Band Decomposition of 2-Manifold Meshes
For Physical Construction of Large Structures
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With the design and construction of more and more unusually shaped buildings, the computer graphics community has started to explore new methods to reduce the cost of the physical construction for large shapes. Most of currently suggested methods focus on reduction of the number of differently shaped components to reduce fabrication cost. In this work, we focus on physical construction using developable components such as thin metals or thick papers. In practice, for developable surfaces fabrication is economical even if each component is different. Such developable components can be manufactured fairly inexpensively by cutting large sheets of thin metals or thin paper using laser-cutters, which are now widely available.

Figure 1: This large sculpture of Bunny is constructed with laser cut poster-board papers assembled with brass fasteners.

Figure 2: Construction elements for bunny. (a) is an example of vertex component that is cut with laser cutter, (b) shows elements of vertex component and (c) shows the process of assembling vertex components with fasteners.

We observe that one of the biggest expenses for construction of large shapes comes from handling and assembling the large number components. This problem is like putting pieces of a large puzzle together. However, unlike puzzles we do not want construction process to be challenging. Instead, we want to simplify the construction process in such a way that the components can be assembled with a minimum instruction by the construction workers who may not have extensive experience.

In this work, we introduce an approach to automatically create such easily assembled developable components from any given manifold mesh. Our approach is based on classical Graph Rotation Systems (GRS). Each developable component, which we call vertex component, is a physical equivalent of a rotation at the vertex $v$ of a graph $G$. Each vertex component is a star shaped polygon that physically corresponds to the cyclic permutation of the edge-ends incident on $v$ (See Figure 2(a)). We engrave edge-numbers with laser-cutters directly on edge-ends of vertex components to simplify finding corresponding edge ends. When we print edge-numbers, we actually define a collection of rotations, one for each vertex in $G$. This is formally called a pure rotation system of a graph.

The fundamental Heffter-Edmunds theorem of GRS asserts that there is a bijective correspondence between the set of pure rotation systems of a graph and the set of equivalence classes of embeddings of the graph in the orientable surfaces. As a direct consequence of the theorem, to assemble the structure all construction workers have to do is to attach the corresponding edge-ends of vertex components. Once all the components are attached to each other, the whole structure will correctly be assembled.

Gauss-Bonnet theorem, moreover, asserts that the total Gaussian curvature of a surface is the Euler characteristics times $2\pi$. Since the structure is made up only developable components, Gaussian curvature is zero everywhere on the solid parts. The Gaussian curvature happens only in empty regions and that are determined uniquely. Since, we correctly form Gaussian curvature of holes, the structures will always be raised and formed in 3-space.

We also develop strategies to simplify finding corresponding pieces among a large number of vertex components. Using this approach, Architecture students have constructed a large version of Stanford Bunny (see Figure 1) in a design and fabrication course in College of Architecture. The costs of poster-board papers and fasteners were very minimal, less than $100. We are currently working on to construct even larger shapes using stronger materials. We are also planning to use the structures obtained by this approach as molds to cast large plaster or cement sculptures.

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