# Modeling Subdivision Control Meshes for Creating Cartoon Faces

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## Abstract

Modeling three-dimensional faces on the computer has been an interesting yet challenging problem. This paper presents a method for creating cartoon faces by using a subdivision scheme. We use set-operations for conceptual design of subdivision control meshes. To ensure the quality of the control mesh, we have eliminated high valenced extraordinary vertices since smoothness of the surface decreases with valence. In addition, we have limited the number of extraordinary vertices to eliminate ripples. We have also ensured an even structure around extraordinary vertices (The size of each quadrilateral in subdivided meshes are roughly similar). We have also developed a user-friendly interface to sculpt the control mesh. Using this interface we have been able to create a variety of cartoon faces.

## **1** Introduction and Motivation

Modeling three dimensional faces on the computer is a very challenging and time consuming problem. There are a wide variety of ways to approach the facial modeling problem. Existing function based methods for creating smooth facial models include parametric, implicit and subdivision surface modeling.

One parametric surface approach is to draw a set of carefully spaced vertical or horizontal contours of the face and then lofting them together. The problem with this approach is that adding detail to one part of the surface will result in the introduction of unwanted detail in other parts. Figure 1 shows an example where addition of detail on the nose results in a dense collection of iso-parametric lines on the forehead and cheek. These extra control points may produce unwanted creases.



Figure 1. An example of parametric model with unwanted creases.

Another approach is to use parametric surface patches [15]. Using this approach the face can be modeled using a group of independent connected patches. With careful planning, a high quality model can be obtained. However, patch modeling is time-consuming and requires a good understanding of 3D space. A major problem with patch modeling is maintaining boundary continuity between the patches. Figure 2 shows an example parametric patch model.

Implicit surfaces can be powerful in modeling solid shapes such as faces, since they inherently provide a simple implementation of geometric operations such as union and intersections by composition of functions [7]. Unfortunately, implicit methods are not computationally efficient since computation of an implicit surface is a root finding problem in 3D. Improvements in computational speed generally require limitations on the shapes [3, 4].

Subdivision surfaces are actually polygonal surfaces that can approach any given parametric surface as a limit surface using an iterative subdivision scheme. The power of subdivision schemes for modeling faces come from the fact that they can smooth out any 2-manifold mesh with arbitrary topology. Subdivision control meshes are not restricted to 4-sided patches like tensor product parametric surfaces.

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Figure 2. An example of parametric patch model.

The goal of this research is to develop a high quality subdivision control mesh for creating cartoon faces. One problem with the subdivision schemes is that smoothness of the surface decreases with valence of extraordinary points of control mesh [35]. (Extraordinary points of are those vertices which have a valence (the number of edges meeting at the vertex) other than four for Catmull-Clark and Doo-Sabin schemes [8, 13].) In addition, extraordinary vertices can create surface ripples. Moreover, there may not be an even structure around extraordinary vertices, i.e., the size of each quadrilateral in the subdivided mesh may be extremely different. It is, therefore, essential to avoid the introduction of extraordinary points wherever possible. Our control mesh for the face is created to keep extraordinary vertices to a minimum.

The facial models from this system are not intended to be photorealistic. This modeling tool is intended for creating a variety of cartoon faces or caricatures. Detail such as nostrils, eyelids and ears must be sculpted later and are not included in the initial control mesh.

One of our contributions is the introduction of an efficient facial modeling interface for sculpting the initial control mesh. This interface allows the user to manipulate the initial control mesh by changing data in front and side views. This interface is both flexible and robust. Users can create highly exaggerated facial models and the resulting subdivision surfaces are almost always smooth.

## 2 Previous Work

The first computer generated 3D faces were created by Parke [27] at the University of Utah in the early 1970s. This facial animation was achieved by collecting facial expression polygon data from real faces using photogrammetric techniques and interpolating between them.

Physically based facial models were first generated in

1980 by Platt [30] at the University of Pennsylvania. In 1987, Waters [31] described a new approach to facial animation using facial muscles. Waters et al. [32] presented a methodology to incorporate geometrically accurate polygonal facial representations constructed by photogrammetry of stereo facial images with generic tissue and muscle models to synthesize faces capable of expressive articulation. In 1993, Akimoto, et al. [1, 2] used the front and side views of a person to automatically modify a generic head model. Their method had two parts; extracting the features from the two views and automatically modifying the generic model. Lee, et al. [22, 23] presented an efficient method to generate a 3D head from image data.

