few hundred time slots.

## 3 The Principal-Agent Framework and Contract Design

We now introduce the *principal-agent model* [12], [13]. One agent acts as the principal who offers one or more contracts to one or more agents. The agent(s) then select(s) one or rejects all. Either the primary or the secondary user could act as the principal, and the other as the agent. In most settings, primary user will be the principal because he owns the spectrum and has "market power." There can, however, be scenarios where the primary user is limited in capabilities and unable to act as an agent. In that case, the secondary user may act as the principal. Thus, in this paper, we will consider both settings.

Denote the power used by the principal as $P_P$ and that used by the agent as $P_A$. Let the *utility* of the principal be its rate $R_P(P_P, P_A)$ and utility of the agent be its rate $R_A(P_A, P_P)$. We ignore other considerations such as the cost of transmission power. Denote by $\lambda(P_A, P_P)$ the payment that the agent makes to the principal. This can be positive or negative. We consider $u_P = R_P + \lambda$ to be the *payoff* of the principal, and $u_A = R_A - \lambda$ to be the payoff of the agent.

**Definition 1.** *A* spectrum contract *is a tuple* $(\lambda(\cdot,\cdot), P_P, P_A)$ *such that the operating point is* $(P_P, P_A)$ *and the payment is* $\lambda(P_P, P_A)$.

Denote agent strategy space $\mathcal{S}_A = [0, \bar{P}_A] \times [0, \bar{P}_P]$, payment function $\lambda : \mathcal{S}_A \rightarrow \mathbb{R}$ and outcome function $f : \mathcal{S}_A \rightarrow \mathcal{S}_A$. First, the principal picks a payment function $\lambda$ and an outcome function $f$. Then, the agent rejects the contract offered, or accepts and picks operating powers in $\mathcal{S}_A$. We will typically have $f(P_A, P_P) = (P_A, P_P)$, i.e., the agent will pick the powers at the operating point, but at times we may restrict this. The principal wants to design a $(\lambda, f)$ that maximizes his payoff $R_P + \lambda$ once the agent has accepted and an operating point $(P_P, P_A)$ has been picked. Let $\bar{R}_P$ and $\bar{R}_A$ denote the reservation utilities (rates) for the principal and the agent that they can derive if the contract is not accepted.

**Definition 2.** *We say that a spectrum contract function* $\lambda$ *is* individually rational for an agent *if there exist feasible* $(P_P, P_A)$ *such that* $R_A(P_P, P_A) - \lambda(P_P, P_A) \geq \bar{R}_A$.

That is, it would be rational for the agent (or the principal) to participate only if he can pick a *feasible* operating point at which his payoff is at least as large as his reservation utility $\bar{U}_A$.

**Definition 3.** *We say that a spectrum contract is* IC for an agent *if at the operating point* $(P_P, P_A)$, $R_A(P_P, P_A) - \lambda(P_P, P_A) \geq R_A(P_A', P_P) - \lambda(P_P, P_A'), \forall P_A' \leq \bar{P}_A$.

That is, (among the IR operating points), the agent will pick an operating point that maximizes its payoff. So, in designing the contracts, the principal should take both the IR and IC constraints into account.

The contract design problem induces a game between the principal and the agent(s). The principal wants to design a contract that maximizes his payoff, but at the same time get accepted by the agent(s). From the set of contracts offered, the agent will accept that contract which will maximize its payoff [13]. The contract that gets accepted finally is called an *equilibrium contract*.

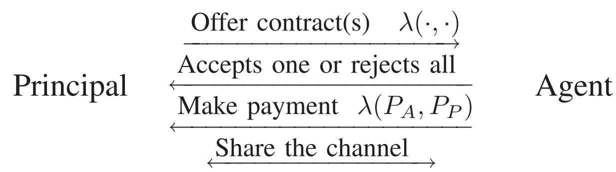


Fig. 2. Contractual mechanism between Principal and Agent.

Now, the principal's contract design problem is given by the following optimization problem:

$$\text{CD-OPT:} \max_{P_P, P_A, \lambda(\cdot,\cdot)} R_P(P_P, P_A) + \lambda(P_P, P_A) \tag{5}$$

$$s.t. \text{ [IR]: } R_A(P_A, P_P) - \lambda(P_P, P_A) \geq \bar{R}_A \tag{6}$$

$$\begin{aligned}\text{[IC]: } & R_A(P_A, P_P) - \lambda(P_P, P_A) \\ & \geq R_A(P_A', P_P) - \lambda(P_P, P_A'), \forall P_A' \leq \bar{P}_A.\end{aligned} \tag{7}$$

The optimization problem CD-OPT above is a nonconvex, variational problem, solving which, in general, is difficult. Existence of a solution can be established using standard arguments. We will, however, argue existence by construction in subsequent discussion.

When there is no information asymmetry between the principal and the agent, a contract is called the *first-best*. In such cases, the principal can make the agent operate at his reservation utility and extract all his *surplus* (the difference between utility with and without sharing). When the principal has incomplete or imperfect information about the agent's type (e.g., the channel coefficients here), a contract is called *second-best*. In such cases, the principal "pays" an *information rent* to the agent. Clearly, the principal's payoff under second-best contracts will be smaller than that in first-best contracts.

The principal and the agent can make contract after they observe their *types*, i.e., after they estimate their channel coefficients. Such contracts are called *ex post* contract. However, sometimes the principal and the agent can contract before they estimate their channel coefficients and such contracts are called *ex ante* contracts. In the subsequent discussions, we will show that if there is no information asymmetry between the principal and the agent, then the optimal contracts are of *ex post* type and if there is an information asymmetry, then optimal contracts are of *ex ante* type.

The timing of the contractual mechanism between the principal and the agent is summarized in the Fig. 2.

We define a *social welfare function* as $S(P_P, P_A) = R_P(P_P, P_A) + R_A(P_A, P_P)$. We say that a rate allocation $(R_P^{**}, R_A^{**})$ is *socially optimal* if it is achieved by a power allocation $(P_P^{**}, P_A^{**})$ that maximizes the social welfare subject to the power constraints. We will say that a spectrum contract is *(socially) optimal* if it achieves a socially optimal rate allocation. We will say that a rate allocation $(R_P^{*}, R_A^{*})$ is *Pareto-optimal* if neither user's rate can be increased without decreasing the other user's rate. Spectrum contracts that yield Pareto-optimal rates will be referred to as *Pareto-optimal contracts*.

Given the channel model and the communication scheme (like interference-limited communication, TDMA or FDMA), our aim is to design a spectrum contract, which achieves a socially optimal rate allocation. In the following paragraphs, we will examine the optimum contract design for two typical system models. In particular, we will argue that the equilibrium rate allocation achieved under those contract mechanism can be far away from the socially optimal rate allocation. This motivates our use of SIC as the communication scheme.

The most obvious choice for a contract mechanism is *paying-for-interference* model. The primary user will allow the secondary users to share the spectrum while treating their signals as noise. Spectrum sharing will decrease the data rate of the primary user who would have enjoyed an interference free channel otherwise. So, the challenge is to design a contract mechanism, which will ensure that the primary user has an incentive to share the spectrum (via ensuring the payment from the secondary users), secondary users accept the offered contract (via ensuring a positive utility), and the rate allocations are socially optimal.

Denote the received power from the transmitter $x$ at the receiver $y$ by $P_{xy} = c_{x,y}P_x$, and without loss of generality, set $N_0 = 1$ (here, $x, y = p$ or $s$). Also assume that (p)rimary acts as the (P)rincipal and the (s)econdary acts as the (A)gent (hence $R_P = R_p$ and $R_A = R_s$). Then, the contract design problem for a *paying-for-interference* model is given by CD-OPT (see (23)-(7)), where $R_p(P_p, P_s) = \log(1 + \frac{P_{p,p}}{1+P_{s,p}})$ $R_s(P_s, P_p) = \log(1 + \frac{P_{s,s}}{1+P_{p,s}})$.

