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Beamforming – DOA Estimation for a Uniform Linear Array

ECE-S435 Final Project

3/21/2008
1. INTRODUCTION

I did my final statistical DSP project on beamforming to find the Direction Of Arrival (DOA) estimation of the received signal from an antenna so that it can be transmitted back to the same location from the transmitting antenna. Beamforming for DOA estimation is part of my senior design project which is to build a prototype of an ultra wide band (UWB) base station operating at a frequency of 3.42 GHz. For a better explanation of what I am trying to accomplish with regards to beamforming, Figure 1 shows a block diagram of the UWB base station we are building for our senior design project (ECE-25). Basically, the received signal from the receiving antenna is connected to the receiver board where the analog signal processing is done and then the digitized signal from the analog-to-digital convertor (ADC) is sent to the FPGA board which does the signal processing. The main algorithm that the FPGA board performs is multi-beamforming to compute the angle of arrival of the received signal so that the processed signal can be sent back to the source using the transmitting antenna with the beams directed at the desired direction of arrival.
Propagating signals contain a lot of information about the sources that produce them. This information includes the temporal and spatial characteristics of the source signal which allows the determination of the source location. This process is called beamforming. The receiver antenna as shown in Figure 1 picks up not just the desired signal but also several other sources in addition to the one of our desired interest and the desired signal is also contaminated by noise. Therefore, beamforming is a signal processing method which gives us a way to focus on the selected signal. This is called spatiotemporal filtering which separates signals according to their direction of propagation. Knowledge of antenna array geometry and its design along with its directivity pattern or beam pattern is essential in beamforming.

Beamforming is very important in a base station because the beam is enabled in the desired direction and the beam pattern of the antenna array can therefore be changed without physically changing the antenna. The algorithm points the antenna array’s beam towards desired directions algorithmically rather than physically. Beamforming generally performs the same operations on an antenna array irrespective of the number of sensors in the antenna array or the amount of noise present in the wave field. Figure 2 shows a general setup of how beamforming works. The star represents a source signal and the black circles represent antenna array elements. Since they are located in a linear fashion, this array geometry is called uniform linear array (ULA) geometry. The area shaded in green represents the correct region, the area represented in yellow represents a small margin of error while the areas in red represent incorrect values. Beamforming computes the desired angle of incidence of the source signal on the antenna arrays so that the processed beam can be directed to the same source at the computed angle of arrival. The parameter that is set while performing beamforming is that the source signal is in the plane in front of the array.
For this final project in DSP, I used uniform linear array (ULA) antenna geometry with 16 antenna elements as shown in Figure 3. M1, M2, ..., M16 represents the individual antenna elements with a uniform distance d between each element and S represents the source signal. Since the antenna array structure is linear, the field we are dealing with is a two dimensional field as shown in the Figure 3. The source signal is incident on the receiving antenna at an angle of incidence $\theta$ with respect to the y-axis with the center of each antenna elements lying on the x-axis. Taking the first antenna element M1 as the reference point, the amount of phase shift at each antenna element can be calculated using the geometry of the uniform linear array. This basically means that the source signal takes extra time to reach each antenna element relative to the first element. Then, time delays are added so that all the signals are perfectly in phase with each other. The question then arises of the amount of time delay to be added. To solve this problem, an iterative approach is used where all the time delays for all possible directions are tested and at the correct value of the delay, the signal is applied and at the rest of the delays, the signals will interfere resulting in a diminished output signal. The resulting output signal’s power is plotted as a function of the incident angle $\theta$ and the value of the angle at which the power output is maximum shows that the beam pattern for the signal arrived from that angle $\theta_0$.

Some other aspects of the antenna design are also important in beamforming. An antenna basically converts electrical signals from the transmitter to electromagnetic waves and vice-versa at the receiver end. The antenna consists of some materials that create electric and magnetic fields in the space around them and if the fields change, they propagate through space as an electromagnetic field at the speed of light ($c = 3 \times 10^8$ m/s). The dimensions of an antenna are specified in terms of wavelength of the radio signal between transmitted and received antennas ($\lambda = c/f$, where f is the carrier frequency of the radio signal). In our senior design project, we are building a UWB base station operating at a frequency of 3.432 GHz. Therefore, the wavelength I used in my beamforming algorithm is $\lambda = c/f = 3e8/3.432e9 = 0.0874$ m/s. The distance between antenna elements of the 16-element uniform linear array was taken to be half wavelength ($d = 0.0437$ m) as per the design of the antenna in our senior design project. The delay of the signal arriving at each element is then equal to $k = 2\pi/\lambda$. In beamforming, the phases of the signals from the array elements is measured and the DOA is calculated from these measurements. Consider the ULA diagram in the Figure 4:
Consider the source signal arriving at an angle of incidence $\theta$ which is incident first on the first left most antenna element. The dotted line shown represents the wavefront which is the locus of point which have the same phase difference. The same signal then arrives at the second antenna element after travelling an additional path distance $\Delta l$. Using geometry, it is seen that $$\sin \theta = \frac{\Delta l}{d} \Rightarrow \Delta l = d \sin \theta.$$ This path difference leads to a phase difference $\Delta \Phi$ between the signals arriving at the two antennas. Therefore, the propagation delay between elements is:

