Two Methods for Creating Chinese Painting

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Abstract

We present two methods to create realistic Chinese painting. The first method is to create 3D Chinese painting animation using existing software packages. The second method is an expressive paint tool which allows an artist to interactively create 2D Chinese painting.

1. Introduction and Motivation

The main goal of this research is to bring Chinese painting to the digital realm, while keeping the tradition and the essence of the original art form [17, 14]. In addition, with the flexible and powerful computer medium, new forms of art can be created — 3D Chinese painting animation and stereoscopic images of Chinese painting. Using the computer medium adds a third dimension to the world of Chinese painting, which until now has traditionally been restricted to two dimensions.

Chinese painters have been using the ink-and-brush medium to create beautiful paintings on paper or silk scrolls for thousands of years. These paintings were drawn using brushes made from very fine animal hair which were dipped in ink and water. The interaction between ink particles, water particles and paper produces different aesthetically appealing effects unique to the ink-and-brush medium. The secret of Chinese painting is in putting the correct tones in the proper parts of the brush and in being able to call them forth by proper handling of the brush.

Chinese painting can be traced back to decorations on pottery and on the floors of thatched huts in the Neolithic period [around 10,000 B.C.]. In the Eastern Zhou dynasty, 2,500 years ago, the use of brush and ink had already developed to such an extent that the basic brush-made shapes have changed little since then. What makes Chinese painting such an exquisite flower in the garden of Chinese civilization is the way the arts of the brush — painting, calligraphy, and poetry — together with the related art of seal engraving, interact to produce so many of the masterpieces [17]. Traditional Chinese painting falls into two broad styles: *gong bi* and *xiao pin* (also known as *xie yi*).

Gong bi painting comprises serious work for which the artist makes comprehensive preliminary drawings before composing a highly finished painting. It is primarily an outline drawing, with colors added where the artist thinks necessary. It is refined and decorative.

Xiao pin means a "simple artistic creation." The artist engages in lighthearted ink play as the brush ceaselessly cavorts back and forth on the paper. It appears to be done with spontaneity, and even careless freedom [14].

In this research, we developed two methods to simulate the *xiao pin* style. We particularly tried to achieve four characteristics of *xiao pin* style Chinese painting that is given below. The followings are the characteristics of *xiao pin* style painting.

- *Qi* The Chinese character for *qi* means "steam" or "air," which is a sort of unseen, moving energy. Gu Kaizhi (345-406), who was one of the first to postulate theories about Chinese painting, said that "form exists in order to express spirit." If drawing with the brush is the skeleton, and ink and color work is the flesh, then the *qi* is the life force [14]. Therefore, each brush stroke in a Chinese painting is unique and filled with spirit. These brush strokes can only be created by the painters who have full control of the appropriate tone of ink, amount of ink and water, as well as the speed and pressure applied to the brush.
- Fei Bai The white space showing through the strokes

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is known as *fei bai*, or "flying white," so named because it conveys a sense of speed and movement [19]. *Fei bai* is created when the brush becomes dry and leaves the paper exposed. This is considered desirable because the energy continues across the gap and suggests great vitality [14]. A highlight effect that is more accidental than controlled, it also suggests the play of shimmering light and imparts a three-dimensional quality to the form.

- Atmospheric Perspective or Depth Atmospheric perspective refers to the fact that objects get blurry and hazy in the distance due to particles in the air. In Chinese painting, objects in the background are usually painted with very diluted ink to give the distant feeling and add depth to the painting [14].
- *Irregularity* No matter how thoughtful a painter is, there is always an element of surprise whenever the brush touches the paper. One can always find many different shades of the same color within a stroke. Also, the edge of a stroke is never perfectly smooth because of interaction between the ink and the paper surface.

Creation of *xiao pin* style paintings requires simulation of the essential materials for Chinese painting. These materials include the followings:

- *Paper* The most common paper for Chinese painting is *xuan*. This is also known as rice paper, but in fact is largely made from bamboo pulp. It is white and soft, and comes in various qualities and thicknesses [14].
- *Ink* Ink normally comes in stick forms. Ink sticks are made of soot from materials such as burned pine wood or lampblack. These are pounded together with glue and camphor and placed in molds to form a solid block. Bottled ink is also available, but rubbing your own ink is essential for the serious painter [14].
- *Ink Stones* The ink stone on which the ink stick is rubbed is hard and non-porous. A little water is added to the stone to make ink [14].
- *Brushes* Because all of the techniques depend on brushwork, the brush is the single most important tool of the Chinese painter. Brushes are made from a wide variety of animal hairs, for example, the soft goat hair and the hard wolf hair [14].
- *Pigments* The basic colors for Chinese painting are as follows: indigo, rattan yellow, umber (raw and burnt), rouge, carmine, cinnabar, mineral blue, mineral green, lead white. The colors are traditionally made from mineral and vegetable pigments [16].