The optimization problem CD-OPT is convex only if the signal-to-interference ratio (SIR) of each user is very high. If the SIR of each user is very high, the achievable rate of user $i$, $\log(1 + \mathrm{SIR}_i)$, will be approximately equal to $\log(\mathrm{SIR}_i)$ and the problem can be solved by using Geometric Programming [18]. However, in practice, most systems operate with a very low SIR and the CD-OPT problem is nonconvex. In [18], it was shown that this nonconvex optimization problem can be solved in an iterative way by using Complementary Geometric Programming methods. However, the solution obtained may correspond to a local maxima rather than the global maxima. Thus, the equilibrium rate allocation achieved by the contract mechanism for *paying-for-interference* model can be far away from the socially optimal rate allocation.

In this paper, we assume that the primary and the secondary user(s), if they agree to a contract, use the "cooperative communication" scheme, SIC [1], either can act as a dominant user or can be the principal, i.e., the one who proposes the contract. Such schemes typically assume that the channel coefficients (i.e., types of the players) are common knowledge, and furthermore, once roles, rates, and transmit powers to be used have been agreed upon, the users adhere to those commitments. We first design contracts for the complete information setting, and then discuss their robustness in the asymmetric/hidden information setting. We investigate the social optimality of the spectrum contract. Furthermore, any of the principal or the agent(s) can deviate from the agreed contract: They can transmit at a higher power, or a greater rate. This can cause suboptimal performance of the cooperative communication scheme. We will provide incentive mechanisms to avoid this *moral hazard problem*.

**Remarks.** An important question is how often the contractual negotiation needs to happen. We note that even if the channel is nonstationary, channel estimation is done in real systems only once every few hundred time slots. The contracts could be renegotiated every time channel estimation is done. However, this is not necessary. The contracted rates and payments can be interpreted as functions of channel gains. Then, the contractual negotiation need only happen once per session, though the payments and rates could vary over time.

## 4 First-Best Contracts for Cognitive Spectrum Sharing

We first assume that there is no informational asymmetry between the principal and the agent. The channel coefficients and the power constraints are known to all the users. This is a first step to understanding the more interesting case of informational asymmetry when channel coefficients are not common knowledge and users can manipulate their estimation. It turns out that the informational asymmetry case becomes really easy to understand once we understand and design contracts for the full information case. We specify the conditions under which 1) a first-best contract exists, 2) the first-best contract results in a Pareto-optimal operating point, and 3) the first-best contract is socially optimal.

Denote the received power from the transmitter $x$ at the receiver $y$ by $P_{xy} = c_{x,y}P_x$, and without loss of generality, set $N_0 = 1$ (here $x, y = p$ or $s$). Now, if the SIC scheme is successful, the dominant user's rate depends only on its received power at its own receiver. For example, when the primary acts as the dominant user, its transmission rate will be

$$R_{dd}(P_p, P_s) := \log(1 + P_{p,p}), \tag{8}$$

where $P_{p,p}$ is the received power of the primary transmitter at the primary receiver. Here, $dd$ refers to from dominant user transmitter to dominant user receiver. The nondominant user must transmit at a rate smaller than both

$$\begin{aligned} R_{nd}(P_s, P_p) &:= \log\left(1 + \frac{P_{s,p}}{1 + P_{p,p}}\right), \\ R_{nn}(P_s, P_p) &:= \log\left(1 + \frac{P_{s,s}}{1 + P_{p,s}}\right). \end{aligned} \tag{9}$$

Here, $nd$ refers to from nondominant user to dominant user receiver and $nn$ refers to from nondominant user to nondominant user receiver. For the SIC scheme to be successful, the nondominant user must transmit at a rate smaller than $R_{nd}(P_s, P_p)$ so that its signal can be decoded by the dominant user's receiver. He must also transmit at a rate smaller than $R_{nn}(P_s, P_p)$ so that its own receiver can decode his own signal.

### 4.1 Spectrum Contracts without Time-Sharing

We first consider the problem of contract design with full information when roles of users are fixed (i.e., the principal offers contracts to the agent in which its' role as a dominant or nondominant user is fixed). We focus on a two user setting for simplicity, one a primary, and the other a secondary user. Extension to multiple secondary users can be done under certain conditions, and is discussed later.

In the two-user setting, there are four cases: Either of the two users can be the principal, and either can be a dominant user. Below, we discuss only one case out of these four cases. Other cases can be analyzed in the same manner with similar results. The exact theorem statements for the other cases are in the appendix, which can be found on the Computer Society Digital Library at http://doi.ieeecomputersociety.org/10.1109/TMC.2012.171 (Theorems 7-9). Their proofs are omitted as they are very similar to that of Theorem 1. We assume that reservation utilities of the users are $\bar{R}_p = \log(1 + P_{pp})$ and $\bar{R}_s = 0$.

*Case A: Primary user is the principal and the dominant user.* Now, if the secondary user accepts a contract, and both cooperate in using the SIC scheme, then the maximum achievable rate of the primary user is given by (8). The secondary user's maximum transmission rate is limited by the minimum of rates $R_{nd}$ and $R_{nn}$ in (9). Define (for principal = Primary, dominant = Primary)

$$\begin{aligned}\lambda_{nd,A}(P_s;\bar{P}_p) &= R_{nd}(P_s,\bar{P}_p),\\ \lambda_{nn,A}(P_s;\bar{P}_p) &= R_{nn}(P_s,\bar{P}_p).\end{aligned} \tag{10}$$

Note that in these contract functions, the power of the primary is fixed at $\bar{P}_p$, i.e., the secondary user (if he accepts) can only pick his own power $P_s$. We now specify sufficient conditions on the channel coefficients under which the primary user offers different first-best contract functions.

Denote

$$\eta_{P,1} := (c_{ss} - c_{sp}) \text{ and } \eta_{P,2} := (c_{sp}c_{ps} - c_{pp}c_{ss}). \tag{11}$$

**Theorem 1.**

1. *If $\eta_{P,1} > \eta_{P,2}\bar{P}_p$, then the (first-best) equilibrium contract is $(\lambda_{nd,A}(\cdot;\bar{P}_p),(\bar{P}_p,\bar{P}_s))$, and the equilibrium rate allocation is $(R_{dd}(\bar{P}_p,\bar{P}_s),R_{nd}(\bar{P}_s,\bar{P}_p))$, which is socially optimal (and Pareto-optimal as well).*
2. *If $\eta_{P,1} \le \eta_{P,2}\bar{P}_p$ and $c_{p,p} \ge c_{p,s}$, then the (first-best) equilibrium contract is $(\lambda_{nn,A}(\cdot;\bar{P}_p),(\bar{P}_p,\bar{P}_s))$, and the equilibrium rate allocation is $(R_{dd}(\bar{P}_p,\bar{P}_s),R_{nn}(\bar{P}_s,\bar{P}_p))$, which is socially optimal (and Pareto-optimal as well).*
3. *There exist channel conditions (in case: $\eta_{P,1} < \eta_{P,2}\bar{P}_p$, $c_{p,p} < c_{p,s}$) under which the (first-best) equilibrium contract does not yield Pareto-optimal rates.*

**Proof.**

1. Consider the contract design problem CD-OPT in (23) with $p = p(rimary)$, and $a = s(econdary)$. In addition to the [IR] and [IC] constraints there, here we will also have the following:

$$R_p(P_p,P_s) = R_{dd}(P_p,P_s), \tag{12}$$

$$R_s(P_s,P_p) \le R_{nd}(P_s,P_p) = \log\left(1 + \frac{P_{s,p}}{1+P_{p,p}}\right), \tag{13}$$

$$R_s(P_s,P_p) \le R_{nn}(P_s,P_p) = \log\left(1 + \frac{P_{s,s}}{1+P_{p,s}}\right). \tag{14}$$