$$\Delta \Phi = \frac{2\pi}{\lambda} \cdot \frac{\Delta l}{d} = \frac{2\pi d \sin \theta}{\lambda}.$$ 

In the same way, for $L$th element in a ULA consisting of $L$ elements, the phase difference is equal to

$$\Delta \Phi = \frac{2\pi}{\lambda} \cdot \frac{(L-1) \Delta l}{d} = \frac{(L-1)(2\pi d \sin \theta)}{\lambda}.$$ (See Figure 5). The incident signal first impinges on the right-most element in this case and travels an additional path of $d \sin \theta$ to reach each subsequent element.

The angle of arrival can then be calculated from the phase difference:

$$\theta = \sin^{-1} \left( \frac{\Delta \Phi \lambda}{2\pi d} \right).$$

Therefore, the steps in beamforming for DOA estimation are to first find the steering vector $a(\theta)$ of the antenna array. A signal at an angle $\theta$ results in a scalar multiple of the steering vector. For an antenna array with $L$ elements, the steering vector is equal to $a(\theta) = [a_1(\theta), a_2(\theta), ..., a_L(\theta)]^T$ and for a ULA geometry, the steering vector is equal to

$$a_{ULA}(\theta) = [1, e^{jk d \cos \theta}, ..., e^{-j(L-1)k d \cos \theta}]^T.$$
where \( L \) is the number of elements in the antenna array, \( d \) is the distance between the elements in the antenna array, \( \theta \) is the angle of incidence of the desired signal, and \( k \) is the delay between each element. This gives the steering vector of the signal incident on all of the antenna elements.

The problem of estimating the DOA of the source signal impinging on a receiving antenna \( x(t) \) can be simplified by taking a finite data set instead of having infinite observation time. In practice, only sample estimates of the data are available and the second step in DOA estimation is to find the sample covariance matrix \( R \) of the input data. The covariance matrix of the input signal on all the antenna elements reveals information about the inter-effect among the antenna elements. In DOA estimation, the array is steered in one direction at a time and the power at that angle is calculated. Therefore, the power in all the directions from \( \theta = 0 \) to \( 180^\circ \) is calculated and the steering location which results in maximum power yields the DOA estimate.

The different signal sources are modeled as independent of each other as well as independent of the noise in DOA estimation. The array response is steered by forming a combination of the output and the power is equal to \( P(w) = W^H R W \), where \( R \) is the sample covariance matrix and \( W \) is the weighting vector with \( W^H \) being its Hermetian matrix. The weighting vector \( W \) is calculated using different beamforming techniques like Bartlett’s algorithm, MUSIC algorithm, ESPRIT algorithm to name a few.

For my project, I chose Bartlett’s algorithm which is also called Conventional Beamforming (CBF) because of its robustness to noise and computational ease. This algorithm gives a good estimation of the DOA even when the source signal is corrupted with interference and noise. The weight vector in Bartlett’s algorithm is given by

\[
W_{BF} = \frac{a(\theta)}{\sqrt{a''(\theta)a(\theta)}}
\]

and therefore, the power of the signal at a particular direction \( \theta \) is equal to

\[
P_{BF}(\theta) = \frac{a''(\theta)\hat{R}a(\theta)}{a''(\theta)a(\theta)}.
\]

The power of the signal is found by sweeping the angle \( \theta \) in all directions from 0 to \( 180^\circ \) and the direction at which the power is maximum gives the DOA of the received signal.

2. RESULTS

The uniform linear array I used for the project has 16 elements and an inter-element spacing of \( d = \text{lambda}/2 = 0.0437 \) meters. The input signal is first generated which is incident on each element of the antenna array. To test if the program works correctly, the input signal was generated at an angle of \( 89^\circ \). Then, additive Gaussian noise (AWGN) was generated with an amplitude of 1 and six interfering signals or jammers were generated with random arrival angles from 0 to \( 180^\circ \) and random amplitude till 5. These signals were summed up and the resulting signal was the incident signal on the receiving antenna.