• Seals - One always notices the distinctive red imprint of the seal or seals appearing on most Chinese paintings. The seals usually bear personal names and may be used alone or with the signature of the artist. It is also a common practice for owners to add their seals to the paintings [19]. The seals are also necessary to complete the meaning and composition of a painting. Very often, the seals themselves become fine works of art. The thick and sticky seal paste is made of cinnabar (mercuric oxide) which gives the red color.

2. Previous Work

In this chapter, previous work related to 2D ink and brush simulation and 3D non-photorealistic rendering techniques are discussed.

In computer graphics, Strassmann first modeled hairy brushes in 1986 [13]. He represented the model with four objects: the brush, the stroke, the dip and the paper. This is the software model that our 2D paint tool is based on. For implementation, he represented each stroke with a cubic spline and rendered the stroke using polygons. We observed that his approach is not practical for simulating Chinese painting since it can sometimes create the annoying "bow-tie" problem.

A few years later, Quo and Kunii modeled the diffuse paintings of *sumie*, which is the Japanese word for "black ink painting" [5]. Their model took into account the absorbency of paper, the change in liquid density and the characteristics of liquid flow to produce very realistic diffuse effects. Recently, Kunii, Nosovskij and Vecherlinin have developed a very realistic diffusion model for diffuse ink painting [10].

In 1995, Guo has developed models for generating realistic calligraphy [6, 7]. In 1997, a brush model for synthesizing calligraphic characters was developed by Ip and Wong [8] to model the position and orientation of the brush. It is important to know that Chinese calligraphy and Chinese painting compliment each other and almost every Chinese painting contains some form of calligraphy.

In the area of three-dimensional non-photorealistic rendering, Meier developed a particle-based brush stroke system which produced a realistic rendering of Monet's haystack painting [11]. This system is good for painting styles that use a lot of similar brush strokes. In Chinese painting, each stroke is unique that creates an expressive element in response to the changes in pressure and direction of the brush. Therefore, the fundamental methodology of Meier's research cannot be applied to Chinese painting.

Most recently, Zhang et. al [20] used simple cellular automaton-based simulation of ink behavior to render 3D images of Suibokuga(or sumie)-like trees. This is the first paper which actually showed images of a 3D object in inkand-brush style.

Besides developing an extensive software system to simulate traditional media, procedural shaders had also been used to shade 3D objects for producing non-photorealistic imagery. Examples are the woodblock print shader and cartoon shader created with Pixar's RenderMan shading language [1].

In this work, we developed two methods to create realistic Chinese painting. The first method is to create 3D Chinese painting animation using existing software packages. The second method is an expressive paint tool which allows an artist to create 2D Chinese painting. The next section presents our method for creating 3D Chinese painting animations.

3. Animated 3D Chinese Painting

Creating a 3D Chinese painting animation requires development of techniques for the modeling, shading and lighting processes. Modeling for a 3D Chinese painting can be simplified by observing that each painting consists of a set of brush strokes. The essence of Chinese painting can be preserved by modeling brush strokes as 3D geometric objects that form a 3D scene. Modeling shapes of each of these 3D geometric object is a challenge since these strokes need to be modeled in such a way that they will look interesting from every angle. Figure 1 shows a bird model which is made up of separate pieces of geometry.



Figure 1. The 3D model of a bird where different parts of the bird (main body, wings, beak, eyes, tail parts) are modeled as separate pieces and the pieces intersect each other.

The major part of implementing 3D Chinese painting animation is shader development since each stroke must look like brush strokes of chinese painting from every possible angle of view. Moreover, shaders should provide the following characteristics of Chinese painting.

• Uniqueness - Each Chinese painting is composed of many unique brush strokes. This purpose is partly

satisfied by using individual geometry for each brush stroke. To preserve the uniqueness of each stroke, we also need to vary the shading on each piece of geometry. This can be achieved by writing different procedural shaders for each stroke or using the same shader for the same type of stroke but with each individual stroke having different parameters.