Another approach to facial modeling is to allow a user to specify parameters for the facial geometry. In 1974 Parke [28] created the first parametrized facial model. Patel [29] offers a set of deformation parameters closely tied to the structure of the head. Dipaola [12] provides a set of localized volumetric deformations to extend the range of facial types. DeCarlo et al. [9] describe a method that automatically generates varied geometric facial models based on anthropometric statistics. Forsey uses Hierarchical B-splines [16] for face design. Multiresolution methods have also been used for the facial design and manipulation [34, 21, 19].

In this work, we use subdivision schemes to obtain smooth facial models. The 1998 Academy Award winning short film Geri's Game was one of the first movies from a major studio which successfully used subdivision surfaces for modeling faces [11].

Subdivision schemes assume that the users provide the initial control meshes. These initial control meshes can either be created by direct modeling or obtained by scanning a sculpted real object. A smoother version of this initial mesh without changing the original topology is obtained by subdivision operations.

All subdivision schemes can be expressed by a set of linear difference equations. More formally, each new point is computed as a linear combination of a set of points in a local topological region. The scheme can be written as a linear system  $p_{n+1} = Ap_n$  where  $p_n$  and  $p_{n+1}$  are the vectors of respectively the old points and the new points in the local topological region and A is the transformation matrix [35]. Note here, the local topological region should correspond to a simple disk (topologically). This implies that the underlying structure must be a valid 2-manifold (or 2-manifold with boundary). The initial control mesh for a subdivision scheme should not have artifacts that commonly exist in computer graphics models such as wrongly-oriented polygons, intersecting or overlapping polygons, missing polygons, cracks, and T-junctions [5].

The existing subdivision schemes can be classified based on three criteria [35]: (1) the type of refinement rule (e.g. vertex insertion or corner cutting), (2) the type of generated mesh (e.g. triangular or quadrilateral), (3) whether the scheme is approximating or interpolating. For instance, among commonly used subdivision schemes, Catmull-Clark is a vertex insertion, quadrilateral and approximating scheme [8], Loop is vertex insertion, triangular and approximating scheme [25] and Doo-Sabin is a corner cutting quadrilateral and approximating scheme [13].

In this work, we use a vertex insertion, quadrilateral and approximating (VIQA) scheme, which is a generalization of Catmull-Clark subdivision scheme. This scheme provides continuity control with an additional tension like parameter [33]. In a VIQA scheme the initial 2-manifold mesh can be arbitrary. For regular meshes, VIQA can provide tensor product B-Spline surfaces which are widely used in modeling.

In a VIQA subdivision scheme, the vertices with a valance other than 4 are called extraordinary vertices. (The valance is the number of edges meeting at the vertex.) To obtain high-quality VIQA surface, it is necessary to avoid extraordinary vertices since they may create uneven polygon sizes and ripples. Use of polygons which are not 4-sided also results in extraordinary vertices as shown in Figure 3. To avoid such secondary extraordinary vertices, non-quadrilateral faces should be avoided.



Figure 3. Vertex Insertion type subdivision on a mesh containing a 6-sided face, resulting in the introduction of an extraordinary point.

# **3** Designing The Initial Mesh

Since the quality and topology of the smooth surface resulting from subdivision rules depend greatly on the initial control mesh, theoretical assurance of the quality of the initial control mesh is extremely important. In other words, the process of obtaining the initial control mesh must be robust and guarantee valid 2-manifolds.

The method adopted in this work is to use set operations to design an initial control mesh for VIQA subdivision. Set operations are very intuitive in designing such an initial mesh. Any convex region on a face, such as the nose or chin, can be approximated by performing an intersection operation of its side, top and front views. Once we have an approximate shape for each convex region of the face, we can perform a union operation of all these individual shapes to get a final model that will be used as the initial control mesh.

However, the problem with set operations is that they can result in non-manifold topologies, non-quadrilateral faces and lamina topology. They can even create free points and floating edges. Moreover, many existing data structures in mesh modeling are specifically developed so that they can represent non-manifold surfaces resulting from set operations. They do not guarantee valid 2-manifold surfaces. Because of this fundamental problem, the process of obtaining the initial control mesh can result in unwanted artifacts such as wrongly-oriented polygons, intersecting or overlapping polygons, missing polygons, cracks, and T-junctions.

Another problem with set operations is that they can result in many extraordinary vertices and non-quadrilateral faces. For instance, as shown in Figure 4 even in an inter-



Figure 4. Incorrect alignment of meshes resulting in introduction of non-quadrilaterals.

section of two simple shapes (extruded polygons), we can end up with non-quadrilateral polygons. However, with a careful alignment of the same shapes we can obtain a shape consisting of only quadrilaterals as shown in Figure 5 (Note that we cannot avoid extraordinary vertices completely).



