21st Century Space Frame System

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In Memory of Marc Pelletier 1959-2017

Abstract

Marc Pelletier, who is one of the co-inventors of Zometool, died in November 6, 2017. This article is a tribute to his contributions in geometry. I would like to thank Paul Hildebrandt, my daughter Crystal Rose Buhler, Steve Baer, Clark Richert, Bob Nickerson, Jay Bonner, Michael Stranahan, Sir Roger Penrose, Chris Maslanka, Ergun Akleman and Carla Hilger for their help, interviews, encouragement, and contributions to this paper.

Introduction

Marc Pelletier was a genius proponent in understanding and visualizing higher dimensional geometry [Coxeter1963, 1967]. Marc Pelletier had dialogues and became close friends with exceptional geometers such as Steve Baer, Clark Richert, H.S.M. (Donald) Coxeter [Schattschneider 2005], Roger Penrose, John Conway and Jay Bonner. His proclivity to visualize structures from the higher dimensions lead invention of Zometool.

Mathematical physicist Sir Roger Penrose in January 2018 said that "The loss of Marc Pelletier will be felt deeply by those who see the beauty in geometry and its realization in the amazing material structures that he was able to produce. His creative abilities in this regard were enormous, and a wonder to appreciate. His gentle nature belied his subtle skills in being able to steal ideas from Plato's world of perfect abstract forms and present them to us in actual physical forms that we can touch, feel, play with, and enjoy."

In Loving Memory of Marc Pelletier, co-inventor of Zometool August 4, 1959-November 6, 2017

Marc Pelletier's gift was to give everyone, children and adults, the ability to become the creator, to discover new things at their own fingertips using the products that he designed. The only requirement is to touch it and explore. His invention Zometool crosses economic, social, language, cultural and political barriers.

Some of my favorite quotes from Marc Pelletier are:
"Changing the world, one kid at a time."
"Build genius."
"The structure of number is the structure of space"
"We stand on the shoulders of giants."
"The adventure is just beginning."

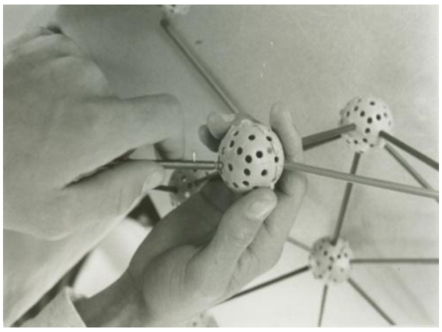
The sheer volume of work and materials that Marc Pelletier left behind presents a challenge both for me and for Paul Hildebrandt, the other co-inventor of Zometool. For instance, Marc managed to meet Paul Donchian's brother and saved Donchian's books, co-inventor of quasi-symmetrical patterns [Loeb 1993 and builder of projections of 4- and higher dimensional shapes

[Coxeter1963, Hildebrandt 2006]. Marc also managed to archive and save many other polyhedron models, books and math relics. We will continue to organize and classify the sheer volume of materials he archived and share with others.

History of Zometool

Marc Pelletier is probably best known for Zometool, which is a system to construct 3-Dimensional shadows of polytopes up to 31-dimensions [Hildebrandt 2005]. For creation of Zometool, he collaborated with Paul Hildebrandt [Hildebrandt 2006, 2007, 2012]. The innovation of Marc and Paul was a shape-coded version of Steve Baer's 31-Zome System. It took more than ten years to accomplish the *Zomeball* mold. All but one machinist, Bob Nickerson of 4-Axis Machining in Denver, Colorado, deemed the injection mold possible. The mold produces hollow balls in one piece and the original design is likely the most complicated injection mold ever built. This mold is said to be more complicated than a car engine mold.

Zoomtool stemmed from Marc’s particular interest in polytopes and hyperspace [Coxeter1973]. Marc Pelletier’s interest with higher dimensional polytopes started with an unlikely event of his middle school burning down. This unlikely event led him to the high school library in New Hampshire where he found a copy of Cundy and Rollett's, Mathematical Models [Cundy 1961], and Coxeter's, Regular Polytopes [Coxeter 1973] to pass the time. In library, Marc also read the Zome Primer by Steve Baer [Baer 1970] and realized that Baer's 31- Zome system could model 3-Dimensional shadows from up to 31-dimensions, including the hyper-cube, the 120- (4-D dodecahedron) and the 600-cell (4-D icosahedron), which were illustrated by the model builder Paul Donchian in Regular Polytopes [Coxeter 1973].



