Local Mesh Operators: Extrusions Revisited

Eric Landreneau
Computer Science Department

Ergun Akleman*
Visualization Sciences Program

Vinod Srinivasan
Visualization Sciences Program

Texas A&M University

Abstract

In this paper, we present a set of generalized "local" mesh operators. Local operators are those that operate on a single face without affecting the rest of the mesh. Boundary edges of the chosen face also stay the same. We have identified two types of local operators: (1) Extrusions that create generalized pipes in which bottom and top polygons have the same number of sides, and (2) Stellations that create generalized pyramids, where there is a top vertex instead of top polygon. Our operators can create extrusions that are regular polyhedra including dodecahedron, icosahedron, octahedron and tetrahedron. The tetrahedron is created using the stellation operator, which is also useful to create generalized versions of Kepler and Poinsot solids.

1. Introduction and Motivation

Classical extrusion (see Figure 1A) is a local mesh operator. We call it a local operator since it can be applied to one face of the mesh without affecting the rest. Classical extrusion has always been one of the most under-rated yet commonly-used operators in modeling. With the advent of Catmull-Clark subdivision [2], classical extrusion became especially useful, since it creates only quadrilaterals when applied to quadrilateral meshes (see Figure 1A). This property is particularly suitable for Catmull-Clark [2, 11] and Doo-Sabin [4, 8] subdivision schemes which work best with quadrilateral control meshes.

In this paper, we have developed new local mesh operators that can be used to create a wide variety of control

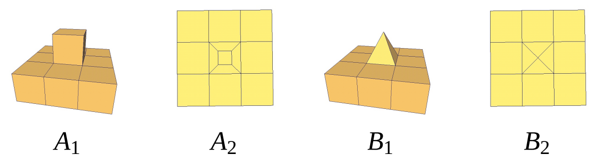


Figure 1. A_1 **Cubical (classical) extrusion and** A_2 **its Shegel diagram.** B_1 **Tetrahedral extrusion (Stellation) and** B_2 **its Shegel diagram.**

meshes suitable for various subdivisions such as Loop [6], Simplest [7], $\sqrt{3}$ subdivision [5] or Pentagonal subdivision [1]. See Figures 14 and 15 for some examples of using extrusions to create control meshes for subdivision surfaces. Our local operators come from a simple observation that it is possible to view local extrusion operations as pasting generalized versions of platonic solids [9, 10] to a face.

Based on this observation, we have identified three new local operators. For consistency we have also renamed two existing local operators. The names come directly from the regular solids [10] they create. The Figures 1, 2 and 3 show how these five extrusions affect a square. The right images are Schegel diagrams of extrusions [10].

- CUBICAL EXTRUSION. This is the classical extrusion that pastes a generalized cube to a face. When applied to a square face, it can give a cube as shown in Figure 1A. We have introduced a new way of calculating positions of the newly created vertices such that the end face of the extrusion approaches a planar convex shape regardless of the shape of initial polygon.
- TETRAHEDRAL EXTRUSION This is a local operator that is provided by most modeling software. This operator is also known as stellation in polyhedral modeling [10]. It pastes a generalized tetrahedron to a face (see Figure 1B). When applied to a triangle, it creates

* Corresponding Author: Visualization Sciences Program, Department of Architecture. Address: 216 Langford Center, College Station, Texas 77843-3137. email: ergun@viz.tamu.edu. phone: +(979) 845-6599. fax: +(979) 845-4491.

a tetrahedron. We have introduced a variety of new usages for this operator including the double stellation shown in Figure 2B).

- OCTAHEDRAL EXTRUSION This is a new operator that pastes a generalized octahedron to a face (see Figure 2A). When applied to a triangle, this operator produces an octahedron.
- DODECAHEDRAL EXTRUSION This is a new operator that pastes a generalized dodecahedron to a face (see Figure 3A). This operator creates a dodecahedron when applied to a pentagon.
- ICOSAHEDRAL EXTRUSION pastes a generalized icosahedron to a face (see Figure 3B). When a applied to a triangle, it produces an icosahedron.

