On a Family of Symmetric, Connected and High Genus Sculptures

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Abstract

This paper introduces a design guideline to construc a family of symmetric, connected sculptures with high number of holes and handles. Our guideline provides users a creative flexibility. Using this design guideline, sculptors can easily create a wide variety of sculptures with a similar conceptual form.

1. Introduction

In this paper, we present a design guideline to create a new sculptural family with interactive topological modeling. Using this design guideline a large set of sculptures that have a similar conceptual form can easily be created (see Figures 1 and 2). We have tested the design guideline in a computer aided sculpting course [2]. We observe that, using the design guideline, students can rapidly create a wide variety of shapes. Although these shapes are completely different; they can be perceived as belonging to the same family and having a similar conceptual form. Figure 5 shows some examples of shapes that were created by some of the students using our design guideline, as one of the biweekly assignments of the computer aided sculpting course.

Figure 1: Photographs of 3D prints of two of the first symmetric high genus shapes (shapes with high number of handles and holes) designed by Ergun Akleman. For each sculpture, we took two photographs from slightly different point of views. The sculptures are photographed on a mirror. Background is eliminated. These shapes are made from ABS plastic and printed using a Fused Deposition Machine (FDM). They are later painted using an acrilic paint.

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2. Motivation

Development of new methods to create new aesthetic forms are essential for artistic applications. In computer graphics and mathematics, it is very common to find methods to create aesthetically pleasing visual results which has never been created before. Although, most researchers usually view their results as proof-of-concepts, those results can be aesthetically pleasing and considered to be artworks. The methods developed by researchers are also useful for artists since they are reproducible and allow a wide variety of results to be produced. For instance, the fractal methods to create Julia sets are widely used by non-mathematicians to create artworks [1].

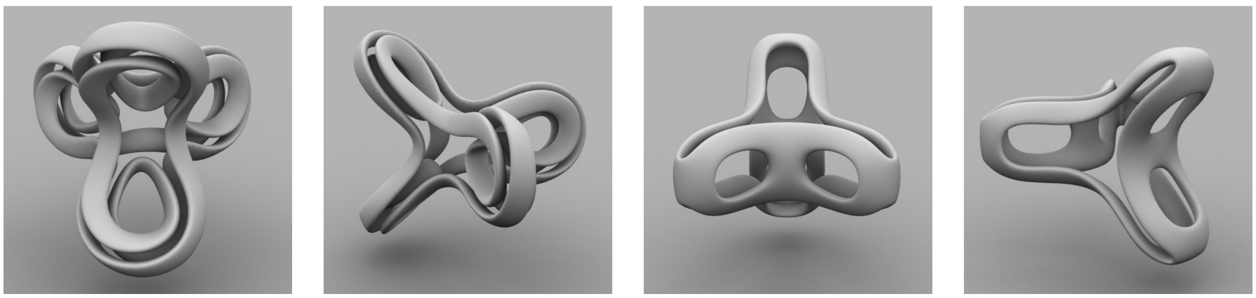


Figure 2: Computer rendered versions of the models from which sculptures in Figure 1 are printed. Images are rendered by Ozan Ozener. The images show two views of the same shape. The ribbon shape on the left is really a genus-1 surface, but, during the construction of this surface, genus changed several times.

With the advance of computer graphics, many artists have also begun to use mathematics as a tool to create revolutionary forms of sculptures. There are many contemporary sculptors such as George Hart [14], Helaman Ferguson [11, 10], Bathseba Grossman [12], Brent Collins, and Carlo Séquin [16] who successfully combine art and mathematics to create unusual and high-genus sculptures. These mathematical sculptors develop their own methods to model, prototype and fabricate an extraordinary variety of shapes. Most of these contemporary sculptors who successfully combine art and mathematics are, in fact, mathematicians or engineers who have developed their own methods. Identification of methods to create new aesthetically pleasing sculptural forms can eventually be a major direction for solid and shape modeling research.

One of the most exciting aspects of sculpting has always been the development of new methods to design and construct unusual, interesting, and aesthetically pleasing shapes. One of the most interesting sculptural shapes are high-genus surfaces [19, 20, 10, 17, 5]. Our goal in this work is to develop a reproducible guideline to design high-genus sculptures. We also want to provide creative flexibility to users for constructing a wide variety of sculptures that can still be perceived as belonging to the same family. We also want our resulting shapes to be physically realizable. In other words, we should be able to 3D print the resulting shapes. In this paper, we present a design guideline that satisfy all the goals to create new aesthetically pleasing sculptural forms.