   We will call this the CD-OPT-PP, the contract design optimization problem for case A. It is now easy to check that for $\eta_{P,1} > \eta_{P,2}\bar{P}_p$ constraint (13) is tight. This can be done easily by comparing the RHS of inequalities (13) and (14). Since $\bar{R}_s = 0$, an obvious solution for the optimal payment function $\lambda^*(\cdot,\cdot)$ is $R_s(\cdot,\cdot)$. With this, the payoff of the agent (the secondary user) is zero at any operating point. Thus, the IR and IC constraints would be satisfied. Substituting the $\lambda^*(\cdot,\cdot)$ in the objective function and maximizing will give the optimum power allocation of the users as $P_p = \bar{P}_p$ and $P_s = \bar{P}_s$. Now, assuming that the secondary user prefers to get a higher rate with zero payoff to a lower rate with zero payoff, he will then pick $P_s = \bar{P}_s$. Note, however, that the secondary user can maximize his rate by making the interference from the primary user arbitrarily small, which will not be optimal for the primary user. Thus, the primary user offers contracts with his transmission power fixed at $\bar{P}_p$. The secondary only gets to pick his own transmission power $P_s$ and the corresponding payment $\lambda_{nd,A}(P_s;\bar{P}_p)$. Thus, offering $\lambda_{nd,A}(\cdot;\bar{P}_p)$ by the primary user, and picking $P_s = \bar{P}_s$ by the secondary user is an equilibrium. It is easy to check that the corresponding rate allocation maximizes the sum rate $R_p + R_s$, and is, thus, Pareto-optimal as well.
2. As before, consider the CD-OPT-PP problem, and assume that the channel coefficients are such that $\eta_{P,1} \le \eta_{P,2}\bar{P}_p$ and $c_{p,p} \ge c_{p,s}$. Then, for any feasible $P_p \in [0,\bar{P}_p]$, the RHS of the inequality (14) is always smaller than the RHS of the inequality (13). Thus, the inequality (14) will be tight (an equality) at the optimum. Furthermore, since $\bar{R}_s = 0$, an obvious solution for $\lambda^*(\cdot,\cdot)$, as before, is $R_s(\cdot,\cdot)$. With this, the payoff of the agent (the secondary user) is zero at any operating point and the IR and IC constraints would be satisfied. Substituting the $\lambda^*(\cdot,\cdot)$ in the objective function and maximizing will give the optimum power allocation of the users as $P_p = \bar{P}_p$ and $P_s = \bar{P}_s$. Also, the secondary will pick $P_s = \bar{P}_s$ as before. The principal again fixes his transmission power at $\bar{P}_p$ since given the choice, the secondary user will pick $P_p = 0$. Thus, the secondary only gets to pick $P_s$ and the corresponding payment $\lambda_{nn,A}(P_s;\bar{P}_p)$. Thus, offering $\lambda_{nn,A}(\cdot;\bar{P}_p)$ by the primary user, and picking $P_s = \bar{P}_s$ by the secondary user is an equilibrium with corresponding rate allocation being sum rate and Pareto-optimal.
3. When $\eta_{P,1} < \eta_{P,2}\bar{P}_p$, $c_{p,p} < c_{p,s}$, and $c_{pp}(1 + c_{ps}\bar{P}_p + c_{ss}\bar{P}_s + c_{sp}\bar{P}_s + c_{ps}^2\bar{P}_P^2) < c_{ps}c_{ss}\bar{P}_s$, it can be checked that the optimal solution of the CD-OPT-PP optimization problem will occur at $P_s = \bar{P}_s$ and $P_p < \bar{P}_p$, in which case the corresponding rate pair will be in the interior of the achievable rate region, and cannot be Pareto-optimal. □

We summarize the results of Theorem 1 in Fig. 3. The operating points are the rate pairs, $(R_p, R_s)$, at the equilibrium and are shown in the Fig. 1.

| Channel Conditions | Contract Function | Powers | Operating Point | Optimality |
|---|---|---|---|---|
| $\eta_{P,1} > \eta_{P,2}\bar{P}_p$ | $\lambda_{nd,A}$ | $(\bar{P}_p, \bar{P}_s)$ | B in Fig. 1 | Socially/ Pareto |
| $\eta_{P,1} \leq \eta_{P,2}\bar{P}_p$, $c_{p,p} \geq c_{p,s}$ | $\lambda_{nn,A}$ | $(\bar{P}_p, \bar{P}_s)$ | B in Fig. 1 | Socially/ Pareto |
| $\eta_{P,1} \leq \eta_{P,2}\bar{P}_p$, $c_{p,p} < c_{p,s}$ | $\lambda_{nn,A}$ | $(P_p, \bar{P}_s)$ $P_p < \bar{P}_p$ | Interior in Fig. 1 | non-Pareto |

Fig. 3. Summary of the results for Case A under full information.

Figs. 4 and 5 show the value of the contract function $\lambda(\cdot,\cdot)$ for different values of $\bar{P}_s, \bar{P}_p$. In Fig. 4, channel parameters are $c_{pp} = c_{ss} = 1.0, c_{sp} = c_{ps} = 0.5$. So, $\eta_{P,1} > \eta_{P,2}\bar{P}_p$ for any positive value of $\bar{P}_p$ and by Theorem 1 the optimal contract function is $\lambda_{nd,A}(\bar{P}_s, \bar{P}_p)$. In Fig. 5, channel parameters are $c_{pp} = 0.8, c_{ss} = 0.7, c_{sp} = 0.8, c_{ps} = 0.4$. So, $\eta_{P,1} \leq \eta_{P,2}\bar{P}_p$ when $\bar{P}_p \in [0, 0.42]$ and $\eta_{P,1} > \eta_{P,2}\bar{P}_p$ when $\bar{P}_p \in [0.42, 1.0]$. So, by Theorem 1, the optimal contract function is $\lambda_{nn,A}(\bar{P}_s, \bar{P}_p)$ when $\bar{P}_p \in [0, 0.42]$ and $\lambda_{nd,A}(\bar{P}_s, \bar{P}_p)$ when $\bar{P}_p \in [0.42, 1.0]$.

**Remarks.**

1. The other three cases when the secondary user is either the principal, or the dominant user can also be analyzed in a manner similar to above, and the form of the equilibrium contracts can be obtained. We state some notable observations.
2. Cases B, C, and D are given below:

   a. *Case B: Secondary user is the principal and primary user is dominant.* In this case, the principal only offers an arbitrarily small payment $\lambda_B = \epsilon > 0$ to the primary user with fixed powers $(\bar{P}_p, \bar{P}_s)$, and this results in an equilibrium contract under most channel conditions. The reason for this is that because the primary user is dominant, it would suffer no performance degradation by sharing spectrum using SIC. Thus, the secondary user need not offer it any additional payoff. Details are given in Theorem 7 (in the appendix, available in the online supplemental material).

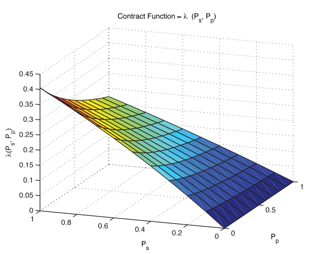

Fig. 4. Plot of the optimal contract function $\lambda(\bar{P}_s, \bar{P}_p)$ against $\bar{P}_s$ and $\bar{P}_p$ for Case A. Channel parameters are $c_{pp} = c_{ss} = 1.0, c_{sp} = c_{ps} = 0.5$.

