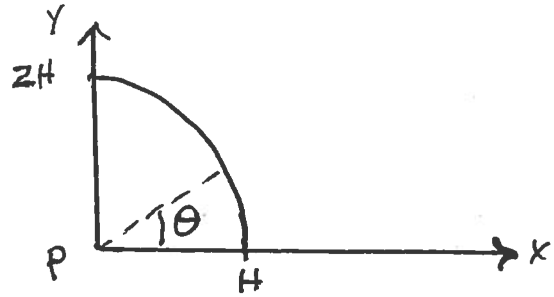
2. (25 points)A small object of mass m moves along a horizontal track. Its distance from some point P is a function of θ, the angular position, and is given by $2H - H\cos\theta$ as shown below. The object has an angular acceleration about P given by $c_1 t$ where c_1 is a known constant. Find the object's angular momentum about the point P and the object's acceleration as a function of time if it is at rest at time $t = 0$ at the position $\theta = 0$.



Free Body Diagrams (If appropriate). Law or Definition

$$r(t) = 2H - H\cos\theta \qquad a_r = \frac{d^2r}{dt^2} - r\omega^2$$

$$\alpha(t) = c_1 t \qquad a_\theta = 2\frac{dr}{dt}\omega + r\alpha$$

$$\vec{L} = \vec{r}\times\vec{p} = mr^2\omega\,(\hat{r}\hat{r})$$

Application

$$\omega = \int \alpha\, dt = \int c_1 t\, dt = \boxed{\frac{c_1 t^2}{2}} + \overset{=0}{\text{Const}} \qquad \theta(t) = \int \frac{c_1 t^2}{2}dt = \frac{c_1 t^3}{6}$$

$$\boxed{a_r = \frac{d^2r}{dt^2} - r\omega^2 \qquad a_\theta = 2\frac{dr}{dt}\omega + r\alpha} \qquad \boxed{\vec{L} = m(2H - H\cos\theta)^2 \frac{c_1 t^2}{2} \odot}$$

$$\frac{dr}{dt} = +H\sin\theta\frac{d\theta}{dt} = +H\sin\theta\,\omega = \boxed{+H\sin\theta\frac{c_1 t^2}{2}}$$

$$\frac{d^2r}{dt^2} = +H\cos\theta\frac{d\theta}{dt}\omega + H\sin\theta\frac{d\omega}{dt} =$$

$$= \boxed{+H\cos\theta\,\omega^2 + H\sin\theta\,\alpha}$$

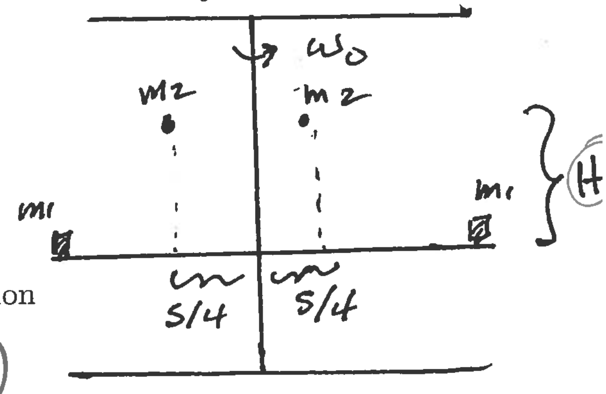
Result

$$a_r = +H\cos\theta\frac{c_1^2 t^4}{4} + H\sin\theta\, c_1 t - (2H - H\cos\theta)\frac{c_1^2 t^4}{4}$$

$$a_\theta = +2H\sin\theta\left(\frac{c_1 t^2}{2}\right)^2 + (2H - H\cos\theta)c_1 t \,; \quad \boxed{\theta = \frac{c_1 t^3}{6}}$$

e fixed already

3. (25 points)A vertical axle is free to rotate. A massless, horizontal rod of length S is attached at its center to the axle as shown. There are two small objects attached to the rod. One of the objects has mass m_1 and is fixed at the end of the rod. The other has mass m_1 and is fixed at the other end of the rod. The axle is given an angular velocity ω_0. At a time defined to be $t = 0$ two small objects, each having mass m_2, are dropped from a height H so that they hit and stick on the rod at the points shown. What will be the angular velocity of the rotating system after the objects hit the rod?