The algorithm was tested by filtering the corrupted signal from noise as well as estimating the DOA on the unfiltered signal as well. To make it easier, a matched filter implementation was used to filter the corrupted signal from AWGN. The covariance matrix of
both the filtered and the unfiltered signal was first found and then the next step was to
generate the steering vector for the 16 element array at each incident angle theta from 0 to
180°. The power at each value of theta was found using the earlier mentioned formula given
here again:

\[ P_{BF}(\theta) = \frac{\mathbf{a}(\theta)\mathbf{R}(\theta)\mathbf{a}^H(\theta)}{\mathbf{a}(\theta)\mathbf{a}^H(\theta)} . \]

The maximum value of power was found for both the filtered and the unfiltered signal case and
the plot of the power in dB versus the incident angle theta was plotted on one graph for both
the filtered and unfiltered source signal case. The plot obtained is shown in Figure 6:

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{plot.png}
\caption{Plot of Power Output vs. Theta. DOA is 89}
\end{figure}

The above plot shows the maximum value of power which is at the angle of incidence \( \theta = 89^\circ \) which is correct since the input signal was generated at an angle of 89°. To test the code,
the program was run for 20 times to find out the probability of error of the DOA algorithm. The
program worked correctly by estimating the exact value of DOA more than 90% of the time
with producing a +/- 1° variation in the correct DOA estimate the rest of the time. Appendix A
gives the m-file for this program and the MATLAB output.

3. FUTURE WORK

This concept of beamforming for DOA estimation can be extended to a planar array
geometry as well where the angle is estimated in a three dimensional system in terms of both
elevation angle \( \theta \) and azimuth angle \( \Phi \) as shown in Figure 7. The planar array shown is a 6x2...
array with 6 elements in the x-direction and 2 elements in the y-direction. The distance between the antenna elements in the x-direction is \( dx \) and in the y-direction is \( dy \).

Then, taking the very first leftmost corner element as the reference element, i.e., the source signal is first incident on this element. Then, there is a phase delay in the source signal arriving at the other elements relative to this reference element. The phase delay in the x-direction is given by 
\[
\Psi_X = -k_X d_X = \frac{2\pi d_X \sin(\theta) \cos(\Phi)}{\lambda}.
\]
and the phase delay in the y-direction is given by 
\[
\Psi_Y = -k_Y d_Y = \frac{2\pi d_Y \sin(\theta) \sin(\Phi)}{\lambda}.
\]

I used the MUSIC algorithm for a planar array system for better accuracy and increased robustness, in which the covariance matrix of the signal is decomposed and the signal and noise subspaces \( E_s \) and \( E_n \) respectively are obtained from the Eigen decomposition of the matrix. The power values for all values of elevation and azimuth angles are found and the MUSIC estimates are then found by making use of the formula:

\[
P(\theta, \Phi) = \frac{1}{A^\times E_s \times A}
\]

The maximum value of power then gives the azimuth and elevation angles of the incoming signal.

Therefore, the steering vector for a planar array is:
The steering vector is swept for both the elevation and the azimuth angles θ and Φ and the power of the signal in all directions is found using the formula for power

\[
P_{BF}(\theta) = \frac{\mathbf{a}^H(\theta)\mathbf{R}\mathbf{a}(\theta)}{\mathbf{a}^H(\theta)\mathbf{a}(\theta)}
\]

The value of the elevation and the azimuth angles θ and Φ where the power is maximum gives the desired DOA estimate of the signal incident on the planar array.

Appendix B shows the m-file for DOA estimation of a 2x2 planar array. The code first generates the source signal arriving at an elevation angle of θ = 27° and azimuth angles Φ = 35° incident on all the four elements of the planar array. The steering vector is then found by sweeping the elevation and the azimuth angles from 0 to 180°. The steering vector is stored in a variable ‘a’ and then the value of power at each elevation and azimuth angle is calculated and plotted.

Due to the matrix multiplications being carried out throughout the program, I vectorized the matrices to obtain a linear steering vector. The results of the beamforming algorithm can be seen in Appendix C. Thus, the elevation and azimuth angles can be estimated simultaneously for a planar array structure with good accuracy for a stationary source.

4. CONCLUSIONS

In communications, beamforming is used to point the transmitting antenna at the signal source without physically moving the antenna so that interference is reduced and communication quality is improved. Beamforming techniques dynamically adjust the array pattern to optimize some characteristics of the received signal. In DOA estimation, beamforming is used to steer the antenna to determine the direction of the signal source. The interfering signals having direction of arrival different from that of the desired signal are rejected. In my project, I illustrated the performance of a Bartlett or Conventional Beamformer for a uniform linear antenna array for a carrier frequency of 3.432 GHz. I estimated the direction of arrival of the source signal incident on a linear array consisting of 16 elements. To prove the robustness of my code, the input signal incident on the antenna consisted of six interfering signals and additive white Gaussian noise along with the desired signal. The program correctly calculated the angle of arrival of the desired signal for random interfering signals and noise generated. The algorithm was robust to the noise and interference in the signal as the correct DOA was computed for the original corrupted signal as can be verified from the plot.