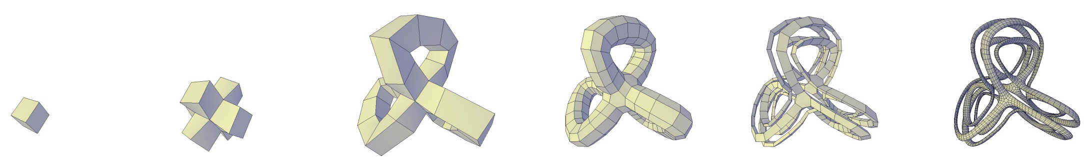
- Using the design guideline it is possible to create a wide variety of shapes that do not specifically resemble any existing sculptural or architectural forms.
- On the other hand, despite the variety all the shapes that are created using the same design guideline must still be perceived as belonging to the same family.
- The resulting shapes can be physically realizable with a minimal effort from the user as shown in Figure 1.

3. Design Guideline

The design guideline is based on a set of topological mesh modeling operators. These operators guarantee that the resulting shape is topologically 2-manifold. If the user avoid self-intersection, which is easy to achieve, the resulting models are 3D-printer-ready, i.e. they can be printed by using a rapid prototyping machine as shown above.

Our design guideline consists of six stages.

- **Stage 0: Initial Shape.** Start with a symmetric convex polyhedral shape [7, 13, 21]. Any platonic solid such as tetrahedron, cube or dodecahedron [18, 22] are perfect candidates for starting shape. In the example shown in Figure 3, the initial shape is a cube.
- **Stage 1: Extrusions.** Apply the same extrusion operation to all faces of the starting shape. Adding a twist or rotation to the extrusions can create an additional effect as shown in Figure 3. Moreover, the recently introduced local mesh operators can also be used in this stage. These local operators can extrude generalized platonic solids and are called tetrahedral, cubical, octahedral, dodecahedral and icosahedral extrusions [15].



Stage 0: Initial Shape	Stage 1: Extrusions	Stage 2: Handle Creation	Stage 3: Doo-Sabin Subdivision	Stage 4: Rind Modeling	Stage 5: Smoothing with Catmull-Clark

Figure 3: The design guideline.

- **Stage 2: Creating Handles.** Connect symmetrically related faces using multi-segment curved handles [17]. This operation, along with cubical extrusions, populate the mesh with quadrilateral faces with 4-valent vertices as shown in Figure 3. After the operation, there will be only a handful of extraordinary points (i.e. vertices with valence other than 4 and faces with sides other than 4).
- **Stage 3: Doo-Sabin Smoothing.** Apply Doo-Sabin subdivision once [8]. This operation will create visibly connected clusters of faces. These clusters of faces are formed as a result of extraordinary points. The faces in each cluster defines a route that connects one extraordinary point into another. These clusters can easily be seen in Figure 3.
- **Stage 4: Rind Modeling.** Rind modeling creates a rind structure by creating a smaller replica of initial shape [5]. In rind modeling, users can punch holes in this ring shape by clicking the faces. There exist two strategies: (1) Deleting all non-cluster faces by making the clusters clearly visible, (2) Deleting all cluster faces by making the non-cluster structures visible. In the example shown in Figure 3 we made clusters visible.
- **Stage 5: Final Smoothing.** Final smoothing is useful to create a simple and smooth surface. Although, in final smoothing any subdivision scheme can be used, we use Doo-Sabin [8] or Catmull-Clark [6] schemes for smoothing.

Using both stages 2 and 3 is not really required. It is even possible to skip either one of them to create different results. As it can be seen in attached examples it is possible to create a wide variety of shapes using this design guideline.

4. Implementation

The operations used in all six stages have already been implemented and included in our existing 2-manifold mesh modeling system, called TopMod [3, 9, 5, 17]. Our system is implemented in C++ and OpenGL. All the examples in this paper were created using this system. You can download TopMod from http://www-viz.tamu.edu/faculty/ergun/research/topology/ for free and use the software for non-commercial applications. TopMod provides only Open-GL based interactive rendering. For high quality rendering, shapes can be exported in obj format and rendered in some 3D modeling and animation system. Our students use Maya and 3D Studio Max for rendering the final models.