Figure 5. Correct alignment of shapes.

Because of the problems we mentioned above, we have used set operations only in the design stage. We designed the initial control mesh with a great care to ensure that the resulting mesh is a 2-manifold and that it has minimum number of extraordinary vertices and nonquadrilateral faces. We have identified convex regions that can be created as an intersection of extruded polygons. We observed that a cartoon face broadly consists of six convex regions; the forehead, the cheek, the nose, the chin, the upper lip and the lower lip. The shape of each convex region can be approximated by an intersection of extruded side, front and top views. After creating these convex regions separately, we carefully placed them together and performed a union operation to get the initial control mesh. The union operation creates polygons with lamina topology at the faces were the regions touch each other. These polygons are deleted from the control mesh to make it subdivision friendly.

### 3.0.1 Designing Convex Regions

As mentioned before, we divide the face into six convex regions. To create a convex region using set intersection, we use side, front and top views. We first determine which view provides the maximum information about the shape of the region. In most cases this is the side view. Next we identify whether intersecting this mesh with the top view or the front view gives us additional information about the shape. The following subsections discuss how each region is constructed.

**Forehead:** The approximate shape for the forehead is obtained by performing an intersection operation of the extruded side view and the extruded front view as shown in Figure 6. Both side and front views are 10 sided convex polygons. The bottom-most section forms the top part of the eye socket. The section above this gives shape to the eyebrow. The next section forms the major plane of the forehead and the back of the head while the top section gives the curve for the top of the head.



Figure 6. Forehead: side view, front view and final mesh.

The front view of the forehead is created so that each vertex lies exactly in line with an associated point on the side view. If any of these vertices do not lie in the same horizontal plane as vertices on the side view, we can end up with non quadrilateral polygons as shown in Figure 4. **Cheek:** The cheek is also obtained by performing an intersection of extruded side and front views as shown in Figure 7. Both side and front views are also 10 sided convex polygons. The bottom-most section connects the lower lip to the back of the jaw. Similarly the next section connects the upper lip to the back of the head. The third section connects the base of the nose tothe back of the head. The final section makes up the bottom part of the eye socket.



Figure 7. Cheek: side view, front view and final mesh.

**Nose:** The nose is an intersection of the extruded side view and extruded top view as shown in Figure 8. The side view of the nose is made up of three quadrilaterals. The section in the middle defines the *hook* of the nose. It also connects the nose to a section of the cheek. The section at the bottom of the side view is used to control the tip of the nose. The section at the top connects the nose to the eye sockets and forehead. Because of the shape of the nose, the approach is to use the top view for further shaping of the nose. The top view helps to taper the nose. This operation helps shape the nose without adding unnecessary complexity.



Figure 8. Nose: side view, top view and final mesh.

This nose however introduces two new vertices to the forehead when they are later attached. As shown in Figure 9 this attachment creates an extraordinary point when the mesh is later subdivided. However this extraordinary point does not create a visible unevenness on the forehead and hence we leave it untouched. While change to the topology of the forehead might remove this extraordinary point, it will also increase the complexity of the mesh.



Figure 9. Extraordinary point on forehead.

**Chin:** The chin is obtained by intersecting the extruded side view and the extruded front view as shown in Figure 10. Both side and front views are also 10 sided convex hexagons. The side view of the chin is made up of two sections. The bottom sectionprovides the roundness of the chin when the control mesh is later subdivided. The top section attaches the chin to the bottom of the cheek.



Figure 10. Chin: side view, front view and final mesh.

**Upper Lip:** The creation of the upper lip is a little more complicated than the previous regions. The upper lip could be modeled as a single block. However to actually be able to recognize the mesh as an approximation for the upper lip, the depression in the middle of the lip is important. To accommodate this depression, one half of the upper lip is modeled and then mirrored to obtain the second half.

To obtain the control mesh for the upper lip we do an intersection operation of the side, front and an angular view. This obtains two sloping surfaces; one sloping from the middle of the upper lip towards the cheek and the other sloping from the base of the nose to the edge of the mouth.

Performing the intersection of the extruded side and front views gives us the slope from the base of the nose to the edge of the mouth as shown in Figure 11.

To get the slope from the middle of the upper lip towards the cheek, we might try intersecting this mesh with the top view. However this introduces an unwanted edge in a very unintuitive direction as shown in Figure 12

Rather, we intersect the mesh obtained from the side and front views, with an angular view, which is oriented at about forty five degrees. This slopes the lip towards the cheek as shown in Figure 13.



Figure 11. Intersection of side and front upper lip views.