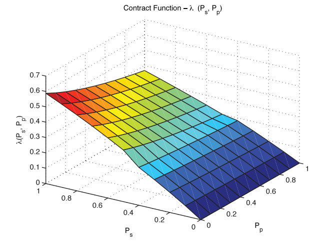

Fig. 5. Plot of the optimal contract function $\lambda(\bar{P}_s, \bar{P}_p)$ against $\bar{P}_s$ and $\bar{P}_p$ for Case A. Channel parameters are $c_{pp} = 0.8$, $c_{ss} = 0.7$, $c_{sp} = 0.8$, and $c_{ps} = 0.4$.

   b. *Case C: Secondary user is the principal and dominant.* In this case, the secondary user fixes $P_p = \bar{P}_p$, and lets the primary user pick its power. Details are given in Theorem 8 (in appendix, available in the online supplemental material).

   c. *Case D: Primary user is the principal and secondary user is dominant.* In this case, the principal fixes $P_p = \bar{P}_p$, and lets the secondary user pick his own power.

3. In all four cases, we have some channel conditions under which no (first-best) Pareto-optimal equilibrium contract exists. Details are given in Theorem 9 (in appendix, available in the online supplemental material).
4. We also note that if the primary user were to insist on using full power, then a contract may fail to materialize in case 3 above.

*Lesson #1.* Under most channel conditions, first-best contracts that are Pareto-optimal at equilibrium can be designed. But under some channel conditions, with fixed roles, it is *impossible* to do so.

## 4.2 Moral Hazard Problems

An obvious question is do either of the users have an incentive to deviate from the contractual agreement. That is, the principal could offer a menu of contracts to the agent. The agent may accept one. Then, the payment is made. But at the time of communication, either of them may use a different rate or transmission power than agreed. This is called the moral hazard problem. And if it happens, how can this be prevented?

The moral hazard problem can indeed happen. Consider the case where the primary user is the principal and dominant with channel coefficients satisfying assumptions in case 2 of Theorem 1. Then, $R_{allowed} = R_{nd}(\bar{P}_p, \bar{P}_s) < R_{achievable} = R_{nn}(\bar{P}_p, \bar{P}_s)$, i.e, the secondary user can increase his rate to above the agreed rate $R_{allowed}$, and get across a higher rate to his own receiver while making a lower payment.

This deviation by the secondary user, however, can be detected by the primary/dominant user because this will cause the SIC to fail at the primary user, who will then get zero rate. To avoid the moral hazard problem, the principal can require the secondary user to make a *refundable access deposit*, say larger than his reservation utility $\bar{R}_p$. If the secondary user deviates, this is forfeited.

In the same way, if the primary user is nondominant, under some channel conditions, it has an incentive to deviate from the agreed rate and powers. Again, such a deviation will cause the SIC scheme to fail at the secondary receiver, who will then get zero rate. It can, thus, be detected. The principal in this case can avoid the moral hazard problem by *deferred payment*, i.e., the secondary user makes the payment only after channel use. If the primary user deviates, the payment is forfeited.

*Lesson #2.* Using simple incentive schemes, the moral hazard problems can always be avoided.

### 4.3 First-Best Spectrum Contracts with Time-Sharing

We now allow for time-sharing between the primary and secondary users. The time-sharing roles are determined as part of the contractual negotiation. Denote by $\alpha$ the time-sharing variable. The principal acts as a dominant user $\alpha$ fraction of the time, and as a nondominant user $\bar{\alpha} = 1 - \alpha$ fraction of the time with $\alpha \in [0, 1]$. The parameter $\alpha$ is picked by the agent. Either the primary or the secondary can act as the principal. Here, we discuss only one case. The other case can be analyzed in the same manner.

**Definition 4.** *A* time-sharing spectrum contract *is a tuple* $(\alpha, \lambda^{TS}(\cdot,\cdot,\cdot), P_p, P_a)$ *such that the operating point is* $(\alpha, (P_p, P_a))$ *and the payment is* $\lambda^{TS}(\alpha, P_p, P_a)$.

*Case: Primary user is the principal and dominant.*The contract design problem of the principal in the time-sharing case is the same as CD-OPT in (5) with $p = p(rimary)$, and $a = s(econdary)$, with the rate functions $R_p$ and $R_s$ given as

$$\begin{aligned} R_p(P_p, P_s) &= \alpha R_{dd}(P_p, P_s) + \bar{\alpha} R_n(P_p, P_s), \\ R_s(P_s, P_p) &= \alpha R_n(P_s, P_p) + \bar{\alpha} R_{dd}(P_s, P_p), \end{aligned} \tag{15}$$

where the rate functions

$$\begin{aligned} R_{dd}(P_x, P_y) &= \log(1 + P_{xx}), \\ R_n(P_x, P_y) &= \min\{R_{nd}(P_x, P_y), R_{nn}(P_x, P_y)\}, \\ R_{nd}(P_x, P_y) &= \log\left(1 + \frac{P_{xy}}{1 + P_{yy}}\right), \text{ and} \\ R_{nn}(P_x, P_y) &= \log\left(1 + \frac{P_{xx}}{1 + P_{yx}}\right). \end{aligned}$$

Equation (15) is obtained by the standard time-sharing formula. The primary user act as the dominant user for $\alpha$ fraction of the time achieving a rate $R_{dd}(P_p, P_s)$ and act as the non-dominant user for the remaining $\bar{\alpha}$ fraction of the time achieving a rate $R_n(P_p, P_s)$. Thus, the total rate of the primary will be $R_p(P_p, P_s) = \alpha R_{dd}(P_p, P_s) + \bar{\alpha} R_n(P_p, P_s)$. Thus,

$$R_n(P_s, P_p) \le R_{nd}(P_s, P_p) := \log\left(1 + \frac{P_{s,p}}{1 + P_{p,p}}\right), \tag{16}$$

$$R_n(P_s, P_p) \le R_{nn}(P_s, P_p) := \log\left(1 + \frac{P_{s,s}}{1 + P_{p,s}}\right), \tag{17}$$

$$R_n(P_p, P_s) \le R_{nd}(P_p, P_s) := \log\left(1 + \frac{P_{p,s}}{1 + P_{s,s}}\right), \tag{18}$$

$$R_n(P_p, P_s) \le R_{nn}(P_p, P_s) := \log\left(1 + \frac{P_{p,p}}{1 + P_{s,p}}\right). \tag{19}$$

Define:

$$\lambda^{TS}_{nd}(\alpha; P_s; \bar{P}_p) := \alpha R_{nd}(P_s, \bar{P}_p) + \bar{\alpha} R_{dd}(P_s, \bar{P}_p), \tag{20}$$

$$\lambda^{TS}_{nn}(\alpha; P_s; \bar{P}_p) := \alpha R_{nn}(P_s, \bar{P}_p) + \bar{\alpha} R_{dd}(P_s, \bar{P}_p). \tag{21}$$

We now give the equilibrium contract functions and rate allocations in the time-sharing case.

Denote

$$\eta_{S,1} := (c_{pp} - c_{ps}) \text{ and } \eta_{S,2} := (c_{ps}c_{sp} - c_{pp}c_{ss}). \tag{22}$$

**Theorem 2.** *Let* $\eta_{P,1}, \eta_{P,2}$ *as defined in (11) and* $\eta_{S,1}$ *and* $\eta_{S,2}$ *as defined above:*