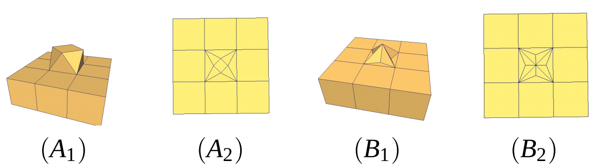


Figure 2. (A_1) **Octahedral extrusion and** (A_2) **its Shegel diagram.** (B_1) **Double stellation and** (B_2) **its Shegel diagram.**

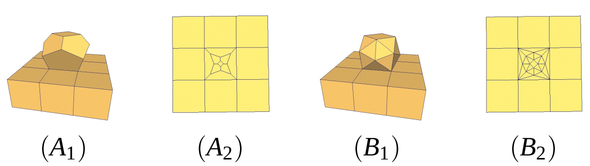


Figure 3. (A_1) **Dodecahedral extrusion and** (A_2) **its Shegel diagram.** (B_1) **Icosahedral extrusion and** (B_2) **its Shegel diagram.**

1.1. Additional Contributions

In addition to new extrusions, this paper also introduces new control parameters for these operators.

- *Relative Size.* In our extrusions, the edge lengths of the extruded shape are a function of the average edge length of the extruded polygon. This approach leads to the size of extruded shape becoming a function of the size of extruded polygon. Since this approach is similar to the replacement approach in fractal geometry, our local operations can be used to create fractal shapes.
- *Polygon Smoothing.* For cubical and octahedral extrusions, we have introduced new controls that are based on the control parameters for the Doo-Sabin subdivision scheme. Using successive applications of Doo-Sabin subdivision, the top surface of the extrusion approaches a planar convex shape regardless of initial polygon.
- Platonic Solid Controls. For octahedral and icosahedral extrusions, we have introduced radically different control parameters. Instead of controlling the size of extruded face, we control dihedral angles and edge lengths. This approach allows us to (1) create regular polyhedra for default values, (2) guarantee planar faces for dodecahedral extrusion, and (3) generate a wide variety of shapes.

1.2. Test-Beds

To illustrate how these extrusions work, we applied the new local operators to the test-beds shown in Figure 4.

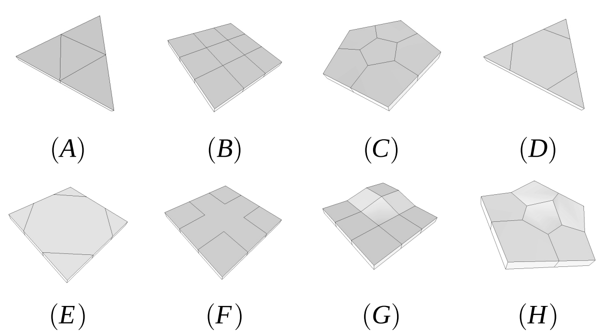


Figure 4. The block shaped test-bed meshes. The blocks in the first row includes regular planar polygons: (A) **equilateral triangle,** (B) **square,** (C) **equilateral pentagon and** (D) **equilateral hexagon.** (E) **includes a non-equilateral but convex octagon,** (F) **includes a non-convex 12-gon.** (G) **and** (H) **include non-convex quadrilateral and pentagon.**

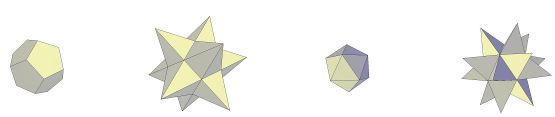
2. Tetrahedral Extrusion

Let f be a face with k vertices. The tetrahedral extrusion, when applied to f, deletes face f and creates k new triangles that form a generalized pyramid.

Tetrahedral extrusion can also be called stellation since for $s = 1$ it can create regular stellated solids (Kepler solids[1]) when applied to all the faces of a dodecahedron and an icosahedron (see Figure 5A and B). Using $s = -1$, tetrahedral extrusion can also create the Great Dodecahedron, a regular Poinsot solid (see Figure 6A).

