Richard Skinnerrls2723Sheet#25
Week #12Problem 7.6.15
http://208.180.24.221/7_5_9.htm

Find a potential function $V(\vec{x})$ so that $\vec{F}(\vec{x}) = -\nabla V(\vec{x})$, if

$$\vec{F}(x, y, z) = y^2 z\hat{i} + 2xyz\hat{j} + xy^2\hat{k}.$$

Solution:
First, we know that

$$\mathrm{V}(\vec{x}) = \left(\frac{\partial}{\partial x} V\right) i + \left(\frac{\partial}{\partial y} V\right) j + \left(\frac{\partial}{\partial z} v\right) k$$

Therefore by integrating the components of F(x), we can find V(x).

$$\int y^2 z \, dx = y^2 z \, x$$

This here is enough to show our final answer. However, for completeness, we can go ahead and calculate the rest of the integrals and make sure they match this one.

$$\int 2 x y z \, dy = y^2 z \, x$$

$$\int x y^2 \, dz = y^2 z \, x$$

You can see that all of the component integrals have the same answer. This is our function for -V(x). Therefore

$$V(x) = -xy^2z$$