1. *If* $\eta_{P,1} \le \eta_{P,2}\bar{P}_p$ *and* $c_{p,p} \ge c_{p,s}$*, then the equilibrium contract is* $(\alpha = 0, \lambda^{TS}_{nn}(.;.;\bar{P}_p), (\bar{P}_p, \bar{P}_s))$*. If* $\eta_{S,1} > \eta_{S,2}\bar{P}_s$*, the equilibrium rate allocation is* $(R_{nd}(\bar{P}_p, \bar{P}_s), R_{dd}(\bar{P}_s, \bar{P}_p))$*, which is Pareto-optimal. If* $\eta_{S,1} \le \eta_{S,2}\bar{P}_s$*, then the equilibrium rate allocation is* $(R_{nn}(\bar{P}_p, \bar{P}_s), R_{dd}(\bar{P}_s, \bar{P}_p))$*, which is Pareto-optimal.*
2. *If* $\eta_{P,1} > \eta_{P,2}\bar{P}_p$ *and* $\frac{c_{p,s} - c_{s,s}}{c_{s,s}.c_{p,p}} < \bar{P}_p$*, then the equilibrium contract is* $(\alpha = 0, \lambda^{TS}_{nd}(.;P_s;\bar{P}_p), (\bar{P}_p, \bar{P}_s))$*. If* $\eta_{S,1} \le \eta_{S,2}\bar{P}_p$ *the equilibrium rate allocation is* $(R_{nn}(\bar{P}_p, \bar{P}_s), R_{dd}(\bar{P}_s, \bar{P}_p))$*, which is Pareto-optimal. If* $\eta_{S,1} > \eta_{S,2}\bar{P}_p$ *the equilibrium rate allocation is* $(R_{nd}(\bar{P}_p, \bar{P}_s), R_{dd}(\bar{P}_s, \bar{P}_p))$*, which is Pareto-optimal.*
3. $\eta_{P,1} > \eta_{P,2}\bar{P}_p$ *and* $\frac{c_{p,s} - c_{s,s}}{c_{s,s}.c_{p,p}} > \bar{P}_p$*, then the equilibrium contract is* $(\alpha = 1, \lambda^{TS}_{nd}(.;P_s;\bar{P}_p), (\bar{P}_p, \bar{P}_s))$*. The equilibrium rate allocation is* $(R_{dd}(\bar{P}_p, \bar{P}_s), R_{nd}(\bar{P}_s, \bar{P}_p))$*, which is Pareto-optimal.*
4. *There exist channel conditions under which the equilibrium contract is not Pareto-optimal.*

**Proof.**

1. As in the proof of Theorem 1, the obvious solution for $\lambda^*(\cdot,\cdot)$ is $R_s(\cdot,\cdot)$. When $\eta_{P,1} \le \eta_{P,2}\bar{P}_p$ and $c_{p,p} \ge c_{p,s}$, the constraint (17) is tight. Then by (15), we can calculate $R_s$. It is easy to observe that in this case, $R_s(P_s, P_p) = \lambda^{TS}_{nn}(\alpha; P_s; P_p)$. With this form of the contract, the net utility of the agent is zero at any operating point. Thus, IR and IC constraints are satisfied. Now, since $R_s(P_s, P_p)$ is monotonic in $P_s$, the agent will pick $P_s = \bar{P}_s$. Furthermore, the secondary user, if given the choice, will pick $P_p$ to make interference from the primary user arbitrary small. But this does not maximize the payoff of the primary user, and who, thus, fixes his transmission power at $\bar{P}_p$, and the secondary user only gets to pick his transmission power $P_s$ and the corresponding payment $\lambda^{TS}_{nn}(\cdot; P_s; \bar{P}_p)$. Thus, offering $\lambda^{TS}_{\underline{nn}}(\cdot;\cdot;\bar{P}_p)$ by the primary user, and picking $P_s = \bar{P}_s$ by the secondary user is an equilibrium. Under $\eta_{P,1} \le \eta_{P,2}\bar{P}_p$, it is easy to check that $R_{dd} \ge R_{nn}$. So, the secondary will choose $\alpha = 0$. Now, when $\eta_{S,1} < \eta_{S,2}\bar{P}_s$, $R_n(\cdot,\cdot) = R_{nn}(\cdot,\cdot)$ and when $\eta_{S,1} > \eta_{S,2}\bar{P}_s$, $R_n(\cdot,\cdot) = R_{nd}(\cdot,\cdot)$. So, the equilibrium rates can be calculated from (15). It is easy to check that the corresponding rate allocation is Pareto-optimal.

2. The argument in this case is very similar to case 1. The only difference is that equilibrium contract is $\lambda_{nd}^{TS}(\cdot;\cdot;P_p)$ with $\alpha = 0$.
3. The argument in this case is again very similar to case 1. The only difference is that equilibrium contract is $\lambda_{nd}^{TS}(\cdot;\cdot;P_p)$ with $\alpha = 1$.
4. $\eta_{P,1} < \eta_{P,2}\bar{P}_p, c_{p,p} < c_{p,s}$ and $\eta_{S,1} < \eta_{S,2}\bar{P}_p, c_{s,s} < c_{s,p}$, whatever be the choice of $\alpha$, the condition is similar to that in step 4 of Theorem 1. So, these channel condition results in an equilibrium contract that does not yield Pareto-optimal rates. □

**Remarks.** Above, we have given forms for equilibrium contracts that allow for time-sharing of dominant role. This is determined itself as part of the contract. The key observation here is that despite allowing for any time-sharing, i.e., $\alpha \in [0, 1]$, at equilibrium, we only see $\alpha \in \{0, 1\}$, i.e., though the dominant role is now determined as part of the contractual "negotiation," but we do not see any actual time-sharing.

*Lesson #3.* While time-sharing contracts are more flexible, the problem of nonexistence of Pareto-optimal first-best contracts under some channel conditions cannot be resolved even with this increased flexibility.

### 4.4 Extension to Multiple Secondary Users

The analysis in the previous sections can be extended to the case of multiple secondary users. We illustrate this by considering a three user case. We show that under some channel conditions, (first-best) equilibrium contract exists and the equilibrium rate allocation is socially optimal.

We assume that the primary is the dominant user and the data from both the secondary users will be decoded by the primary's receiver. Secondary user 1 treats primary users' data as noise, but can decode secondary user 2's data. Secondary user 2 is acting as the nondominant user for the other two users. Being the dominant user, primary user gets an interference free channel and has a rate $R_0 = \log(1 + P_{p,p})$. The rate of secondary user 1 has two constraints: It should be decodable at the primary user's receiver as well as at its own receiver. These rate constraint are given by $R_{s_1,p} = \log(1 + \frac{P_{s_1,p}}{1+P_{p,p}})$ and $R_{s_1,s_1} = \log(1 + \frac{P_{s_1,s_1}}{1+P_{p,s_1}})$, respectively. Similarly, secondary user 2's rate has three constraints: It should be decodable at primary user's receiver, at secondary user 1's receiver, and at its own receiver. These constraint rates are given by

$$R_{s_2,p} = \log\left(1 + \frac{P_{s_2,p}}{1 + P_{p,p} + P_{s_1,p}}\right),$$

$$R_{s_2,s_1} = \log\left(1 + \frac{P_{s_2,s_1}}{1 + P_{p,s_1} + P_{s_1,s_1}}\right), \text{ and}$$

$$R_{s_2,s_2} = \log\left(1 + \frac{P_{s_2,s_2}}{1 + P_{p,s_2} + P_{s_1,s_2}}\right),$$

respectively. Assuming that the primary user is acting as the principal, his contract design problem is as follows:

CD-OPT:

$$\begin{aligned}
&max_{P_p,P_{s_1},P_{s_2},\lambda} R_0(P_p, P_{s_1}, P_{s_2}) + \lambda_1(P_p, P_{s_1}, P_{s_2}) \\
&\quad + \lambda_2(P_p, P_{s_1}, P_{s_2}) \\
&s.t.\ [\mathbf{IR1}]: R_1(P_p, P_{s_1}, P_{s_2}) - \lambda_1(P_p, P_{s_1}, P_{s_2}) \geq 0 \\
&\quad [\mathbf{IR2}]: R_2(P_p, P_{s_1}, P_{s_2}) - \lambda_2(P_p, P_{s_1}, P_{s_2}) \geq 0 \\
&\quad [\mathbf{IC1}]: R_1(P_p, P_{s_1}, P_{s_2}) - \lambda_1(P_p, P_{s_1}, P_{s_2}) \\
&\quad \geq R_1(P_p, P'_{s_1}, P_{s_2}) - \lambda_1(P_p, P'_{s_1}, P_{s_2}), \\
&\quad \forall P'_{s_1} \leq \bar{P}_{s_1} \\
&\quad [\mathbf{IC2}]: R_1(P_p, P_{s_1}, P_{s_2}) - \lambda_1(P_p, P_{s_1}, P_{s_2}) \\
&\quad \geq R_1(P_p, P_{s_1}, P'_{s_2}) - \lambda_1(P_p, P_{s_1}, P'_{s_2}), \\
&\quad \forall P'_{s_2} \leq \bar{P}_{s_2} \\
&\quad R_1 = \min(R_{s_1,p}, R_{s_1,s_1}), \\
&\quad R_2 = \min(R_{s_2,p}, R_{s_2,s_1}, R_{s_2,s_2}).
\end{aligned}$$

We now specify sufficient conditions on the channel coefficients under which the primary user offers socially optimal first-best contracts.