Steve Baer's Zometoy *Shape coding idea visualized* *Zomeball realized*
Original Construction of Zomeballs

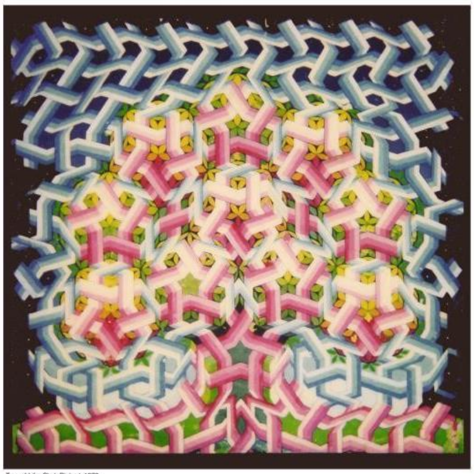
Marc built by hand a 4-D dodecahedron. The paper model, which Marc constructed in high school, took him a short amount of time compared to the first 120-cell model, which Donchian built, took over two years to complete. His father Myles Pelletier, seeing his son's interest, took him to meet Buckminster Fuller, who is inventor of geodesic dome [Fuller 1965, 1969, 1971]. Between Baer's Zome Primer, Coxeter and Fuller, he would look at mathematics and architecture in a different way. It became a lifelong passion.

After finishing high school with a GED, Pelletier began a lifelong adventure, seeking out and meeting his math heroes/mentors. First stop, Steve Baer in New Mexico - a long journey for a New Hampshire boy. Much to Marc's dismay, Baer called the 31-Zome system, "broken glass on the highway". The Zometoy was the construction system, which Baer developed to illustrate the 31-Zome system in the Zome Primer. The kit had wooden struts and plastic balls. Pelletier begged Baer for a Zometoy kit. Resisting Marc was futile. Baer gave Marc a kit. Marc noticed this was not user friendly and a novice could easily put a strut in the wrong hole (all the holes were the same).

Steve Baer introduced Marc Pelletier to Clark Richert. Both Baer and Richert were involved in Drop City, Colorado. Drop City, much like Buckminster Fuller's geodesic domes, utilized architecture, which was not cubic in nature. The first domes in 1965 were geodesic. Baer influenced Drop City by building the first Zome building. The word Zome came from combining the word dome with zonohedron thus creating the word Zome. All zonohedron domes are 3-dimensional shadows from the higher dimensions, which Marc was keen to remind anyone who would listen.

Figure 2. Clark Richert: Zomeworks bus, 1970. Photos September 1970.

Using Steve's Zometoy kit, Richert noticed something new by projecting a tricontahedron. The tricontahedron is a polyhedron with 30 rhombic faces. Using the sun to cast (project) a shadow, Clark noticed the obtuse and acute 5-fold rhombi. He knew that these two rhombi were projections from 6-D space. Clark’s own rule was that edges match each other. He realized that these polygons tile the plane indefinitely and non-repeatedly. This discovery predates Penrose’s publication of these tiles [Gardner 1997]. Marc Pelletier pointed out that Penrose tiles (which cause quasi-periodic tilings via a matching rule) are projected from a 6-dimensional hypercubic lattice and are necessarily non-periodic. Thus, Richert discovered and understood that these 5-fold rhombi tiled quasi-periodically, independent of Penrose’s discovery. Clark Richert painted the Zomeworks (Steve Baer’s company) bus with these 5-fold rhombi in 1970.



