Developable Sculptural Forms of Ilhan Koman

Tevfik Akgün
Faculty of Art and Design
Design Communication Department
Yildiz Technical University
Istanbul, Turkey
akgunbt@yildiz.edu.tr

Ahmet Koman
Molecular Biology and Genetics Department
Boğaziçi University
Istanbul, Turkey
akoman@boun.edu.tr

Ergun Akleman
Visualization Sciences Program,
Department of Architecture
Texas A&M University
College Station, Texas, USA
ergun@viz.tamu.edu

Abstract

Ilhan Koman was one of the innovative sculptors of the 20th century [9, 10]. He frequently used mathematical concepts in creating his sculptures and discovered a wide variety of sculptural forms that can be of interest for the art+math community. In this paper, we focus on developable sculptural forms he invented approximately 25 years ago, during a period that covers the late 1970's and early 1980's.

1 Introduction

$\pi+\pi+\pi+\pi+\pi+$

Hyperform

Figure 1: The photograph $\pi+\pi+\pi+\pi+\pi+$ series is from a Koman exhibition at Gronningen, Copenhagen (Denmark) in 1986. Hyperform is realized in stainless steel for the steel company Atlas Copco, (Entrance hall, Atlas Copco, Sweden, 1973) (Photos by Koman family)

In this paper, we want to present developable forms invented by sculptor Ilhan Koman during the 1970's. The first is the $\pi+\pi+\pi+\pi+\pi+$ a series of forms derived by the increase of the surface of a circle without changing its radius. A second one is called rolling lady and created using four identical cones. The third is the "Hyperform" that he produced by self-joining a single sheet of metal. Extension of the length of the rectangular sheet in a defined manner led to multiple derivatives. Figure 1 shows photographs of $\pi+\pi+\pi+\pi+\pi+$ and Hyperform developable sculptures.

2 Developable Surfaces

Developable surfaces are defined as the surfaces on which the Gaussian curvature is 0 everywhere [20]. The developable surfaces are useful since they can be made out of sheet metal or paper by rolling a flat sheet of material without stretching it [14]. Most large-scale objects such as airplanes or ships are constructed using un-stretched sheet metals, since sheet metals are easy to model and they have good stability and vibration properties. Moreover, sheet metals provide good fluid dynamic properties. In ship or airplane design, the problems usually stem from engineering concerns and in engineering design there has been a strong interest in developable surfaces. For instance, modeling packages such as Rhino provides developable surface analysis [14, 15].

Although, once invented, it is easy to physically construct developable surfaces using sheet metal or paper, it is not that easy to provide computational models to represent developable surfaces. Sun and Fiume developed a technique for constructing developable surfaces [19], but, their method is useful only to represent ribbons and it is hard to use to represent general developable surfaces. Haeberli has recently introduced a method to represent a shape with piecewise developable surfaces and developed a Lamina Design Software [7]. The current results seem to be limited but the Haeberli's approach Lamina has a great potential for developable surface design. Mitani and Suzuki introduced a method to approximate any given shape using developable surfaces to create paper models [11]. Because of the approximate nature of their models, there exists gaps between individual pieces and therefore, their method is not suitable for engineering application.

Sheet metal is not only excellent for stability, fluid dynamics and vibration, but also one can construct aesthetic buildings and sculptures using sheet metal or paper. Developable surfaces are frequently used by contemporary architects, allowing them to design new forms. However, the design and construction of large-scale shapes with developable surfaces requires extensive architectural and civil engineering expertise. Only a few architectural firms such as Gehry Associates can take advantage of the current graphics and modeling technology to construct such revolutionary new forms [12]. Some architectural structures can be easier to construct with developable surfaces. For instance, Fishback and Tuazon introduced Randome, a dome like structure that is constructed from developable surfaces [6].