**Theorem 3.**

1. *When the channel conditions are such that $c_{p,p} > c_{p,s_1}$, $c_{p,p} > c_{p,s_2}$, $c_{p,s_1} > c_{p,s_2}$, and $c_{s_1,s_1} > c_{s_1,s_2}$, then there exists a (first-best) equilibrium contract. The contract functions are of the form $\lambda_1(\bar{P}_p, \cdot, P_{s_2}) = R_1(\bar{P}_p, \cdot, P_{s_2})$, $\lambda_2(\bar{P}_p, P_{s_2}, \cdot) = R_2(\bar{P}_p, P_{s_2}, \cdot)$. The equilibrium rate allocation is socially optimal (and Pareto-optimal as well).*
2. *There exist channel conditions under which the (first-best) equilibrium contract does not yield Pareto-optimal rates.*

**Proof.** As in the two users case, we will assume that the IR constraints are tight, find the $\lambda_1$ and $\lambda_2$, substitute them into the objective function and solve by treating it as an unconstrained optimization problem. Now, the exact expression for $\lambda_1$ and $\lambda_2$ depends on $R_1$ and $R_2$, and which of the constraints on $R_1$ and $R_2$ are tight. This depends on the channel conditions. So, there can be six different forms for the objective functions. By taking each case separately, it can easily be shown that the conditions given in the theorem are sufficient for the existence of a first-best contract. Here, we give one particular case for illustration.

Assume, $R_1 = R_{s_1,s_1}$ and $R_2 = R_{s_2,s_2}$. Then, the optimization problem reduces to

$$\max_{P_p,P_{s_1},P_{s_2}} \frac{(1 + P_{pp})(1 + P_{p,s_1} + P_{s_1,s_1})}{(1 + P_{p,s_1})} \cdot \frac{(1 + P_{p,s_2} + P_{s_1,s_2} + P_{s_2,s_2})}{(1 + P_{p,s_1})(1 + P_{p,s_2} + P_{s_1,s_2})}. \tag{23}$$

It is straightforward to show that the above function is monotonic in $(P_p, P_{s_1}, P_{s_2})$ under the conditions given in the theorem. So, each user will transmit at their maximum power and the sum rate will be on the Pareto-optimal boundary.

Now, consider the same case as above, except that $c_{p,p} < c_{p,s_1}$, instead of $c_{p,p} > c_{p,s_1}$. And, similar to case 4 in Theorem 1, it can be shown that the solution for the above optimization problem will be $P_{s_1} = \bar{P}_{s_1}, P_{s_2} = \bar{P}_{s_2}$, but $P_p < \bar{P}_p$, in which case the corresponding rate pair will be in the interior of the achievable rate region, and cannot be Pareto-optimal. □

*Lesson #4.* With multiple secondary users, first-best contracts that are Pareto-optimal can be designed. But under some channel conditions, it is *impossible* to do so.

## 5 Hidden Information: Second-Best Cognitive Spectrum Contracts

In the channel estimation phase, each user sends a pilot signal, all receivers calculate the channel strength by estimating received power and then feedback the channel strength to the corresponding transmitter. Clearly, this requires the cooperation from all the users. In the previous section, we assumed that the channel information is common knowledge and correct. However, when the users are selfish and rational, each one may want to maximize his own utility. So, each user may manipulate the channel measurements if he can increase his own payoff. Now the question is, are the contracts designed in the previous section robust with *hidden information*? If not, can one design contracts that are robust and still Pareto-optimal at equilibrium?

Our main result below is that hidden information does not always hurt: When the primary user is dominant, neither the agent nor the principal will manipulate the channel measurement (see Theorem 4). Thus, it is possible to design an *ex post* second-best contract, which achieves the first-best outcome. However, when the secondary user is dominant, it is impossible to design an *ex post* second-best contract, which achieves the first-best outcome. For those cases, we propose an *ex ante* second-best contract, which achieves the first-best outcome. In the following discussion, we analyze Cases A-D and propose the optimal contract for each case. We present only case A in detail. Other cases can be found in the appendix, available in the online supplemental material.

*Case A: Primary user is the principal and the dominant user.*

**Theorem 4.** *Let contract functions $\lambda^A_{nd} = \lambda_{nd,A}$ and $\lambda^A_{nn} = \lambda_{nn,A}$ be as defined in (10). In Case A, the agent will report the channel coefficients truthfully. The equilibrium contracts and rate allocations are the same as in the complete information case (as in Theorem 1).*

**Proof.** The basic idea behind this theorem is that, the primary user can observe the secondary user's rate here. Since the primary is the dominant user here, the secondary user's data are decoded at the primary user's receiver. For this decoding to work, the primary receiver should know the exact rate at which the secondary's data are encoded at. So, in particular, if the secondary user tries to increase its rate from the agreed upon rate $R_s$ in the contract, it will lead to SIC decoding failure as explained in Section 4.2 and hence the deviation will be detected by the primary. This means that the secondary user cannot increase his rate from the agreed upon rate $R_s = \min\{R_{nd}(P_s, P_p), R_{nn}(P_s, P_p)\}$ (see (9)).

Suppose that $R_{nn} = \min\{R_{nd}(P_s, P_p), R_{nn}(P_s, P_p)\}$, the maximum rate that the secondary can decode at its own receiver. Now assume that the secondary reports $R'_{nn}$ where $R'_{nn} < R_{nn}$. Thus, the "contract rate" is $R_s = R'_{nn}$. If he tries to transmit at a higher rate, say at $R_{nn}$, that deviation will be immediately detected by the primary as described above. The secondary will lose his *refundable access deposit* (as explained in the Section 4.2 about moral hazards) and hence it will not try this deviation. Now assume that the secondary reports $R''_{nn}$, where $R''_{nn} > R_{nn}$ and, hence, the "contract rate" is $R_s = R''_{nn}$. Thus, it would have to make a higher payment compared to that of reporting $R_{nn}$. But the secondary's own receiver cannot decode this transmission because $R''_{nn}$ is greater than the maximum possible decodable rate $R_{nn}$. Thus the secondary will end up in a negative utility and, hence, he will never try this deviation. So, combining both arguments, the secondary will not misreport the value of $R_{nn}$.

Now, suppose that $R_{nd} = \min\{R_{nd}(P_s,P_p), R_{nn}(P_s,P_p)\}$. In the same way as above, we can argue that the secondary will not misreport the value of $R_{nd}$. Thus, in either case, the secondary user has no incentives to misreport his channel coefficients. Thus, the equilibrium contracts and the rate allocations will be the same as in the complete information scenario. □

*Case B: Secondary user is the principal, Primary user is dominant.*

Similar to Case A, the primary is the dominant user here and the secondary user's data are decoded at the primary's receiver. So, by Theorem 4, neither the primary nor the secondary user will have an incentive to manipulate the channel measurements. So, the contract design will be the same as in the complete information scenario.