Double stellation is the application of stellation operation twice. In double stellation, the first operation creates a pyramid by using a positive s value. Then, a second stellation with a negative s value is applied all newly created triangles. The s values for the second stellation are chosen in such a way that the tips of the newly created inverted pyramids (tetrahedrons) return back to a planar surface that approximates the initial polygon f.

When double stellation is applied to a face, it creates a star shaped extrusion as shown in Figure 6B. When applied to all faces of a dodecahedron, it produces the Great Icosahedron, another Poinsot solid. Using this concept, we can also create generalized versions of Kepler and Poinsot solids.

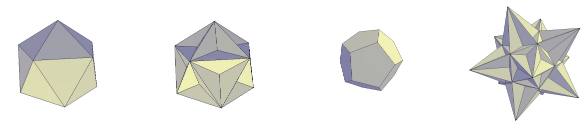


(A_1) (A_2) (B_1) (B_2)

Figure 5. (A_1) **Dodecahedron and** (A_2) **Small Stellated Dodecahedron created by applying stellation operator to all faces of the dodecahedron.** (B_1) **Icosahedron and** (B_2) **Great Stellated Dodecahedron created by by applying stellation operator to all faces of the icosahedron.**

3. Cubical Extrusion with Doo-Sabin Smoothing

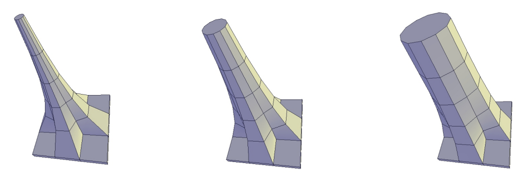
This is a useful extrusion since it converts non-planar and/or non-convex faces into convex and planar faces as shown in Figure 7. Topologically, this operation is the same as cubical extrusion except for the computation of vertex positions of the newly created polygon. Figure 7 illustrates the effect of Doo-Sabin smoothing in extrusions. As seen in in these examples, with Doo-Sabin smoothing a few ap-



(A_1) (A_2) (B_1) (B_2)

Figure 6. (A_1) **Icosahedron and** (A_2) **Great Dodecahedron created by applying stellation operator with negative** s **to all faces of the icosahedron.** (B_1) **Dodecahedron and** (A_2) **Great Icosahedron created by applying double stellation operator to all faces of the dodecahedron.**

plications of cubical extrusion create a convex, almost planar and regular looking top polygon regardless of the shape of initial polygon.



$s_1 = 1.25$ $s_1 = 1.5$ $s_1 = 1.75$

Figure 7. An example of application of Doo-Sabin cubical extrusion to a non-convex polygon. In this case, we used the 12-gon shown in Figure 4F. We applied Doo-Sabin cubical extrusion 5 **times. The height parameter** s_2 **is the same for all four examples.**

4. Octahedral Extrusion with Doo-Sabin Smoothing

This is a useful operation since it can create structures that are stable but appear to be unstable as seen in Figure 8. In octahedral extrusion, the user can control the size and height of the extrusion with two control parameters, s_1 and s_2. Figure 9 shows an extension of octahedral extrusion that is created by automatically eliminating the edges of intermediate polygons. With Doo-Sabin smoothing a few applications of octahedral extrusion create a convex, almost planar and regular looking top polygon regardless of the shape of initial polygon.

1 Note that these are not real Kepler or Poinsot solids. The real ones self-intersect.

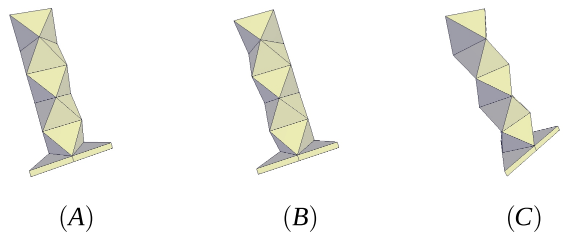


Figure 8. Three views of a shape that is created by applying five iterations of octahedral extrusion to an equilateral triangle.