Developable surfaces are particularly interesting for sculptural design. It is possible to find new forms by physically constructing developable surfaces. Recently, very interesting developable sculptures, called D-forms, were invented by the London designer Tony Wills and introduced by Sharp, Pottman and Wallner [16, 13]. D-forms are created by joining the edges of a pair of sheet metal or paper with the same perimeter [16, 13]. Pottman and Wallner introduced two open questions involving D-forms [13, 5]. Sharp introduced anti-D-forms that are created by joining the holes [17]. Ron Evans invented another related developable form called Plexagons [4]. Paul Bourke has recently constructed computer generated both D-forms and plexons [2, 4] using Evolver developed by Ken Brakke [1].

3 Ilhan Koman Biography

Ilhan Koman was born in 1921, Edirne, Turkey, studied at the Art Academy in Istanbul, opened his first workshop and exhibition in Paris, 1948, moved to Sweden in 1958, where he taught at the Konstfack School of Applied Art in Stockholm until his death in 1986. Figure 2 shows two photographs of Ilhan Koman. At right Koman is photographed in front of his and architect Chet Kanra's sculpture, called *"From Leonardo To..."* in Stockholm. His works cover a wide spectrum of styles and materials, including 12 public monuments in Sweden and 4 in Turkey. He is represented in several museums including Moderna Museet, Stockholm, Museum of Modern Art, New York, Musée d'Art Moderne de la Ville de Paris. During the last twenty years of his life, he worked on inventing diverging geometrical forms which he developed as prototypes to be realized in large-scale projects. Among the geometrical shapes he developed, tetraflex was a flexible polyhedron which he registered at the Swedish Patent Office in 1971. At the time he had created and proposed these

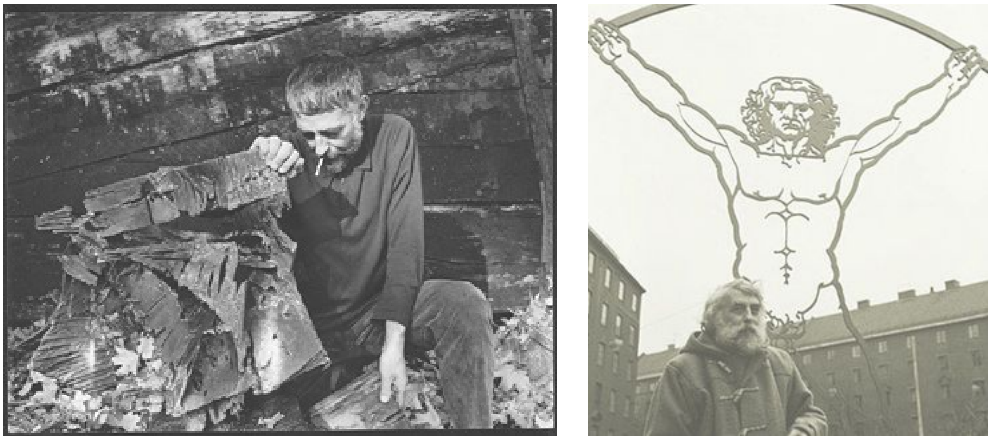
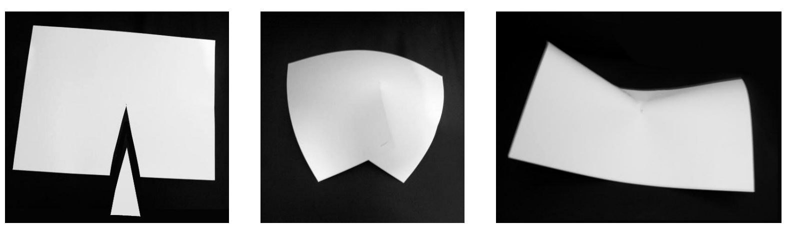


Figure 2: Two photographs of Ilhan Koman (left photo by Christer Strmholm, 1970’s; right photo by Tayfun Tunçelli, 1980’s).

structures as modules for architecture, constructions in space and aviation fuel tanks [9, 10]

4 Vertex Angle Deflections and $\pi+\pi+\pi+\pi+\pi+$ Form

Ilhan Koman’s developable sculptures in $\pi+\pi+\pi+\pi+\pi+$ form [8, 10] vividly illustrate vertex angle deflections which are less known but very useful mathematical devices to understand local behavior of surfaces [3]. Vertex deflection is a measure to show how much a given region is deflected from plane. In practice, local behavior can be changed by vertex deflections with nip and tuck. For instance, By simply take out some angle (nipping or pinching) we can create a convex or concave region with paper (See Figure 3). To create saddle regions we can add angles to a planar paper surface by tucking as shown in Figure 3.