**Corollary 1.** *If the primary user is dominant, then the agent will report the true channel coefficients to the principal. The principal can design a second-best contract which achieves the first-best outcome. The equilibrium contract functions and the rate allocations will be the same as in the complete information case.*

*Case C: Secondary user is the principal and the dominant user.*

**Theorem 5.** *In Case C, it is impossible to design an* ex post *second-best contract, which achieves the first-best outcome.*

**Proof.** The reservation utility of the agent (the primary user) is $\log(1 + c_{p,p}\bar{P}_p)$. So, the IR constraint of the optimization problem CD-OPT is $R_p(\cdot,\cdot) - \lambda(P_p, P_s) \geq \log(1 + c_{p,p}\bar{P}_p)$, where $R_p(\cdot,\cdot)$ is the rate of the primary and $\lambda(P_p, P_s)$ is the payment that it receives from the secondary. So, for the primary user to have an incentive to share spectrum, the payment should be $\lambda(\cdot,\cdot) \geq \log(1 + c_{p,p}\bar{P}_p) - R_p(\cdot,\cdot)$. Since the principal (secondary) has to depend on the agent (primary) to know the value of $c_{p,p}$, the agent (primary) can report a higher value of $c_{p,p}$ than the actual. This is equivalent to reporting a higher reservation utility than the actual and this false reporting cannot be detected by the principal. Thus, any contract function that depends on the reported value of $c_{p,p}$ will not yield a first-best outcome. On the other hand, if the payment offered does not satisfy the IR constraint of the agent (primary), it may not share the spectrum at all. So, it is important for the principal (secondary) to know the value of $c_{p,p}$ to design the contract. Thus, the principal cannot design an *ex post* contract, which makes the agent operate at its reservation utility and first-best outcomes cannot be achieved. □

Since no *ex post* contract which achieves the first-best outcome is possible in this case, we propose an *ex ante* contract design to overcome this problem. An *ex ante* contract is an agreement between the principal and the agent before either of them know any of the channel coefficients' realization, but do know their distribution, which is common knowledge. The agent will accept the contract, if his expected utility (*ex ante* utility) is at least equal to his expected reservation utility. So, the IR constraint in the optimization problem CD-OPT changes to $\mathbb{E}[R_a(P_a, P_p) - \lambda(P_p, P_a) - \bar{R}_a] \geq 0$. Once the contract is accepted, the channel coefficients are measured and the agent will pick his power allocation. In the following theorem, we establish that in an *ex ante* contract, neither the primary nor the secondary user will have an incentive to manipulate the channel coefficients. Define

$$\lambda_{nd}^C(P_p; \bar{P}_s) = \mathbb{E}\big[\log(1 + c_{p,p}\bar{P}_p)\big] - R_{nd}(P_p, \bar{P}_s), \qquad (24)$$

$$\lambda_{nn}^C(P_p; \bar{P}_s) = \mathbb{E}\big[\log(1 + c_{p,p}\bar{P}_p)\big] - R_{nn}(P_p, \bar{P}_s), \qquad (25)$$

where the expectation is with respect to the distribution of the channel coefficients.

**Theorem 6.** *In case C:*

1. *The agent (primary user) will report the channel coefficients truthfully. The principal (secondary user) will be able to calculate $\eta_{S,1}$ and $\eta_{S,2}$ (defined in (22)) correctly, based on the reported channel coefficients.*
2. *If $\eta_{S,1} > \eta_{S,2}\bar{P}_s$, then the equilibrium* ex ante *second-best contract which achieves the first-best outcome is $(\lambda_{nd}^C(\cdot; \bar{P}_s), (\bar{P}_p, \bar{P}_s))$, and the equilibrium rate allocation is $(R_{nd}(\bar{P}_s, \bar{P}_s),\ R_{dd}(\bar{P}_s, \bar{P}_p))$, which is Pareto-optimal as well.*
3. *If $\eta_{S,1} \leq \eta_{S,2}\bar{P}_p$ and $c_{s,s} \geq c_{s,p}$, then the equilibrium* ex ante *second-best contract which achieves the first-best outcome is $(\lambda_{nn}^C(\cdot; \bar{P}_p), (\bar{P}_p, \bar{P}_s))$, and the equilibrium rate allocation is $(R_{nn}(\bar{P}_p, \bar{P}_s), R_{dd}(\bar{P}_s, \bar{P}_p))$, which is Pareto-optimal.*
4. *There exist channel conditions under which the (first-best) equilibrium contract does not yield Pareto-optimal rates.*

**Proof.**

1. From the proof of Theorem 5, it is clear that the primary user has an incentive to report false channel coefficients because the IR constraint depends on the actual value of $c_{p,p}$. However, in an *ex ante* contract, the IR constraint is $\mathbb{E}[R_a(P_a, P_p) - \lambda(P_p, P_a) - \bar{R}_a] \geq 0$ which doesn't depend on the actual value of $c_{p,p}$ but only on their distributions. So, the primary user has no incentive to manipulate the channel coefficients.

   Now, the secondary user can increase his utility only by decreasing his payment to the primary. From (24), the general form of the contract function is $\mathbb{E}[\log(1 + c_{p,p}\bar{P}_p)] - R_p(\cdot,\cdot)$, where $R_p(\cdot,\cdot)$ is the rate of the primary user and $R_p(\cdot,\cdot) = \min(R_{nd}(\cdot,\cdot), R_{nn}(\cdot,\cdot))$. If the secondary user manipulates the channel measurements to increase the transmission rate of the primary user (and decrease the corresponding payment), the primary user's rate will be higher than the maximum possible decodable rate at the secondary's receiver. So, the SIC scheme will fail, and hence, secondary user's own decoding will fail. Thus, the secondary user would not manipulate the channel measurements to lower the payment. Since the reported channel coefficients are correct, the principal can calculate the parameters $\eta_{S,1}$ and $\eta_{S,2}$.

   The rest of the proof is very similar to that of Theorem 1. So, we keep it short.
2. If $\eta_{S,1} > \eta_{S,2}\bar{P}_s$, then $R_{nd}(\cdot,\cdot) \leq R_{nn}(\cdot,\cdot)$, and hence, $R_p = R_{nd}(\cdot,\cdot)$ and the contract function $\lambda_{nd}^C(\cdot,\cdot)$ satisfies the *ex ante* IR constraint with equality. So, the payoff of the agent (primary) is zero at any operating point. Also the solution of the optimization problem CD-OPT with $\lambda(\cdot,\cdot) = \lambda_{nd}^C(\cdot,\cdot)$ is $P_s = \bar{P}_s, P_p = \bar{P}_p$. So, by the same arguments as above, the principal (secondary) will offer the contract $\lambda_{nd}^C(\cdot; \bar{P}_s)$ and the agent (primary) will pick $P_p = \bar{P}_p$. The corresponding rate allocation maximizes the sum rate and is Pareto-optimal.
3. When $\eta_{S,1} \leq \eta_{S,2}\bar{P}_p$ and $c_{s,s} \geq c_{s,p}$, it can easily be shown that $R_{nn}(\cdot,\cdot) \leq R_{nd}(\cdot,\cdot)$, and hence, $R_p = R_{nn}(\cdot,\cdot)$ and the contract function $\lambda_{nn}^C(\cdot,\cdot)$ satisfies the *ex ante* IR constraint with equality. So, the payoff of the agent (primary) is zero at any operating point. The solution of the optimization problem CD-OPT with $\lambda(\cdot,\cdot) = \lambda_{nn}^C(\cdot,\cdot)$ gives the optimal power allocation as $P_s = \bar{P}_s, P_p = \bar{P}_p$. So, the principal (secondary user) fixes his power at $\bar{P}_s$ and offers the contract $\lambda_{nn}^C(\cdot; \bar{P}_s)$. Now, assuming that the primary user prefers to get a higher rate with zero payoff to a lower rate with zero payoff, he will pick $P_p = \bar{P}_p$. It is easy to check that the corresponding rate allocation maximizes the sum rate $R_p + R_s$, and is Pareto-optimal as well.
4. When $\eta_{S,1} < \eta_{S,2}\bar{P}_s, c_{s,s} < c_{s,p}$ and $c_{ss}(1 + c_{sp}\bar{P}_s + c_{pp}\bar{P}_p + c_{ps}\bar{P}_p + c_{sp}^2\bar{P}_s^2) < c_{sp}c_{ss}\bar{P}_p$, it can be checked that the optimal solution of the CD-OPT-PP optimization problem will occur at $P_p = \bar{P}_p$ and $P_s < \bar{P}_s$, in which case the corresponding rate pair will be in the interior of the achievable rate region, and cannot be Pareto-optimal. □

We summarize the results of Theorem 6 in Fig. 6.