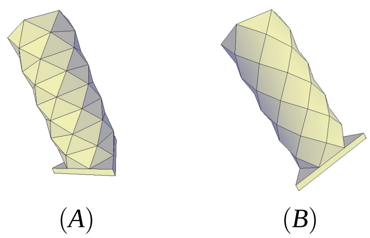


Figure 9. (A) **A shape created by applying a series of octahedral extrusions to a regular hexagon.** (B) **is created by automatically eliminating the edges of intermediate polygons. This design is common in some architectural towers.**

5. Dodecahedral Extrusion

The algorithm for dodecahedral extrusion involves the creation of three "tiers" of vertices which are then connected appropriately to produce the final extrusion (see Figure 10). An angle θ controls dihedral angle of extruded shape and three real numbers s_1, s_2 and s_3 control the positions of the first, second and third tier of vertices. The default values are $\theta = \arccos(-\frac{\sqrt{5}}{5}) \approx 116.5$ and $s_1 = s_2 = s_3 = 1$. These are chosen such that the operator creates a regular dodecahedron when applied to a regular pentagon. Figures 11 and 12 show examples of dodecahedral extrusion applied to a variety of polygons.

6. Icosahedral Extrusion

The algorithm for icosahedral extrusion also involves the creation of three "tiers" of vertices which are then connected appropriately to produce the final extrusion (see Figure 13). Similar to dodecahedral case, an angle θ controls dihedral angle of extruded shape and three real numbers s_1,

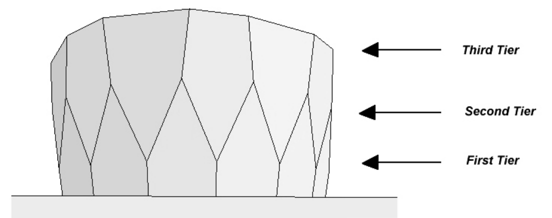


Figure 10. First, second and third tier of vertices in an dodecahedral extrusion.

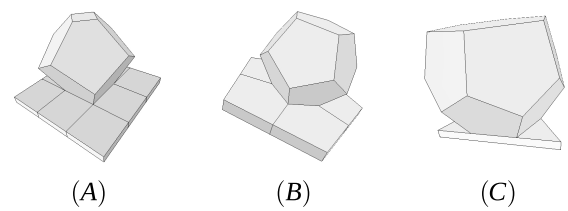


Figure 11. A shape created by applying dodecahedral extrusion to (A) **a square;** (B) **an equilateral pentagon;** (B) **an equilateral pentagon. In these examples** $\theta = 118^0$**,** $s_1 = s_2 = s_3 = 1$**.);**

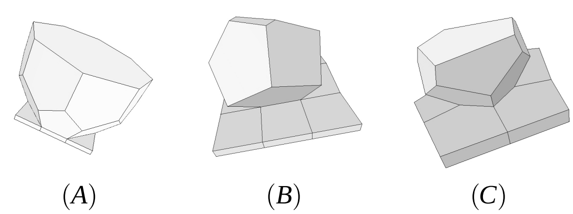


Figure 12. A shape created by applying dodecahedral extrusion to (A) **an octagon (**$\theta = 118^0, s_1 = s_2 = s_3 = 1$**);** (B) **a non-planar quadrilateral (**$\theta = 115^0, s_1 = s_2 = s_3 = 1$**);** (C) **an non-planar pentagon (**$\theta = 118^0, s_1 = 1, s_2 = 0.5, s_3 = 0.5$**).**

s_2 and s_3 control the positions of the first, second and third tier vertices. The default values are $\theta = \arccos(-\frac{\sqrt{5}}{3}) \approx 138.2$ and $s_1 = s_2 = s_3 = 1$. These are chosen such that the operator creates a regular icosahedron when applied to a equilateral triangle. Icosahedral extrusion has turned out to be a very interesting operator. We have been able to create unexpected patterns such as the flower patterns shown in Figure 15 by varying the control parameters. Such patterns can be created with successive application of icosahedral extrusion with $s_1 \approx s_2 \gg s_3$.