Figure 12. Intersection of side, front and top upper lip views results an unwanted edge.



Figure 13. Intersection of side, front and angular upper lip views.

The final mesh for the upper lip is shown in Figure 14.



Figure 14. Upper lip: side view, front view, angular view and final mesh.

While the control mesh we obtained is made up of quadrilaterals, a few new extraordinary points are introduced when the upper lip is combined with the rest of the face. One such extraordinary point is underneath the nose. The others are at the corner of the mouth and at the corner of the nose. The one underneath the nose is hidden from view in most cases.

**Lower Lip:** The lower lip is easier to create than the upper lip. Performing an intersection between the extruded side view and extruded front view gives a mesh for the lower lip. This mesh is again made up of quadrilaterals. Figure 15 shows the final control mesh for the lower lip.



Figure 15. Lower lip: side view, front view and final mesh.

#### 3.0.2 The Base Mesh

Once we have the individual control meshes for each region, they are positioned as shown in Figure 16. The individual control meshes are placed so that their surfaces fall exactly on associated surfaces of the neighbouring mesh. Thus, the bottom of the forehead falls exactly on the top of the cheek. Similarly, the bottom of the cheek falls exactly on the top of the chin. Even a very small misalignment can result in the introduction of extra polygons which may not be quadrilaterals. The base mesh is then created by performing a union operation of all six convex regions. This, however, creates polygons with lamina topology at the faces were the regions meet. These polygons are deleted from the control mesh to make it subdivision friendly.



Figure 16. Positioning of regions to create final mesh.

Although the base mesh is mostly quadrilaterals, there are a few polygons which are not 4-sided. However, these are either well hidden or are at positions where they do not create major problems when the mesh is subdivided. The base mesh satisfies all the conditions required for successful conversion to a subdivision surface. Figure 17 shows the final VIQA subdivision control mesh for the face.

This base mesh contains sixty-nine polygons. Other than quadrilaterals, the control mesh contains one pentagon, two hexagons and one heptagon. These polygons introduce extraordinary points on the mesh. In addition, there exist four vertices with a valence of six. Fortunately, these are either at positions where they are fairly hidden, such as underneath the nose, or at places where they did not create much problems, such as the corner of the mouth.



Figure 17. The control mesh.

#### **3.1** Developing the Interface

Set operations were used while constructing the initial control mesh because it was easier to visualize each region in two dimensions rather than in three dimensions. By using this organized approach, the mesh can be reproduced more easily than if it was constructed by laying out points in space. While set operations are useful during the construction of the initial mesh, they do not work well in an interactive environment. There are several cases where set operations can fail when the regions are being manipulated. Because of this, the subdivision control mesh is created as one single polygonal model whose vertices can be directly manipulated in the interface.

One of the contributions of this work is the introduction of a user-friendly interface. In this approach users can manipulate controls directly on profiles of the side and front views of the mesh. Since the topology of the mesh remains the same no matter how much the user exaggerates the regions on the face, the mesh is guaranteed to remain subdivision friendly. The conceptual design of this interface is shown in Figure 18.



Figure 18. Design for modeling interface.

## 4 Implementation

There were several alternatives for implementing this facial modeling tool. One option was to implement it using C++ and OpenGL. However this would require spending most of the time writing basic display operations unrelated to the core idea. Another option was to use an existing modeling package for the basic operations. The facial modeling tool could then be written as a plug-in. Alias/Wavefront's Maya is such a modeling and animation package. Maya has the advantage of having a scripting language, which supports the development of plug-ins.

#### 4.1 The Modeling Interface

The initial approach for the interface was to provide the user with a set of sliders which would control the shapes of the different regions on the face. This was soon discarded because it was not intuitive. We realized that artists generally prefer working directly on the model. So a new interface was developed which allows the user to directly move points on the generic mesh. A dialog box also provides additional functionality to the user. Figure 19 shows the developed Maya interface.



Figure 19. Modeling interface.

This interface consists of two parts. The first part is a set of three windows where the user manipulates the generic mesh by dragging locators. There are two groups of locators. Locators for manipulating the side view and locators for manipulating the front view. Each locator in turn is parented to two clusters of vertices. One cluster contains relevant points on the generic polygonal mesh. The second cluster contains relevant point(s) on a curve that defines the profile of the mesh. A cluster is used just so that we can group a set of vertices and parent it to a locator.

The locators in the side panel are allowed motion only along the z and y axes. The locators in the front panel are allowed to move only along the x axis. Another level of control is provided to the user by creating a group of larger locators in both the side and front panels. Each of the larger locators are connected to a small group of the original locators. For example there is a large locator connected to the two smaller locators defining the tip of the nose in the side panel. A user therefore has a higher level of control in resizing the tip of the nose by dragging this locator.