- Atmospheric Perspective or Depth To achieve the atmospheric perspective or depth effect, our shaders detect the distance between the camera and the point being shaded. If the distance is large, meaning the object is farther away, the point is shaded with less opacity: therefore it looks more transparent. A linear function is used to interpolate the opacity between the farthest and closest points which are specified by the artist.
- *Fei Bai* To mimic this effect, irregular white lines are generated by the procedural shaders based on how much light is hitting the surface. The brighter parts of the surface get more white lines and these lines are also thicker.
- *Irregularity* When developing the procedural shaders, different noise functions [4] like the fractal and brownian noise functions are used to create the randomness. It is the artist's choice to select the appropriate noise function for the desired look. Also, the edges of the geometries are shaded so that they are not perfectly smooth.

To create the look of brush strokes, we have used procedural shaders. By observing real brush strokes, one can find that there are features which can be translated into layers of procedural shaders. The shader developer can then mix different layers together to get the desired effects. We have developed the following shader layers.

- *Base Color* A basic diffuse color is used as the base color of the stroke. See Figure 2 for an example of the diffuse color shader rendered on a sphere.
- Fei Bai The *fei bai* effect is created by adding irregular white lines to the surface. Based on the diffuse color, the lighter area gets more lines and also the lines are thicker. The lines are created using a pulse function, whereas a noise function is used to make the lines irregular. Figure 2 shows an example of the *fei bai* shading effect.
- *Split End* The ends of the brush stroke shape [1], are made transparent with a pulse function applied to the opacity. A fractal noise is used to give the splitting effect. Figure 3 shows an example of the split end shading effect.



Figure 2. The figure on the left shows a basic diffuse color shader. The figure on the right shader that implements the *fei bai* effect.



Figure 3. The shader on the left has one split end while the shader on the right does not.

- *Pressured End* Same idea as the split end, but instead of making the end of the shape transparent, it is made darker. Pulse function and fractal noise are used. Figure 4 shows an example of the pressured end shading effect.
- *Outline* By determining the angle between the normal of the surface and the viewing vector, we can create an outline for the shape. The bigger the angle, the thicker is the outline. The noise function is used to make the line irregular. Figure 4 shows an example of the outline shading effect and Figure 5 shows a bird that uses different shaders for different parts of its body.

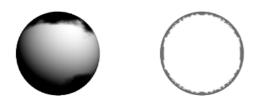


Figure 4. The figure on the left shows a shader that implements the pressured ends. The figure on the right shows a shader that implements the outline.

• Irregular edge - Similar to the outline layer, the edge



Figure 5. The body of the bird uses the outline layer and the split end layer. The wings of the bird combines the diffuse color with noise and the split end layers.

of the object is detected and a noise function is applied to the opacity at the edge to make it look irregular. This effect needs to be used very carefully because if used too much, the animation will become very noisy and distracting. Figure 6 shows the effect when shader layers are combined.



Figure 6. An example of combining the layers: diffuse color, *fei bai*, pressured end and irregular edge.

We used a simple lighting model to get *fei bai* effect at the correct locations. In general, only one directional light is enough to get the desired effect. Figure 7 shows different views of an object with changing *fei bai* effect created by one directional light.

Any off-the-shelf 3D modeling and animation package, together with a rendering system that incorporates procedural shaders and lighting utilities can be used to create 3D Chinese painting animation with our approach.

In our case, Maya 2.0 was used for modeling, animating and lighting. RenderMan Toolkit 3.9 and RenderMan Artist Tools 4.0 were used for shading and rendering. Fi-



Figure 7. The same bamboo stalk as seen in Figure 6, viewed at different angles. The effect of *fei bai* is affected by how much light is hitting the surface.

nally, Composer 5.0 was used for compositing. The hardware platform used was an SGI O2 machine.

An animation *Autumn Bamboo* was created using this approach. This animation is shown in Siggraph Animation Theater in 2000 [2]. Figure 8 shows the beginning frame from the animation.



Figure 8. A frame from the animation *Autumn Bamboo*.

One side bonus of creating 3D Chinese animation with this method is that stereoscopic Chinese paintings can easily be created from 3D models. All one needs to do is to render two images of a scene with a slight change in camera position. When the two images are viewed by both eyes in a stereoscopic viewer, an interesting result occurs and we can see a Chinese painting image in 3D. Figure 9 shows a pair of images used to create a stereoscopic Chinese painting.

4. Expressive Painting Tool

This section explain the second method; an expressive paint tool which allows an artist to create 2D Chinese painting. the first method.



Figure 9. When the left image is viewed by the left eye and the right image is viewed by the right eye, a stereoscopic image is created.

