Compressive Sensed Signal Reconstruction Design
For Sub-Nyquist Rate Sampling

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Texas A&M University, https://cesg.tamu.edu/, 2014

Overview

This work presents a digital signal processing (DSP) implementation to reconstruct compressive data sensed slower than Nyquist sampling rate. The proposed digital design recovery data which is randomly mixed and sampled by specified analog-to-digital converter (ADC).

A modification on the iterative hard thresholding (IHT) reconstruction algorithm is made to adapt unknown and varying sparsity of the signal. The modification is studied empirically and implemented in a hardware-friendly fashion.

The reconstruction ability of fixed-point model is analyzed. The register-transistor level (RTL) of the DSP design is implemented and verified with the fixed-point model in Matlab. The RTL is then synthesized by Synopsys Design Compiler with TSMC 45nm technology.

The post-synthesis implementation consumes 165 mW and is able to reconstruct data with information sparsity of 4%, at equivalent sampling rate of 1 giga-sample-per-second (GSPS).

Compressive Sensing Model

Given a length N vector of time-discrete signal: \( \mathbf{x} \in \mathbb{R}^N \)

\[ \mathbf{y} = \sum_{i=1}^{N} \mathbf{a}_i \psi_i^T \mathbf{x} \]

Where \( \psi_i \) is the \( i \)th column of known orthogonal basis. \( K \) is the number of non-zero element in \( \mathbf{a} \).

The ratio is defined as \( K \times N \).

The Problem Specification: According to comprehensive sensing (CS) theory we should be able to recover \( \mathbf{y} \) with \( M \) random projection samples of \( \mathbf{y}^T \).

The reconstruction algorithm (Iterative hard thresholding (IHT)) [1], [2]

IHT is a simple and efficient algorithm that iteratively approaches the solution of specified problem as:

\[ \mathbf{a}^{(i+1)} = \mathbf{H}_K(\mathbf{a}^{(i)} + \mathbf{B}^T(\mathbf{y} - \mathbf{B} \mathbf{a}^{(i)})) \]

\( \mathbf{H}_K(\mathbf{z}) \) is a non-linear operation which set all elements except the \( K \) largest ones of vector \( \mathbf{z} \) to zero.

\[ \mathbf{B} = \mathbf{\psi}, \mathbf{B}^T = \mathbf{\psi}^T \mathbf{\phi}^T \]

If \( \mathbf{\phi} \) is FFT then \( \mathbf{\phi}^T \) is FFT.

Modification on IHT

In our modified-IHT (MIHT) algorithm, the sparse signal can be reconstructed without knowing the sparsity. IHT iteration function is modified as follows:

\[ \mathbf{a}^{(i+1)} = \mathbf{H}_K(\mathbf{a}^{(i)} + \mathbf{B}^T(\mathbf{y} - \mathbf{B} \mathbf{a}^{(i)})) \]

Where \( \mathbf{H}_K(\mathbf{z}) \) set all elements whose Norm-2 values are not less than \( \tau_0 \times \max \| \mathbf{z} \|_2 \) to zero.

\[ \mathbf{\mu} = \mathbf{H}_K(\mathbf{\mu}), \mathbf{\mu}_j = \begin{cases} 0, & \| \mathbf{z}_j \|_2 < \tau_0 \times \max \| \mathbf{z} \|_2 \\ \mathbf{z}_j, & \text{else} \end{cases} \]

Where \( j \) is the index of element, \( \mathbf{\mu} \) is the fraction number between 0 and 1. It is initially \( 0 \) and does decrease monotonously with the number of iterations.

Estimation error:

\[ e = \| \mathbf{\mu} - \mathbf{\mu} \|_2 \]

To acquire an efficient convergence, our threshold function is selected based on an insight drawn from the results reported in [2].

Threshold function:

\[ \tau_0 = \sqrt{2} \mathbf{a}^{(i)} \]

We define:

\[ \text{SRNR} = \frac{\mathbf{\mu}^T \mathbf{\mu}}{2 | \mathbf{\mu}^T (\mathbf{I} - \mathbf{\mu}) |} \]

Results

Comparison between Modified IHT with \( t_0 = 0.82 \) and original IHT at 25% Nyquist Sampling Rate.

Reconstruction performance among Norm-1, Norm-2, floating point model and fixed-point model.

Concluding remarks

We implemented the fixed-point model in RTL coding, which can reconstruct compressive sensed 4%-occupancy sparse signal of 7 ENOBs by SNRR of 30 dB.

Our design consumes 165 mW at 88 MHz. It takes 22 cycles, latency of 0.25µs to reconstruct one set of samples with consuming 165 mW as equivalent Nyquist Sampling rate of 1 GSPS.

Future works

Current effort is being paid on several questions such as higher reconstruction fidelity, higher recoverable occupancy and lower-power exploration on circuits implementation to make proposed system applicable to commercial devices.

References