Different types of objects can be hidden in the different

panels in Maya. However, it is not possible to hide different groups of the same object in different panels. This causes the locators that control the side view to be visible in the front panel and vice versa. This might confuse the user. The problem is solved by scaling the locators in the side view to zero in the x direction. Similarly all the locators in the front view are scaled to zero in the z direction so that they are not visible in the side panel.



The second part of the interface is a helper dialog box. As shown in Figure 20 this dialog box offers several features to assist the modeling process.

interFACE	•
Create Subdivision Surface	
Continuity	0.5
Divisions	3
Smooth Mesh Reset Mesh	
Side Image Plane	esis/MEL/sourceimages/charls.jpg
Front Image Plane	esis/MEL/sourceimages/charlf.jpg
Default Environment	Export Model



The Create Subdivision Surface button converts the polygonal mesh into a subdivision surface. The MEL command that converts the polygonal mesh into a subdivision surface takes two important parameters. maxPolyCount sets the maximum number of faces the original surface can have to successfully convert it to a subdivision surface. In our modeling tool the number of faces on the base mesh always remains constant at sixty nine faces and hence this value is easily set. The second parameter is maxEdgesPerVert, which sets the maximum number of edges each vertex in the base mesh can have to successfully convert to a subdivision surface. While most of the vertices have four edges coinciding there are a few unavoidable extraordinary vertices. The corner of the mouth where the upper lip, lower lip and the cheek meet has six edges coming together. This is the maximum number of edges coming together at a vertex on our base mesh. Figure 21 shows a subdivision surface created from the generic face model.

The Smooth button allows the user to convert the polygonal base mesh into a smoother polygonal mesh. Maya modifies the topology of the polygonal object by smoothing out vertices and their connected edges. The result is a polygonal mesh subdivided using the VIQA scheme. However this object will remain a polygonal mesh and Maya's



# Figure 21. Control mesh and corresponding subdivision surface.

subdivision operations cannot be performed on this surface.

The important inputs to the MEL command that does polygonal smoothing are divisions and continuity. The flag divisions specifies the number of times Maya performs the smoothing operation. Maya allows this parameter to be one, two, three or four. The higher the value, the smoother the object. Continuity determines the degree of smoothness. This can be any value from 0.0 to 1.0. While modeling a childs face, its a good idea to increase both continuity and divisions. This will ensure that the model is as smooth as possible. While modeling an adult male face, these parameters whould be kept as low as possible. The Figure 22 shows the base mesh smoothed with different values of continuity and divisions.



Figure 22. Smooth polygonal mesh showing different values for divisions and continuity.

The Reset Mesh button is provided so that the user can revert back to the original control mesh if the subdivi-

sion surface or smoothed polygonal mesh is unsatisfactory.

Two edit boxes are provided to allow the user to load side and front image planes as reference while modeling.

The user might unintentionally change settings or layout in Maya. The Default Environment button will override changes and take the user back to the original environment.

Once the user is satisfied with the model (subdivision surface or smoothed polygonal mesh), it may be exported using the Export Model button.

A specific subdivision control mesh along with a corresponding modeling interface was created so that a variety of highly exaggerated cartoon faces could be rapidly created. The model and interface are flexible enough to create the face of a child as well adult faces with equal ease. The Figure 23 shows some of the faces produced by our system.



Figure 23. A sample of facial meshes created using the system.

# 5 Conclusion and Future Work

In this work, we have introduced a set operation based method for designing vertex-insertion, approximat-

ing, quadrilateral control meshes for creating cartoon faces. The mesh satisfies several conditions which guarantee smoothness of the subdivided mesh. We also introduced a user-friendly interface for sculpting this control mesh. The control mesh and interface are flexible enough to create a wide variety of cartoon faces.

The initial control mesh contains a few polygons which are not quadrilaterals. These polygons introduce some extraordinary points in the mesh. Future work on the control mesh involves a reduction of extraordinary points in the control mesh.

In the future, we are also planning to add a neck to the control mesh. The facial model as it exists also lacks detail such as ears, eyelids and nostrils. Currently these features have to be sculpted on to the surface once its converted to a subdivision surface. Controls could also be provided to position and resize the eyeballs.

Currently, the user has total freedom in manipulating the mesh. This freedom allows the user to manipulate the mesh into highly exaggerated shapes which might cause polygons to intersect. An important improvement to the interface would be automatic tests to see if self-intersections are introduced in the mesh.

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