5. Results

Figures 4 show a student (Cem Yuksel) work that illustrate creative flexibility that is provided by our guideline. Each figure show the steps that is used by Cem and final virtual sculptures. By adding handles after completing the basic guideline Cem created two conflicting forms that exists in the same sculpture. Lauren Simpson, to create the shape shown in Figure 5, added tetrahedral extrusion, which is stellation operation, before adding handles. This particular step allowed her to create handles with triangular cross-sections. Finally, rind modeling on handles with triangular cross-sections provided a simple and elegant connections. Audrey Wells' sculpture shown in 5 is based on octahedral extrusions that also allow her to create handles with both quadrilateral (thicker) and triangular (thinner) cross-sections.

6. Conclusions and Future Work

Sculpting and Architecture can provide major research directions for solid and shape modeling. In both, the precision, which is very important for engineering shape design, is not a major concern. In this work, we have introduced a new sculptural approach with a motivation coming from strong aesthetics concerns. Similar to results from computer graphics research, our results are reproducible and allow even novice users to create interesting sculptures with a creative flexibility. Because of their strong symmetry, these shapes can possibly be built using a few building blocks. We are currently investigating physical construction of large versions (medium size shapes such as ones larger than $1m^3$) of such complicated shapes using low-cost materials such as concrete.

We have used the terms such as family and conceptual form to introduce the idea. Unfortunately, such terms that are related to aesthetic or perception are not mathematically well-defined. An interesting research direction is to provide rigorous definitions of such aesthetic related terms using quantitative studies.