Free Body Diagrams (If appropriate). Law or Definition

$$\vec{\tau}_{ext} = \frac{d\vec{L}_{tot}}{dt} \; ; \quad L_z(\text{initial}) = L_z(\text{final})$$

Application

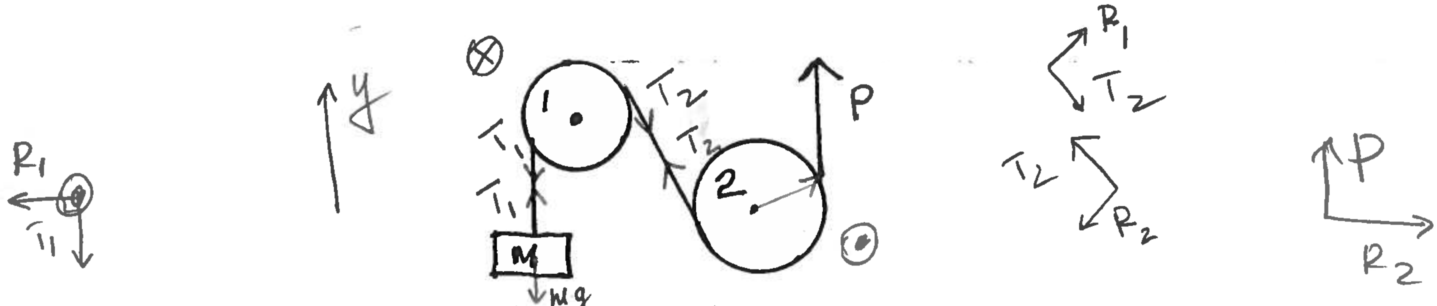
$$m_1\left(\frac{S}{2}\right)^2 \omega_0 + m_1\left(\frac{S}{2}\right)^2 \omega_0 = m_1\left(\frac{S}{2}\right)^2 \omega_{new} + m_1\left(\frac{S}{2}\right)^2 \omega_{new} + 2m_2\left(\frac{S}{4}\right)^2 \omega_{new}$$

$$\omega_{new} = \frac{2m_1\left(\frac{S}{2}\right)^2}{2m_1\left(\frac{S}{2}\right)^2 + 2m_2\left(\frac{S}{4}\right)^2}\,\omega_0 \quad \text{CCW}$$

$$= \frac{m_1}{m_1 + \frac{m_2}{4}}\,\omega_0 \quad \text{CCW}$$

Result

4. (25 points)A block of mass M is suspended by a massless, unstretchable rope from the pulley system shown. The pulleys rotate freely about fixed horizontal axles. The rope moves around the pulleys without slipping. Pulley 1 has a moment of inertia about its center of I_1 and a radius R_1. Pulley 2 has a moment of inertia about its center of I_2 and a radius R_2. If a constant force of magnitude $\vec{P}$ is applied to the rope as shown, what will be the acceleration of the block?



Free Body Diagrams (If appropriate). Law or Definition

$$F_x = m a_x \qquad a_{cm} = R\alpha \ (rhr)$$

$$F_y = m a_y$$

$$\vec{\tau}_{ext} = I\alpha \ (rhr)$$

Application

$$\begin{cases} T_1 - mg = m a_y \ (1) \\ -T_1 R_1 + T_2 R_1 = I_1 \alpha_1 \ (2) \\ -R_2 T_2 + R_2 P = I_2 \alpha_2 \ (3) \\ a_y = R_1 \alpha_1 \ (4) \\ a_y = R_2 \alpha_2 \ (5) \end{cases}$$

$$T_2 - T_1 = \frac{I_1 a_y}{R_1^2}$$

$$+ \quad P - T_2 = \frac{I_2 a_y}{R_2^2}$$

$$P - T_1 = \left(\frac{I_1}{R_1^2} + \frac{I_2}{R_2^2}\right) a_y$$

$$T_1 = P - \left(\frac{I_1}{R_1^2} + \frac{I_2}{R_2^2}\right) a_y$$

$$P - \left(\frac{I}{R_1^2} + \frac{I_2}{R_2^2}\right) a_y - mg = m a_y$$

Result

$$\boxed{a_y = \frac{P - mg}{\frac{I_1}{R_1^2} + \frac{I_2}{R_2^2} + m}}$$