Chinese painting is a spontaneous and expressive form of art. Every brush stroke in a painting conveys meaning to the viewer. To preserve these aspects of Chinese painting, we need to develop a special user interface that guarantees interactivity, a software model for simulating the brush, the ink as well as the strokes.

The overall user interface design goal is to make it very simple and straight-forward to use. The experience of using this tool should be very similar to the actual painting experience. The brush used in Chinese painting is very responsive to the slightest movement the hand makes, giving great flexibility to the kind of brush strokes one can make. To accomplish this, we chose to use a pressure-sensitive pen and tablet as input devices instead of the mouse.

Our brush model tries to mimic a real brush in many different aspects. The brush model consists of many bristles and they are arranged somewhat randomly within a circular area. The circular area is divided into small squares and each bristle is positioned randomly within that square as shown in Figure 10. This preserves the general shape of the brush while adding some irregularity for more realistic strokes. One can experiment with different sizes and arrangements of the squares to get different kinds of strokes. The bristles are responsible for the actual painting. In order for a bristle to paint, it has to have ink on it and also the pressure applied to the brush needs to be sufficient for the bristle to touch the paper (see Figure 11). Therefore, each bristle needs to carry two pieces of information: the amount of ink on the bristle and the pressure threshold.

Although black ink is used most often in Chinese painting, colors are also used for dramatic effect. Therefore, our paint tool needs to provide a way for artists to select different ink colors and apply the colors to different brush strokes. Also, water plays an important part in Chinese painting. Adding more water makes the ink more transparent. The transparency should be noticeable when brush strokes overlap with each other as shown in Figure 12.

In our model, the artist is able to control how much ink

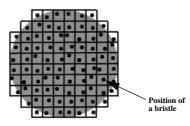


Figure 10. Cross-section of a brush. Bristles are positioned randomly within each square.



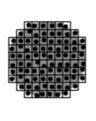
Figure 11. The pressure threshold of the longer bristles in the middle part of the brush is smaller, i.e. less pressure is needed for them to touch the paper while the pressure threshold of the shorter bristles on the side is larger.

is added to a brush and also how much is removed from a brush. These two are important procedures that affect how a stroke will look on paper. When ink is added to a brush, some bristles are randomly selected and their ink content is increased by a random amount. Also, our model implements the "ink stealing" effect [13]. As the artist paints a stroke, a bristle can steal ink from neighboring bristles. This allows the possibility of *fei bai* effect because some bristles can dry up completely and then regain ink by stealing from others. Figure 13 shows an example of the *fei bai* effect.



Figure 13. Fei bai effect shown in this stroke is created by adding ink to randomly selected bristles and ink stealing.

In our brush model, each bristle draws a line on the paper. Combining all the lines that each bristle draws make a stroke. The thickness of the lines can be varied to get different effects. Also, the bristle does not stay at a fixed position all the time. It keeps moving to draw a more irregular line. Therefore, the bristles need to be repositioned frequently. When more pressure is applied to the brush, the bristles are more spread out (see Figure 14).



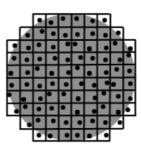


Figure 14. Each small square shows the general area that a bristle occupies. When less pressure is applied to a brush, the area covered by the brush is smaller (as shown on the left) and the bristles are closer to each other. When more pressure is applied the bristles are more spread out.

Our painting system, which we call *CPaint* was developed using object-oriented software design by using C++, together with OpenGL, FLTK, Xlib and Standard Template



Figure 12. Each stroke's transparency depends on how much water is in the ink.

😑 CPaint – Ink and Brush 🔹
File
Brushes
Ink Mix
Color
Black Ink
Water
100
Ink Stone
• -
Name Seal
Load your own seal
Undo

Figure 15. The tools panel for the artist to control different tools for painting.

Library. The hardware platform used was an SGI O2 machine with a Wacom^(R) pressure-sensitive pen and tablet as input devices.

FLUID, a graphical editor that produces FLTK source code was used to design the software user interface. The interface consists of two windows, one is the paper window on which the artist paints and the other one is the tools panel that is shown in Figure 15. By using tools panel, an artist can select different brushes by clicking on the brush buttons. Figure 16 shows the effect on paint strokes when different sizes of brush are used. The amount of water can also be specified. The more water the artist uses, the more transparent is the ink. Figure 17 shows the effect on paint strokes when there is different amount of water on the brush. Ink Stone allows to add and remove ink from a brush. Figure 18 shows the effect on paint strokes when there is different amount of ink on the brush. The artist can also load and use a name seal. The Undo button removes the most current brush stroke. There is no upper limit on how many times one can undo. The Clear button clears the whole paper.