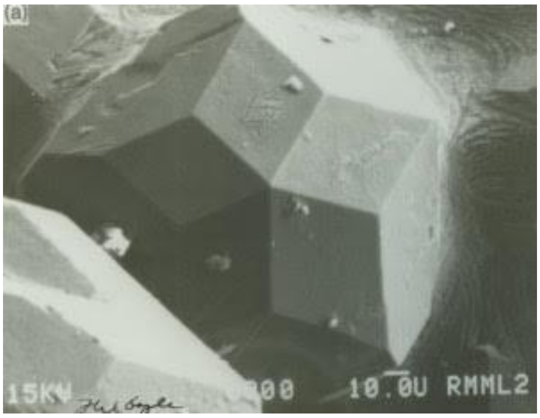
Tree of Life, Clark Richert, 1970

Clark Richert: Tree of Life, 1970

Clark Richert introduced Marc Pelletier and Paul Hildebrandt in 1980, saying, "You two should get along real good", they clearly had a lot in common. Together, they designed a new version of Steve Baer's Zometoy. Their original company was dubbed "Bio Crystal Inc", it is now known as *Zometool*. The shape coding is an elegant solution using the lesser rhombicosadodecahedron as the node. This makes Steve Baer’s original Zometoy much more user friendly. The new ball (node) design has 30 rhombi (in this case, golden rectangles associated with the rhombic tricontahedron), 20 equilateral triangles (associated with the icosahedron) and 12 pentagonal holes (associated with the dodecahedron) for the struts. The struts are not only shape coded, but color coded, as well: blue struts fit in the rectangular hole, yellow struts into the triangular hole, and the red struts into the pentagonal hole. It was also their intention to create an architectural system. This paper formally introduces the space frame system, which was dreamed up before the first Zomeball came out of the injection mold.

Steve Baer designed and constructed homes including his own using quasi-crystal geometry years before Daniel Shechtman discovered the existence of quasi-crystals [Shechtman 1951, Cahn 1986]. Shechtman later received the Nobel Prize for quasi-crystals [Clery 2011].

Quasicrystal architecture by Steve Baer presiding the discovery of the quasi-crystal



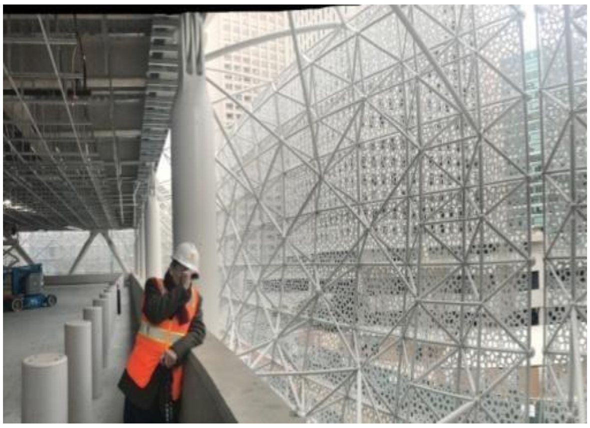
Quasi-Crystal (Photo by Frank Gayle)

Shechtman teaching with a Zometoy model by Pelletier

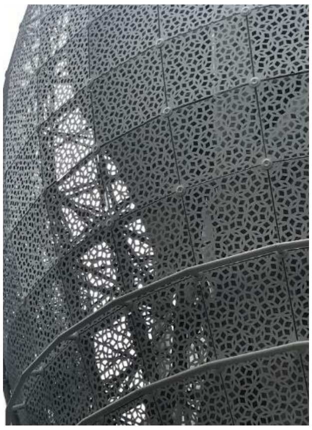
When Pelletier heard of Shechtman’s discovery, Marc visited Shechtman and a number of scientists at the National Bureau of Standards in 1984. Marc modeled the structure of quasi-crystals for the group using Baer's Zometoy kit. The model was a rhombic tricontahedron. Hence, the picture that made it worldwide has Pelletier's model sitting on the table.

Zometool as a Space Frame System

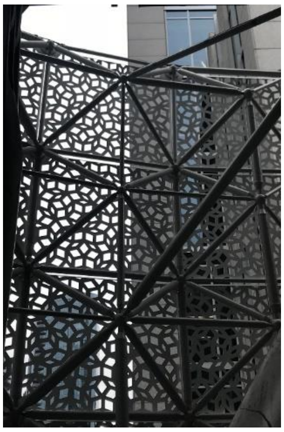
Despite the advances, the current space frame systems still reflect the cubic system of the 20th and previous centuries, paradigm. The Transbay Transit Center in San Francisco utilizes a "Penrose tiling" four city blocks long, this pattern is still supported by a space frame cubic lattice. Instead if Marc Pelletier's space frame system had been used, the 5-fold pattern from the tiling would have been reflected/mirrored in the 15-Dimensional pattern in the space frame supports. Combining the 2-D tiling with the new space frame system would have harmonized the ornament with the support structure. It would have also been able to carry more of a load then the existing spaceframe structure.