Initial Paper **Maxima** **Saddle**

Figure 3: Creation of maxima and saddle with subtracting and adding angles to a flat surface.

To construct $\pi+\pi+\pi+\pi+\pi+$ sculptures, Koman started from a circular sheet of plastic or metal and added angles (tucking) by constructing a series of wild saddle structures by forcing the angle deflection in a given vertex much smaller than -2π [8, 10]. As shown in Figure 4, when the angle decreases lower than -2π, it is possible to create a wide variety of saddle shapes. We observe that Koman’s developable structures can give very useful visual intuition about the local behavior at a vertex for piecewise linear meshes.

Vertex angle deflections can formally defined around vertices of surfaces that are piecewise linear or piecewise developable. A mesh M is called *piecewise linear* if every edge of M is a straight line segment, and every face of M is planar. Piecewise linear meshes are useful since the surface of each face is well-defined [**?**]. Most common piecewise linear meshes are triangular meshes. However, piecewise linear meshes are a significant generalization of triangular meshes, which not only allow non-triangular faces, but also allow faces to have different face valence.

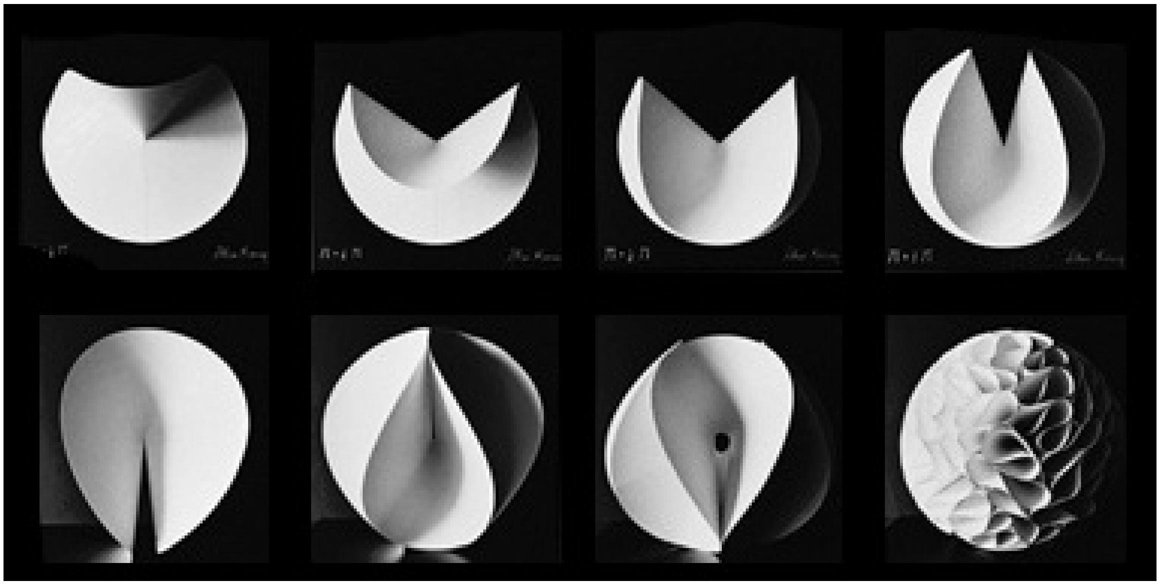


Figure 4: Ilhan Koman's Developable Sculptures. These were constructed with sheet metal in the 1980's. In these sculptures, the connections are almost invisible (photos by Tayfun Tunçelli).