Figure 16. Brush strokes painted using different brush sizes.



Figure 17. Brush strokes painted using increasing amount of water added to the ink from left to right.

The paint tool is designed using an object-oriented approach. The structure of the core classes is shown in Figure 19. To render paint strokes in real time, a special procedure is developed¹ In our procedure, we continuously get pen information from the tablet. When the pen pushes down on the tablet, one is either starting to draw a stroke or placing a seal on the paper. If a stroke is being drawn, a new Stroke object is created with a new Sample that contains the pressure and the x, y coordinates. If a seal is being placed, the image in the paper window is redrawn by calling the drawImage function of the Paper class and then the bitmap seal image is drawn. The drawImage function performs the following steps:(1) It clears the color buffer and depth buffer.(2) It draws the background paper texture. (3) It draws an array of pixels stored in memory which contains all the strokes and seals except the stroke that is currently being drawn. (4) It draws the current stroke. The current stroke is drawn in two parts. First, all the line segments generated by the old samples are redrawn. These line segments are stored in the memory. Then, if there is a new sample being added, a new set of line segments are drawn.

When the pen is lifted from the tablet, that means the artist has finished drawing a stroke. At this point, the

¹Further acceleration is made possible by using double-buffering in OpenGL [15].



Figure 18. Brush strokes painted using decreasing amount of ink on the brush from top to bottom.

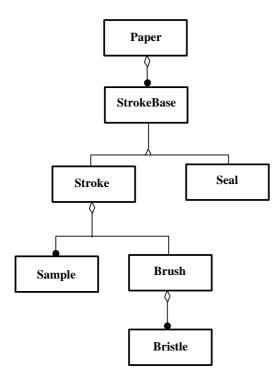


Figure 19. Class diagram of the main components of *CPaint*. Notations of the diagram is based on the book by Rumbaugh [12]. drawImage function is called and the current image pixels are read into the memory. The image in the memory thus contains all the completed strokes and seals. Figure 20 illustrates the process of redrawing the image as the artist paints. Figures 21, 22 and 23 are example paintings created by different artists with our system.

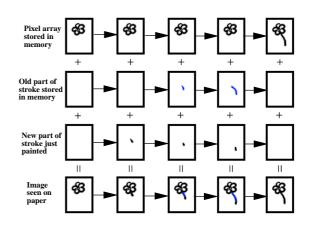


Figure 20. Each time the artist paints a new segment of a stroke, the program first draws the pixel array stored in memory, then the old part of the stroke stored in memory, and then the new part of the stroke. This diagram shows what each component contains at different times and the final image on paper.

5. Conclusion and Future Work

The main goal of this research is to bring Chinese painting to the digital realm, while preserving the tradition and essence of the original art form. In this paper, two computational approaches for creating realistic Chinese painting were discussed.

We created a 2D expressive paint tool that enables artists to create Chinese painting spontaneously. This tool is easy to use and has an interface similar to the real Chinese painting interface. A brush model which contains many bristles is used to mimic a real brush. Each bristle draws a line on the paper and all the lines together form a stroke. This tool was developed using an object-oriented approach. Future changes to this tool can be easily done by modifying each individual object. Although this tool was designed to create Chinese painting, creative artists can use it to generate any image they want. It is possible to make sketches and gestural drawings because of the responsiveness of the brush and



Figure 21. Pumpkin and Beetle.



Figure 22. Noble Orchids.



Figure 23. Summer Bamboo.

the simple user interface.

We also developed an approach to create 3D Chinese painting animation using commercially available 3D modeling and animation packages. We used a pipeline that is commonly used in 3D animation production and developed techniques for the modeling, shading and lighting processes. The modeling method we used was efficient and preserved the uniqueness of brush strokes. Procedural shaders were developed to create the realistic ink rendering. With this approach, a third dimension is added to Chinese painting which until now has been restricted to two dimensions. We can also create stereoscopic Chinese painting with this technique.

Future work can be developed for real-time 3D Chinese painting, Chinese painting on the internet, and virtual reality in a 3D Chinese painting environment. More features can be added to the 2D expressive paint tool, for example, simulating the interaction between ink and paper, as well as different effects when an artist tilts the brush. It is our goal to introduce the art of Chinese painting to more people, from professional animators to the general public and push the traditional art of Chinese painting to a different level.

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