**Corollary 2.** *If the secondary user is dominant, it is impossible to design an* ex post *second-best contract, which achieves the first-best outcome. An* ex ante *contract which achieves the first-best outcome can be designed, which yields Pareto-optimal outcomes at equilibrium under* most *channel conditions.*

| Channel Conditions | Contract Function | Powers | Operating Point | Optimality |
|---|---|---|---|---|
| $\eta_{S,1} > \eta_{S,2}\bar{P}_s$ | $\lambda_{nn}^C$ | $(\bar{P}_s, \bar{P}_p)$ | A in Fig. 1 | Socially/ Pareto |
| $\eta_{S,1} \leq \eta_{S,2}\bar{P}_s$, $c_{s,s} \geq c_{s,p}$ | $\lambda_{nn}^C$ | $(\bar{P}_s, \bar{P}_p)$ | A in Fig. 1 | Socially/ Pareto |
| $\eta_{S,1} \leq \eta_{S,2}\bar{P}_s$, $c_{s,s} < c_{s,p}$ | $\lambda_{nn}^C$ | $(P_s, \bar{P}_p)$ $P_s < \bar{P}_s$ | Interior in Fig. 1 | non-Pareto |

Fig. 6. Summary of the results for Case C under hidden information.

*Lesson #5.* The main lesson learned from the above analysis is that when information about channel coefficients is hidden, it is better for the primary user to be the dominant user. We then have *ex post* second-best equilibrium contracts with first-best, Pareto-optimal outcomes.

## 6 Numerical Examples

Fig. 7 shows the results of complete information Case A as proved in Theorem 1. When $c_{ps} < 0.5$, it can be calculated that $\eta_{P,1} > \eta_{P,2}\bar{P}_p$ and by Theorem 1, $R_s = R_{nd}(\bar{P}_s, \bar{P}_p)$ (in (9)). Since $R_{nd}$ does not depend on $c_{ps}$, secondary user's rate $R_s$ remains constant till $c_{ps} < 0.5$. When $c_{ps} \geq 0.5$ it can be checked that $\eta_{P,1} \leq \eta_{P,2}\bar{P}_p$ and by Theorem 1, $R_s = R_{nn}(\bar{P}_s, \bar{P}_p))$ (in (9)). So, the secondary user's rate decreases with $c_{p,s}$ from this point on. Primary user's rate $\log(1 + c_{pp}\bar{P}_p)$ does not depend on $c_{ps}$ and, hence, remains constant.

Fig. 8 shows the results of hidden information case C proved in Theorem 6. We assume that $c_{pp}$ is a Rayleigh random variable with mean 0.6. The rates and contract function are given by Theorem 6. Fig. 8 shows one realization where channel parameters are $c_{ss} = 1.0, c_{pp} = 0.4, c_{ps} = 0.8$ and vary $c_{sp}$ from 0 to 1. We also fix $\bar{P}_p = 1.0, \bar{P}_s = 1.0$. It can be checked that $\eta_{S,1} \leq \eta_{S,2}\bar{P}_s$ for $c_{sp} > 0$.

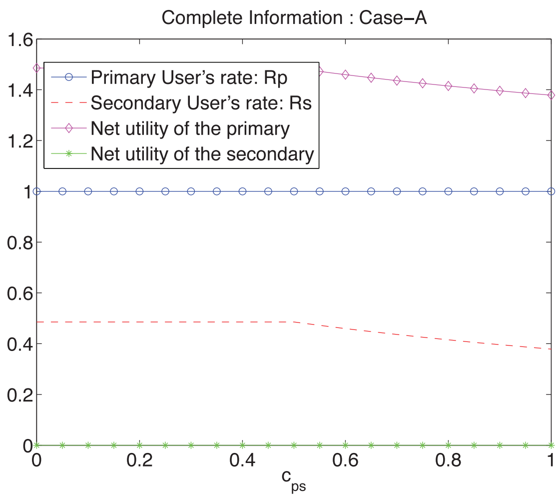


Fig. 7. Complete information Case A: $c_{pp} = 1.0$, $c_{ss} = 0.6$, $c_{sp} = 0.8$, $\bar{P}_p = 1.0$, and $\bar{P}_s = 1.0$.

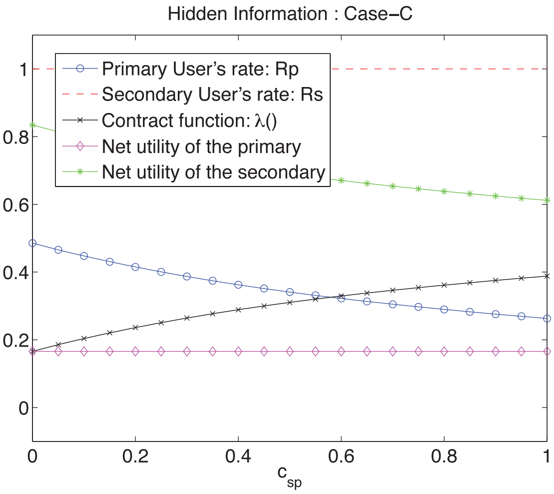


Fig. 8. Hidden information Case C, $c_{ss} = 1.0$, $c_{pp} = 0.4$, $c_{ps} = 0.8$, $\bar{P}_p = 1.0$, and $\bar{P}_s = 1.0$.

## 7 Conclusion and Future Works

This paper presents a new approach to "incentivized" spectrum sharing for licensed bands in cognitive radio systems using "cooperative (or multiuser) communication" schemes. We first considered a full information setting, in which we specified the equilibrium contracts that happen to be Pareto-optimal and, in fact, sum-rate maximizing under most channel conditions. However, there are channel conditions under which no first-best Pareto-optimal contracts exists. This can be seen as an *impossibility theorem*, and hence a negative result. Nevertheless, we are able to characterize sufficient conditions on the channels under which the first-best contract is indeed Pareto-optimal at equilibrium. While there exist incentives for deviating from agreed contracts, we showed that using simple incentive schemes, this *moral hazard* problem can be easily avoided. When we allow for time-sharing as part of the contract, we have concluded that at equilibrium there actually will not be any time-sharing of roles. We then showed that while *hidden information* can lead to nonexistence of an equilibrium contract, this can be avoided if the primary user is assigned a dominant role, which yields Pareto-optimal equilibrium outcomes.

Our setting can be extended in many ways to more realistic scenarios. We have given a complete analysis of the case with a single secondary user. But the results can be extended to settings with multiple secondary users (contracts with time-sharing do present a challenge with multiple secondary users) though the first-best equilibrium contract formats increase in complexity with more users. We have also illustrated a case with two secondary users. We considered utilities of users to be their rates. Contract design with a general concave utility function is a very interesting but difficult problem. In future work, however, we will consider contract design for concave utility function that is (approximately) Pareto-optimal. We have also assumed a static, flat fading channel. In general, channels can have frequency selective fading, and can vary over time. In that setting, a repeated game setting can be considered, and would require more sophisticated, dynamic contracts. Dynamic mechanism design and contract theory is an area of very active research, and this will be considered in the future.

We present the proposed contracts mechanisms in solving incentive issues in spectrum sharing among cognitive radios as "proofs of concept." Implementation in real systems would require attention to many practical communication-theoretic aspects because we assume perfect channel estimates and infinite-block length coding. Moreover, there is also an issue of tracking and managing payments. While important, we regard these as outside the scope of this paper.

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