As the number of elements in the antenna array increases, the side lobes become smaller. The spacing between the antenna elements also plays an important factor. The optimal spacing between antenna elements is equal to half wavelength. As the spacing is increased to more than half wavelength, large side lobes appear in the radiation pattern. Therefore,
beamforming works more efficiently if the antenna is designed so as to optimize the number of elements in the array and the spacing between the array elements. One of the limitations of Bartlett’s algorithm is that its resolving power is decreased when two sources are spaced closer than a beam width of 0.63. However, the algorithm works very well in the presence of noise and any number of interfering signals as long as the interfering signals are separated by at least a 0.63 beam width from the desired signal.

Appendix A shows the MATLAB file for DOA estimation of a uniform linear array antenna geometry with 16 elements. The code generates random input data and estimates the direction of arrival of the input signal. Appendix B shows the MATLAB code for DOA estimation of a uniform linear array where the input data is read from a file which contains the input signal for a 16 elements ULA in txt file format. Appendix C shows the to-date work on the MATLAB code for DOA estimation of a planar 2x2 array.

5. REFERENCES

6. APPENDIX A

M-File for the Project: (ULA_DOA.m)

% Farah Rasheed  
% ECE-S435 Final Project  
% Beamforming - DOA Estimation for Uniform Linear Array

MATLAB Output:
Run 1: The power is maximum at 89. Therefore, DOA is 89 degrees. 
Run 2: The power is maximum at 89. Therefore, DOA is 89 degrees. 
Run 3: The power is maximum at 90. Therefore, DOA is 90 degrees. 
Run 4: The power is maximum at 89. Therefore, DOA is 89 degrees. 
Run 5: The power is maximum at 89. Therefore, DOA is 89 degrees. 
Run 6: The power is maximum at 89. Therefore, DOA is 89 degrees. 
Run 7: The power is maximum at 89. Therefore, DOA is 89 degrees. 
Run 8: The power is maximum at 89. Therefore, DOA is 89 degrees. 
Run 9: The power is maximum at 89. Therefore, DOA is 89 degrees. 
Run 10: The power is maximum at 89. Therefore, DOA is 89 degrees. 
Run 11: The power is maximum at 89. Therefore, DOA is 89 degrees. 
Run 12: The power is maximum at 89. Therefore, DOA is 89 degrees. 
Run 13: The power is maximum at 89. Therefore, DOA is 89 degrees. 
Run 14: The power is maximum at 89. Therefore, DOA is 89 degrees. 
Run 15: The power is maximum at 89. Therefore, DOA is 89 degrees. 
Run 16: The power is maximum at 89. Therefore, DOA is 89 degrees. 
Run 17: The power is maximum at 89. Therefore, DOA is 89 degrees. 
Run 18: The power is maximum at 89. Therefore, DOA is 89 degrees. 
Run 19: The power is maximum at 89. Therefore, DOA is 89 degrees. 
Run 20: The power is maximum at 89. Therefore, DOA is 89 degrees.

The number of times correct DOA was detected out of 20 times is 19.

Probability that the algorithm works 100 percent right is 95.

Probability that the algorithm works correctly with a +/- 1 degree variation is 100.
Plot of Power Output vs. Theta. DOA is 89

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7. APPENDIX B

M-File for the Beamforming-DOA Estimation of Uniform Linear Array: (ULA_DOA_file.m)

% Farah Rasheed
% ECE-S435 Final Project
% Beamforming - DOA Estimation for Uniform Linear Array (input data from file)

% On running the program, a window opens up and asks for the data file.
% The data file must be a text file which consists of the input signal
% incident on the antenna array consisting of 16 elements

clc; clear all; close all
8. APPENDIX C

**M-File for the Beamforming-DOA Estimation of Planar Array: (PLANAR_DOA.m)**

```matlab
% Farah Rasheed
% Senior Design Team-25
% Beamforming - DOA Estimation for Planar Array
% ECE-25
% Beamforming DOA Estimation
```

The input signal was generated in MATLAB and was simulated as arriving from an elevation angle of $27^\circ$ and azimuth angle of $35^\circ$. The MATLAB result in Figure E.25 below shows the plot of the power magnitude in dB versus the azimuth angles on x-axis and elevation angles on y-axis. As seen from the plot, the maximum value of power equal to 165.9 dB is obtained at $\theta = 27^\circ$ and $\phi = 35^\circ$.

![MUSIC Algorithm DOA Estimation](image)

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