Spring 1998

Maple Lab, Week 25

Two-Dimensional Motion of a Projectile with Drag

Background: Consider a point mass moving through the air under the influence of gravity and a drag force due to air resistance. We will take the drag force to be proportional to the square of the speed and acting in the direction opposite to the velocity. Applying Newton's second law, the equation of motion in vector form is:

$$m\frac{d\vec{\mathbf{v}}}{dt} = -k\vec{\mathbf{v}}|\vec{\mathbf{v}}| - m\vec{\mathbf{g}}. \tag{1}$$

Actually, this first-order system of differential equations for the velocity vector $\vec{\mathbf{v}}(t)$ is a second-order system for the position vector $\vec{\mathbf{r}}(t) = x(t)\vec{\mathbf{i}} + y(t)\vec{\mathbf{j}}$. In fact, if we write $\vec{\mathbf{v}}(t) = v_x(t)\vec{\mathbf{i}} + v_y(t)\vec{\mathbf{j}}$, then we must have $x'(t) = v_x(t)$, $y'(t) = v_y(t)$. In addition, $|\vec{\mathbf{v}}| = \sqrt{v_x(t)^2 + v_y(t)^2}$. Consequently, equation (1) gives rise to the following system of four first-order differential equations:

$$x'(t) = v_x(t), \quad y'(t) = v_y(t), \tag{2}$$

$$v_x'(t) = -\frac{k}{m}v_x(t)\sqrt{v_x(t)^2 + v_y(t)^2}, \quad v_y'(t) = -\frac{k}{m}v_y(t)\sqrt{v_x(t)^2 + v_y(t)^2} - g.$$

We wish to use Maple's `dsolve` command to obtain a numerical solution to this problem. First, we assume the point mass is at the origin at time $t = 0$ and is projected at speed $v_0 = 100$ ft/s at an angle $\alpha = \pi/4$ radians above the horizontal. Also, we take $k/m = 0.0025$, in consistent units. (Remark: There are examples in the on-line help showing the use of `dsolve` for solving systems of equations. See also Lab 26.)

Exercises: Use Maple's `dsolve` command with the `numeric` option, together with the `plot` command, to produce the following graphs. [Hint: Look ahead to Lab 26 (second page) for examples of using the `plot` and `display` commands for graphs like these.]

1. Plot the trajectory of the solution to the system (2) in the (x, y) plane, for $0 \leq t \leq 4$.

2. Plot on one coordinate system the above graph together with the trajectory for the solution to the same problem without drag.

3 (extra credit). Experiment with various values of v_0 and α, searching for initial conditions that will make the projectile with drag land ($y = 0$) at the same point x where the projectile without drag landed with the original initial conditions.