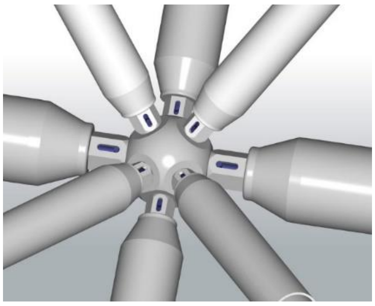
Penrose at Transbay Transit Center, Jan. 2018



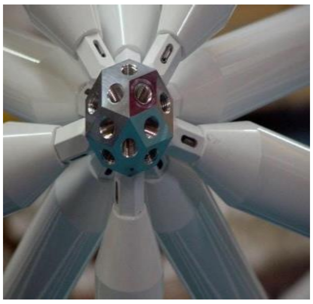
Transbay Transit Center: Street view



Interior view with traditional spaceframe



Traditional Mero node and struts



Tricontahedron node and struts

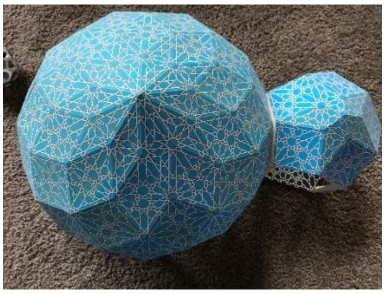
Marc Pelletier liked to think "outside the box". He has later developed an advanced space frame system has a tricontahedron node with 30 threaded rhombic faces in 15 directions/planes. The research and development included testing the strength of parts, materials, glues, CNC machining and strut designs.

A strut design was chosen that would not unscrew when the opposing side was tightened. The screws are retractable and can be engaged independently. This design utilized a Mero type strut design. The strength of the struts were tested at the University of Colorado's Mechanical Engineering Lab in Boulder, Colorado. It took over 10,000 kg of direct force before failure. The 3M Scotch Weld epoxy used in the strut construction was expected to fail, first. Surprisingly, it was the retractable 12 mm stainless steel screw, which sheared.

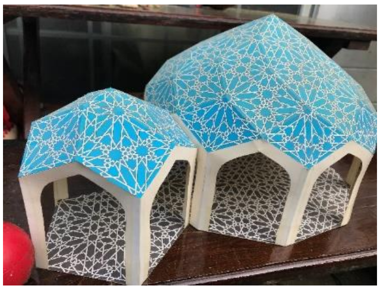
Each tricontahedral node is CNC 5-Axis machined in two complete operations. Each face of the node has a 12 x 1.5mm threaded hole that connects with the stainless-steel screws, which engage with the struts. The struts currently have four lengths, which are kin to the Zometool system. Each screw is also machined. The strut cones are turned on a lathe. The results were stunning. The space frame system was unveiled at the Bridges Conference, Towson, Maryland, USA in 2001. Pelletier created a sculpture, which Reza Sarhangi , who is founder and organizer of Bridges conference, called, "Magnus Magnified", referring genius model builder Magnus Wenninger [Wenninger 1968, 1974, 1978, 1979, 1990, 2003].

Islamic Patterns

Marc Pelletier's last works are a huge number of 2 and 3-level Islamic patterns which repeat/iterate in a fractal like manner on each level for zonohedral domes of various symmetry groups, echoing the Moroccan style. Floors reflect the ceiling though of another symmetry group. The Zomes also cluster, some with courtyards. Fountains using his technique were also designed.



Islamic Geometric Clustering Zome



Floors and roof, different symmetry groups

Jay Bonner, Micheal Baron, and Marc Pelletier.