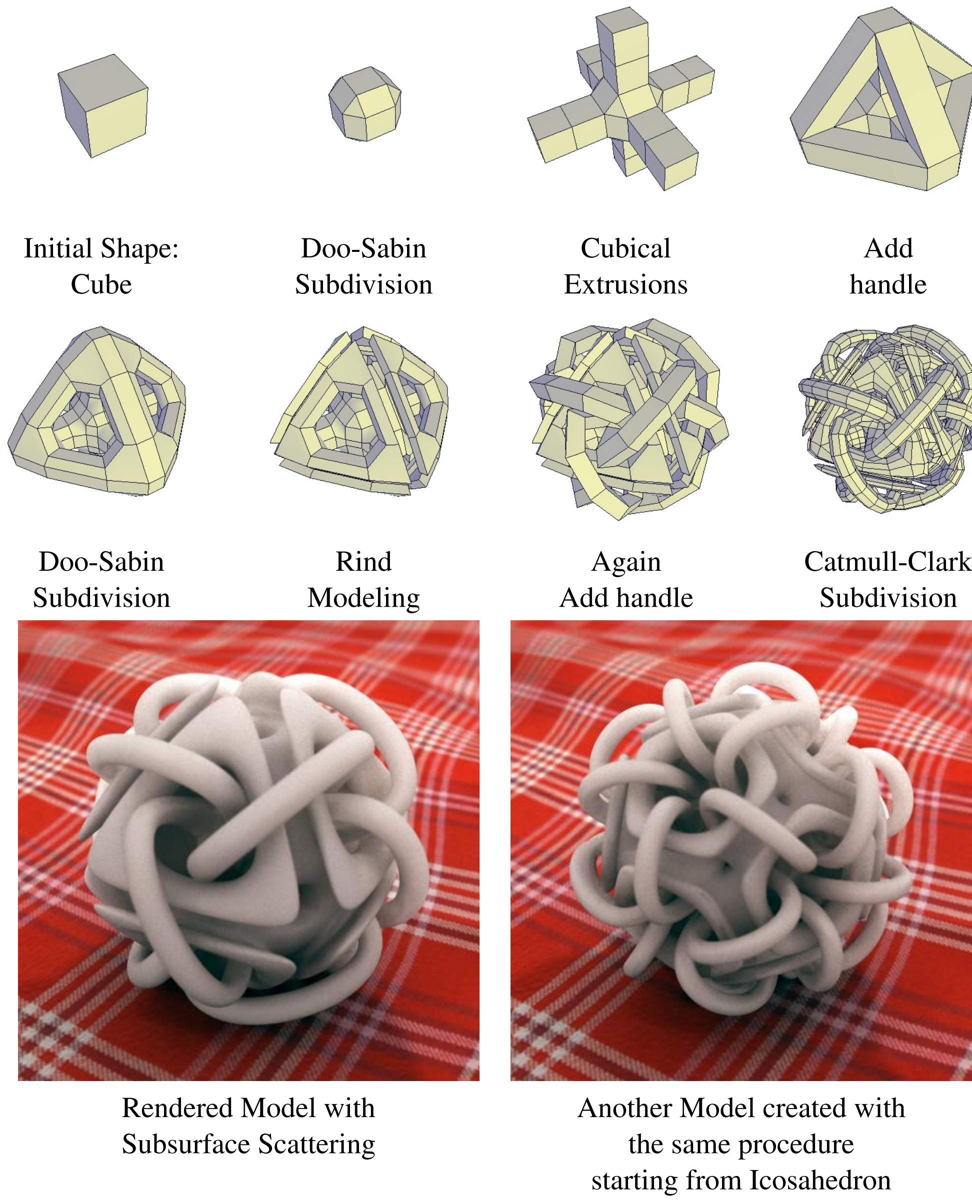


Figure 4: The Procedure used by Cem Yuksel and final virtual sculptures. Both rendered using subsurface scattering to give an illusion of ABS plastic.

References

[1] E. Akleman. Viza 656 - image synthesis; see volume rendering homeworks spring 2004-2005,. http://www-viz.tamu.edu/courses/viza656/index.html, 2005.

[2] E. Akleman. Viza 657 - computer aided sculpting,. http://www-viz.tamu.edu/courses/viza657/index.html, 2005.

[3] E. Akleman and J. Chen. Guaranteeing the 2-manifold property for meshes with doubly linked face list. *International Journal of Shape Modeling*, 5(2):149–177, 1999.

[4] E. Akleman and V. Srinivasan. Honeycomb subdivision. In *Proceedings of ISCIS'02, 17th International Symposium on Computer and Information Sciences*, volume 17, pages 137–141, November 2002.

[5] E. Akleman, V. Srinivasan, and J. Chen. Interactive rind modeling. In *Proceedings of the International Conference on Shape Modeling and Applications*, pages 23–31, 2003.

[6] E. Catmull and J. Clark. Recursively generated b-spline surfaces on arbitrary topological meshes. *Computer Aided Design*, (10):350–355, 1978.

[7] P. Cromwell. *Polyhedra*. Cambridge University Press, 1997.

[8] D. Doo and M. Sabin. Behavior of recursive subdivision surfaces near extraordinary points. *Computer Aided Design*, (10):356–360, 1978.

[9] J. Chen E. Akleman and V. Srinivasan. A minimal and complete set of operators for the development of robust manifold mesh modelers. *Graphical Models Journal, Special issue on International Conference on Shape Modeling and Applications 2002*, 65(2):286–304, 2003.

[10] H. Ferguson, A. Rockwood, and J. Cox. Topological design of sculptured surfaces. In *Proceedings of SIGGRAPH 1992*, Computer Graphics Proceedings, Annual Conference Series, pages 149–156. ACM, ACM Press / ACM SIGGRAPH, 1992.

[11] Heleman Ferguson. Sculptures,. http://www.helasculpt.com, 2005.

[12] Batsheba Grossman. Sculptures,. http://www.bathsheba.com/, 2005.

[13] B. Grunbaum and G. Shephard. *Tilings and Patterns*. W. H. Freeman and Co, NY, 1987.

[14] George Hart. Sculptures,. http://www.georgehart.com, 2005.

[15] E. Landreneau, E. Akleman, and V. Srinivasan. Local mesh operations, extrusions revisited. In *Proceedings of the International Conference on Shape Modeling and Applications*, pages 351 – 356, 2005.

[16] Carlo Sequin. Sculptures,. http://www.cs.berkeley.edu/ sequin/, 2005.

[17] V. Srinivasan, E. Akleman, , and J. Chen. Interactive construction of multi-segment curved handles. In *Proceedings of 10th Pacific Conference on Computer Graphics and Applications*, pages 429–435, 2002.

[18] I. Stewart. *Game, Set and Math: Enigmas and Conundrums*. Penguin Books, London, 1991.

[19] S. Takahashi, Y. Shinagawa, and T. L. Kunii. A feature-based approach for smooth surfaces. In *Proceedings of Fourth Symposium on Solid Modeling*, pages 97–110, 1997.

[20] W. Welch and A. Witkin. Free-form shape design using triangulated surfaces. In *Proceedings of SIGGRAPH 1994*, Computer Graphics Proceedings, Annual Conference Series, pages 247–256. ACM, ACM Press / ACM SIGGRAPH, 1994.