Vertex angle deflections are defined based on the corner angles around a vertex (μ_i) as

$$\bar{\theta}(\mu_i) = 2\pi - \sum_{j=1}^{m_i} \theta_j$$

where θ_j is the internal angle of the j-th corner of vertex μ_i and m_i is the valence of the vertex μ_i. Our recent result show that sum of vertex angle deflections

$$\sum_{i=1}^{v} \bar{\theta}(\mu_i) = 2\pi(2-2g)$$

where g is the genus (number of holes and handles) and $2-2g$ is the Euler Characteristic of the surface. This result is in sync with Gauss-Bonnet theorem [21] which states that the integral of the Gaussian curvature over a closed smooth surface is equal to 2π times the Euler characteristic the surface.

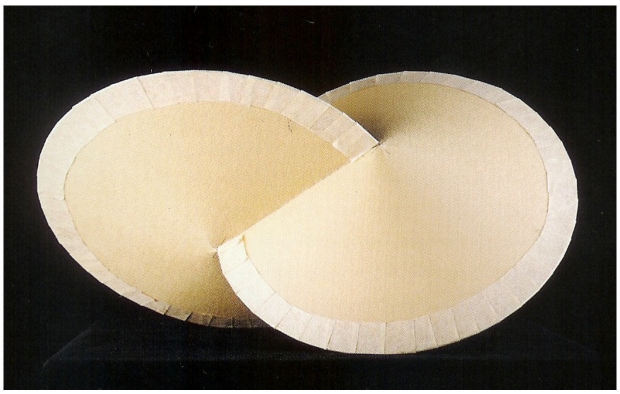
Vertex angle deflection can provide a good intuition about surface curvature. It is interesting to point out that the value of $\bar{\theta}(\mu_i)$ is somewhat related to curvature and, in fact, Calladine used vertex angle deflections to estimate curvature in triangular meshes [3].

- $\bar{\theta}(\mu_i) > 0$: then vertex μ_i is either convex or concave.
- $\bar{\theta}(\mu_i) = 0$: then vertex μ_i is planar.
- $\bar{\theta}(\mu_i) < 0$: then vertex μ_i is a saddle point.

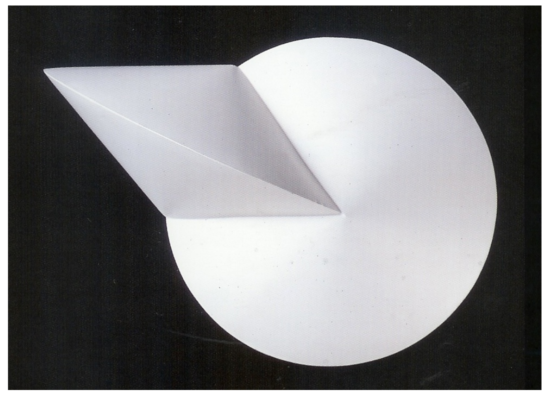
Ilhan Koman's sculptures are easy to make and they can be great tool to teach the effect of vertex angle deflections. Koman was planning to create an extremely large version of one of his wild $\pi+\pi+\pi+\pi+\pi+$ with a platform in the center such that people will go the center of the resulting spherical shape will only see space curve. Unfortunately, he could not complete this dream. It is interesting to note that, Sharp created a simple saddle shape from $\pi+\pi+\pi+\pi+\pi+$ series during his Bridges talk (A saddle with vertex deflection is around $-\pi$) [18]. It is highly possible that $\pi+\pi+\pi+\pi+\pi+$ form is passed from people to people and became an anonymous discovery. It is also possible that it exists several co-discoveries.

5 Rolling Lady

Koman discovered *Rolling Lady* form in early 1980's. This form consists of four cones that are pasted together to create two wheels. As the name suggest this sculpture can roll with a beautiful motion. These sculptures can be created with four identical cut cones as shown in Figure 5. During his Bridges talk, Sharp also mentioned a similar structure that can be constructed from two perpendicular circles [18]. It is also interesting in this case to find out whether or not there is a co-discovery. It is highly possible that Rolling Lady, because of its extreme simplicity and elegance, was quickly created by many people and became an anonymous discovery.