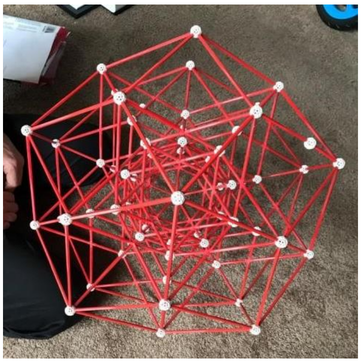
Islamic Zome Clusters,2002, Crestone, Colorado

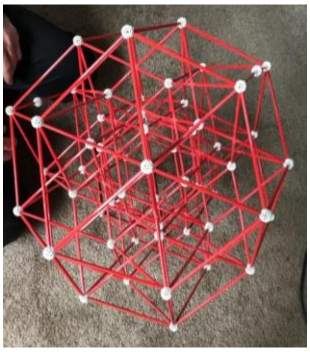
In 1999, Marc Pelletier and I formed a team to promote and create hands-on products to educate students and the public about the wonders of mathematics. It was the path that we both were on when we met. Marc participated in our collaboration up until his last day. I introduced Marc Pelletier and Jay Bonner in 2000, who is an expert on Islamic patterns [Bonner 2003, 2017]. In 1970, Bonner sat under the Zome at the Llama Foundation. Bonner had also worked as part of an international team on the domes in Medina. He decorated the Platonic and Archimedean solids with edge matching ornament. Marc Pelletier commissioned Jay Bonner in 2001 to decorate the 36 and 72-degree rhombi, which were natural to the 5-fold system. Bonner had already ornamented an assortment of these rhombi with a variety of edge matching Islamic geometric and floral patterns in the1970's, unaware of Penrose's discovery. Marc introduced Jay Bonner to the idea of placing these patterns onto zonohedron domes. The 5-fold rhombi only needed a slight skew to turn them into Zomes. As a student of Lloyd Kahn and Jay Baldwin, who are pioneers of geodesic dome and shelter movements [Kahn 1970, 1973], Jay Bonner helped construct geodesic domes [Bonner 2012, Pelletier 2012].

Latest Works

Marc also commissioned Scott Vorthman to make cross connectors for the Zometool which Marc called, "cross-bobs" (in honor of the machinist Bob Nickerson that made the Zomeball mold, possible). The cross-bobs connect struts without the Zome ball node, allowing the struts to intersect and interconnect. These parts were 3DM printed at Shapeways. Using these cross connecters with the Zometool struts allows for many more hyperspace models to be built using Zometool. The blue cross-bobs are 2-axis, 4 holes. Yellow cross-bobs are straight, 2 holes and cross, 4 holes. The red connectors are 2-way, 4 holes and 3-axis, 6 holes, Pelletier's last Zome model used both of the red cross-bobs to make a 6-D cube. Zomes were never far from his mind.

Red Cross-Bobs





Marc Pelletier's last Zometool model, 6-D cube using cross-bobs, 2017. The outer shell is a tricontahedron. the recurring theme

The 21st Century spaceframe system, which Paul Hildebrandt calls, "Big Zome," is like a super-size powder coated construction kit. The 15-plane space frame system is a refinement of the current space frame system. Today's spaceframe system with its simple cubic lattice is a structural weakling. Since the "Big Zome" triangulates through its body and face-centered diagonals, it becomes a powerhouse. It uses the blue lines from the Zometool system (the 30 faces of the rhombic tricontahedron). These 15 directions/planes can build 3-dimensional shadows from 15-dimensional space. Having a huge blue line construction kit thrilled Marc as many beautiful shapes can be projected from the 15-dimensional hypercubic lattice: three of the five platonic solids (the cube, dodecahedron, and icosahedron), semi-regular polyhedral, zonohedra, ruled surfaces, Islamic patterns, as well as Penrose tilings from many different dimensions. It is possible to project many 15-D shapes on a huge scale using the "Big Zome" system using all four strut lengths.

Spaceframe Zomeball *Inner shell Zomeball, Outer Icosadodeca* *Ruled Surface*

One of Marc's abilities was to simplify and to make tangible elegant solutions to improve and bring these systems into the 21st Century. He was a true genius who was ahead of his time. We all miss him dearly.

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