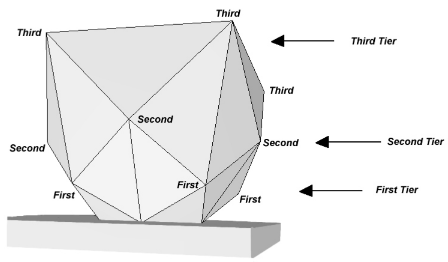


Figure 13. First, second and third tier of vertices in an icosahedral extrusion.

7. Conclusions and Future Work

In this paper we have presented a set of generalized "local" mesh operators that may also be called Regular (Platonic) extrusions. We have introduced a wide variety of ideas that can further be extended as follows:

- *Semi-Regular (Archimedean) Extrusions:* It is possible to extend Platonic extrusions to Archimedean extrusions simply using vertex truncations of extruded solids. The Truncated Octahedron can especially be useful since it is a space filling solid. By applying Truncated Octahedral extrusions to a truncated octahedron, a finite portion of the infinite genus regular solid $(6,4)$ [3] can be obtained. Similarly, using Truncated Tetrahedral extrusions applied to a Truncated Tetrahedron, it is possible to create a subset of the infinite genus regular solid $(6,6)$ [3].
- *Dodecahedral and Icosahedral Extrusions with Doo-Sabin Smoothing:* One problem with angle control is that dodecahedral and icosahedral extrusions cannot be applied directly to non-convex polygons. Thus, instead Doo-Sabin smoothing can be used for Dodecahedral and Icosahedral extrusions. In Dodecahedron case, it may be possible to apply Doo-Sabin smoothing to a group of vertices that are not necessarily in the same tier. Although, we may not necessarily obtain exact planarity, it may be possible to obtain faces that approach planar faces. In Icosahedron case, Doo-Sabin Smoothing can easily be applied to each tier of vertices providing a total of 6 controls to the user.
- *Cubical Extrusions with Planarity Constraint:* Planarity constraint used in Dodecahedral extrusions can also be applied to Cubical extrusions and each face of the extrusion can easily be made planar.
- *Octahedral Extrusions with Angle Control:* Although using the current approach, the user can approximate a regular octahedron, using angle control and relative scaling, it is possible to create precisely regular octahedrons.

These new operators are particularly useful for designing architectural shapes. However, their usage is not limited only to architecture. For instance, since these operators allow to model manifold polygonal meshes with low polygon counts that can be smoothed to create molecule-like shapes as illustrated in Figure 14, extrusions can especially be useful for modeling molecular shapes.

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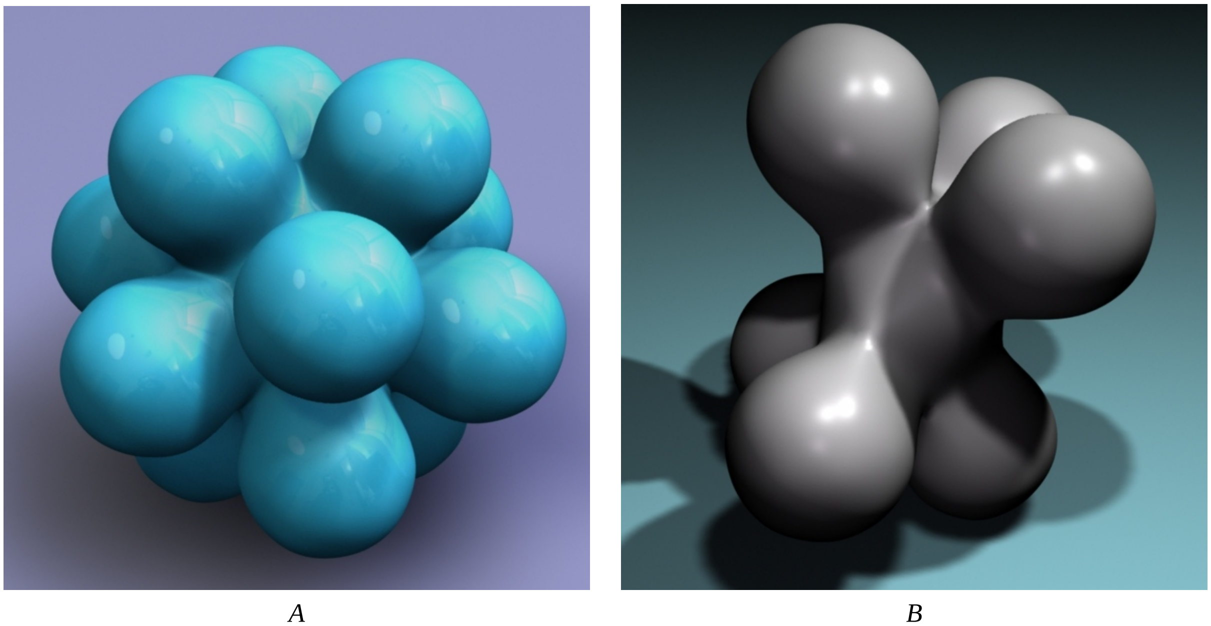