Cardboard (Stockholm, 1980's)



Metal Foil (1983)

Figure 5: Rolling Ladies (photos by Yildirim Arici)

6 Hyperforms

Hyperforms are one of the most elegant and difficult to understand forms Koman discovered. Simplest hyperforms are shown in Figure 6. These simplest hyperforms are created from rectangles formed by four equal squares, twisting the corners by 2π and joining them together. Figure 7 shows how to create these simplest hyperforms.

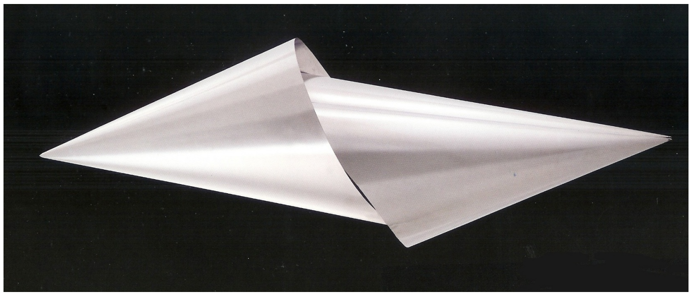
Figures 8 and 9 show more complicated hyperforms. Elegant beauty of these sculptures is even apparent in the photographs.

Unfortunately, we do not have much information how they were really created. Koman wrote that more complicated hyperforms can be constructed from rectangles that consist of more than four equal squares. He simply states that as the number of squares increases, the degree of twist at each level increases accordingly, $(2\pi, 4\pi, 6\pi \ldots)$, creating derivatives of the form. He also provided a number he called proportion for some of these sculptures. It is not clear how proportion is related to these final forms.

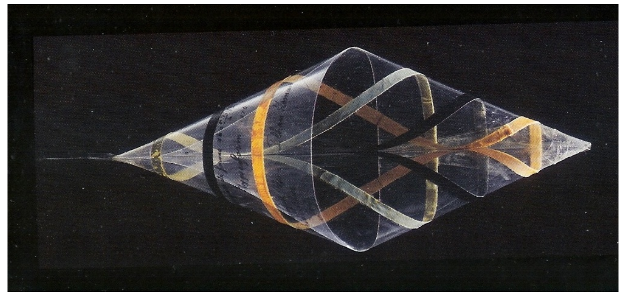
7 Conclusions and Future Work

Ilhan Koman's developable sculptures can easily be created in classroom and can be used to teach an appreciation of the complexity of 3D structures. It will also be interesting to develop mathematical models to create hyperforms using computer graphics.

Ilhan Koman's discoveries do not end with developable sculptures. He has also invented a variety of moveable rigid bodies in the same class as flexagons and kaleidocycles; self-carrying structures; forms



Hyperform (1978)
Stainless Steel
$60 \times 60 \times 145$ cm



Hyperform (1983)
Transparent Plastic
$12 \times 12 \times 27$ cm

Figure 6: Simplest hyperforms that are created from $w = 4h$ rectangles where w is the width and h is the height of the rectangle (photos by Yildirim Arici). (These sculptures are hanged from ceiling. The images are rotated $\pi/2$ counter-clockwise to use less space in paper.)

created from self-similarity and walking sculptures. We hope that we will be able to continue to explore other mathematical sculptures of Ilhan Koman.