[21] D. Wells. *The Penguin Dictionary of Curious and Interesting Geometry*. London: Penguin, 1991.

[22] R. Williams. *The Geometrical Foundation of Natural Structures*. Dover Publications, Inc., 1972.

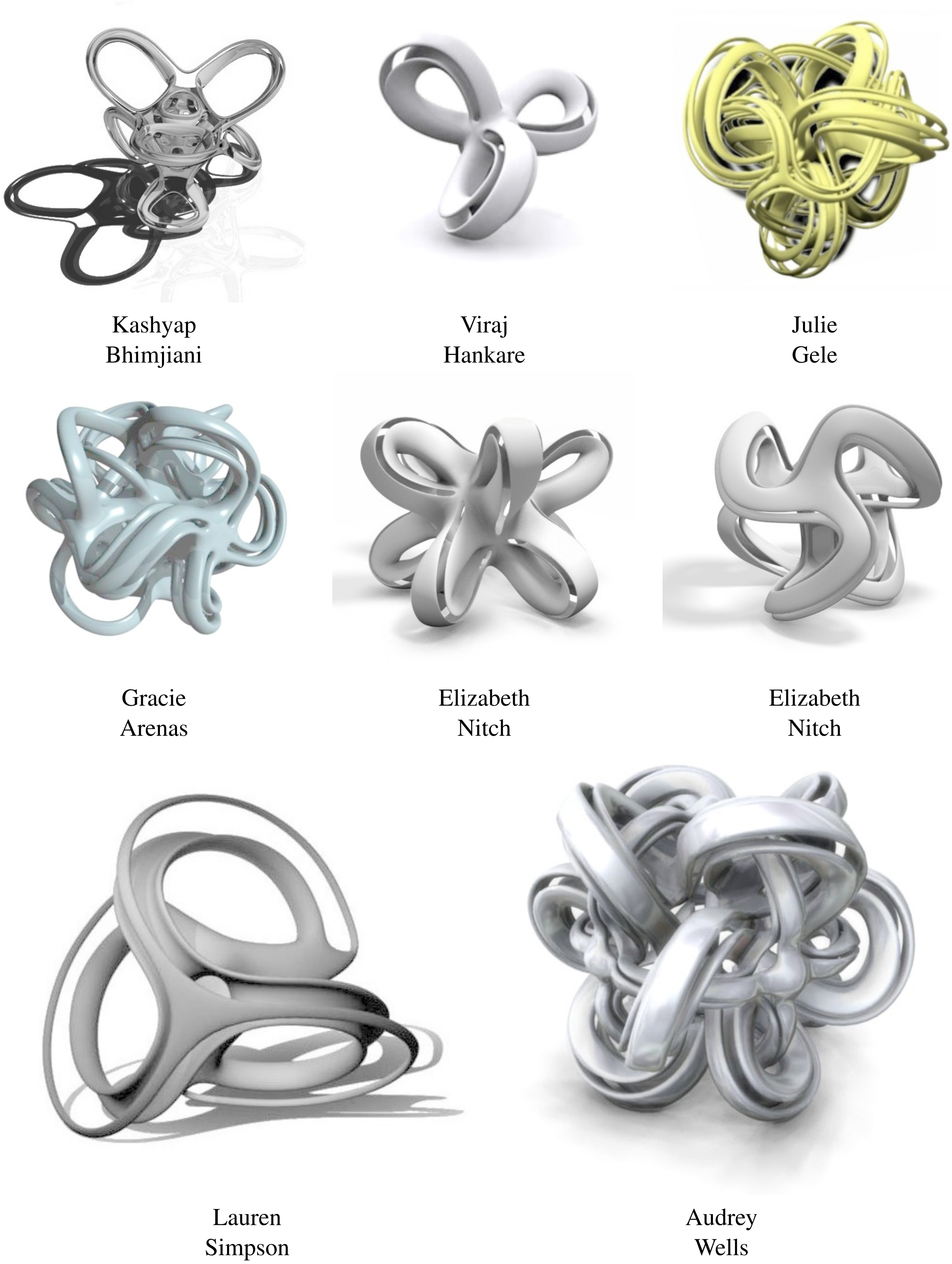


Figure 5: Rendered examples of high genus symmetric sculptures designed by students who take a computer aided sculpture course. Although, these images are created by seven different students; shapes and rendering styles are completely different, all the virtual sculptures can be perceived as same type.