Figure 14. Examples of subdivision smoothed (A) dodecahedral (B) icosahedral extruded surfaces that can provide molecular-like patterns.

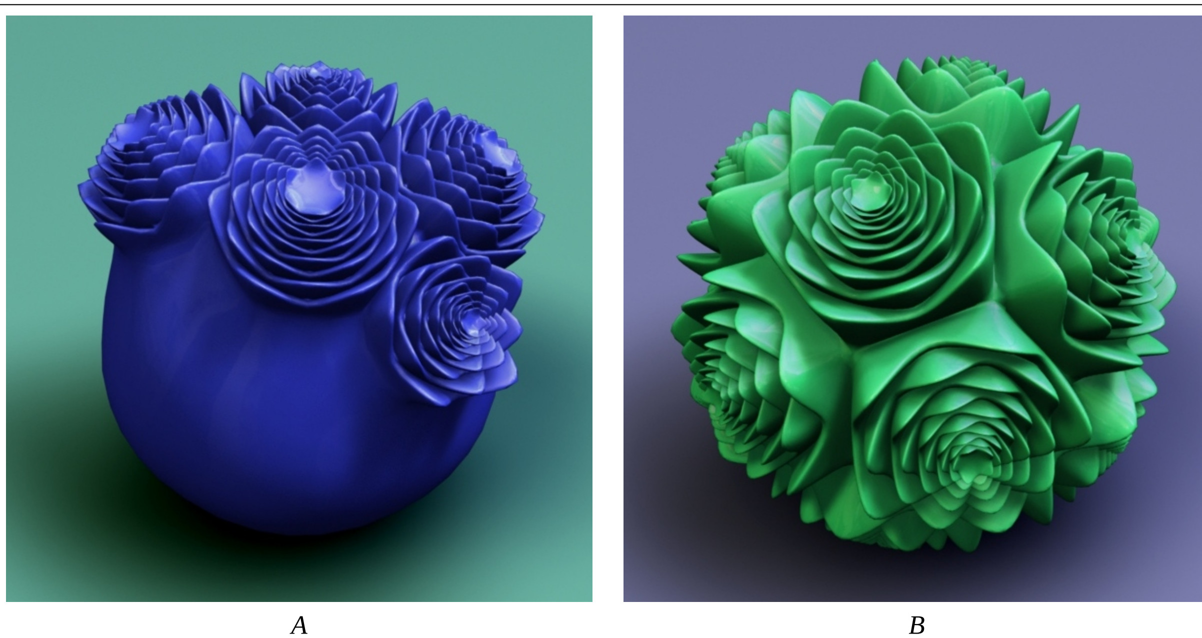

Figure 15. Examples of smoothed icosahedral extruded surfaces that can provide plant-like patterns.