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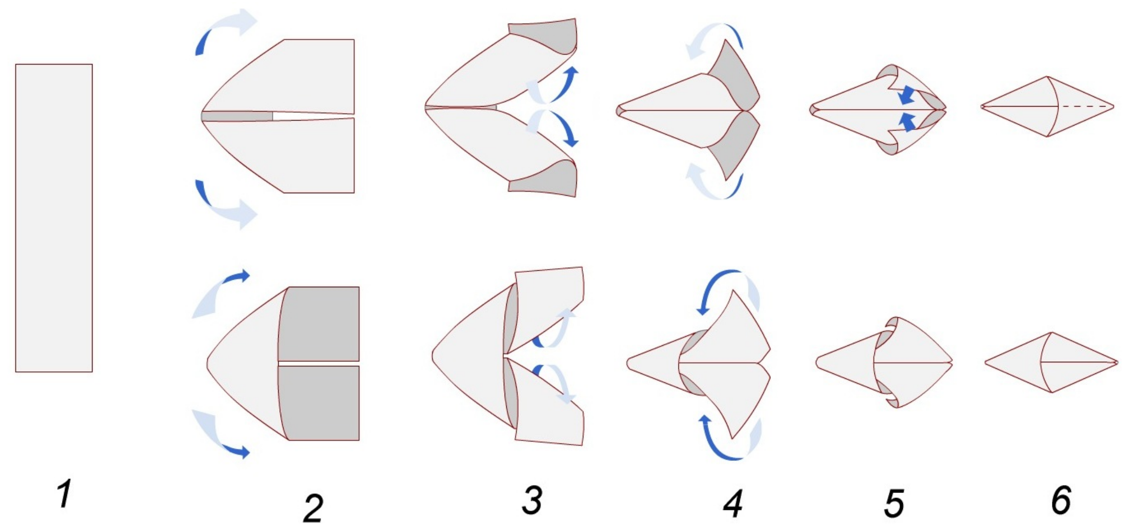

Figure 7: Construction of simplest hyperform from a rectangle formed by four equal squares. To illustrate the process we show both sides in each stage. The final form is obtained by joining edges in stage 5. (Illustration by Özlem Bakir)

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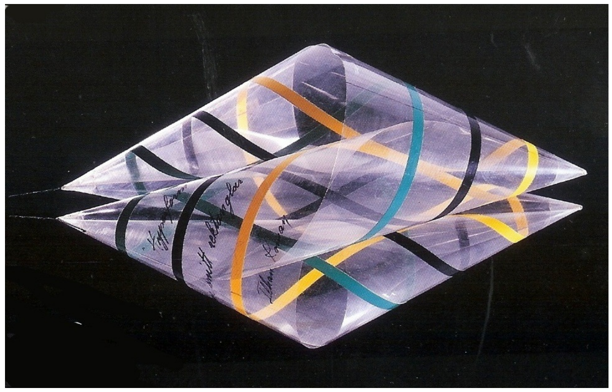
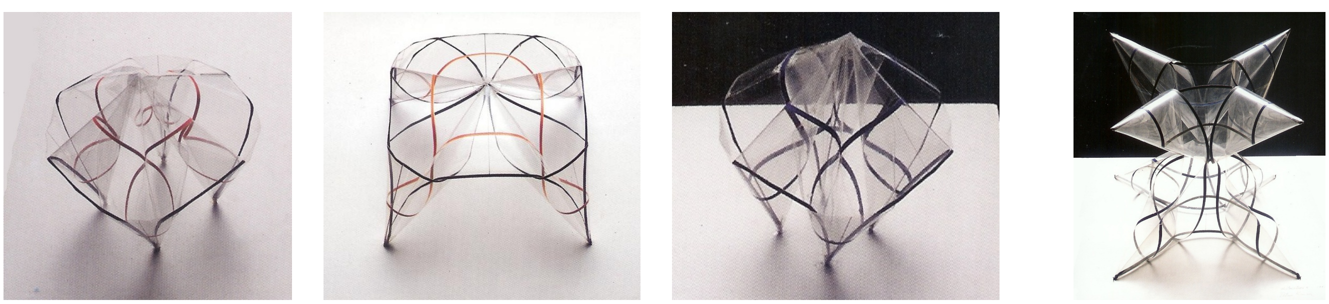


Figure 8: Twin Hyperform that is constructed from transparent plastics (1970-75). The actual dimensions are $24 \times 24 \times 50$ cm. (This sculpture is hanged from ceiling. The image is rotated $\pi/2$ counter-clockwise to use less space.)



Proportion $\sqrt{2}h$ (1983)	Proportion $3/8$ (1983)	Proportion $3/2h$ (1983)	Quadruple Hyperform(1983)
$60 \times 60 \times 15$ cm	$50 \times 50 \times 13$ cm	$60 \times 60 \times 15$ cm	$55 \times 55 \times 50$ cm

Figure 9: Complicated Hyperforms constructed with Transparent Plastic on stand that are constructed (photos by Yildirim Arici).