Spring 1998

Maple Lab, Week 29

Far-Out Integrals II

Based on suggestions by M. L. Platt, CASE Newsletter #28, April 1997

References: Handout for Lab 21 (Far-Out Integrals I);
Improper integrals: Stewart Sec. 7.9;
Infinite series: Stewart Chap. 10

Theme: We continue our investigation of $I \equiv \displaystyle\int_0^\infty \frac{\sin x}{x}\,dx$.

Exercises: **5.** How do we know that the improper integral I is convergent? Study the Comparison Theorem for Integrals, Stewart pp. 493–494. Although Stewart never gets around to saying so, the series theorem on p. 635 has an analogue for improper integrals:

$$\text{If } \int_a^\infty |f(x)|\,dx \text{ converges, then } \int_a^\infty f(x)\,dx \text{ converges.}$$

Therefore, the condition $f(x) \geq g(x) \geq 0$ in the comparison theorem can be replaced by $f(x) \geq |g(x)|$.

(A) Can you apply the comparison theorem directly to the formula above for I? (Why not?)

(B) Integrate by parts to get an integral to which the comparison theorem applies. *Hint:*

$$\int_0^\infty f(x)\,dx = \int_0^\pi f(x)\,dx + \int_\pi^\infty f(x)\,dx.$$

6. Define $a_k = \displaystyle\int_{k\pi}^{(k+1)\pi} \frac{\sin x}{x}\,dx$, and consider $S = \displaystyle\sum_{k=0}^\infty a_k$.

(A) Prove that the series S converges, and that the sum equals I.

(B) Write a Maple procedure to evaluate a_k numerically for any given $k \geq 1$. (The procedure may call `simpson` from the `student` package.)

(C) Evaluate a_0 separately. (You probably already did this if you finished the extra credit Exercise 4 in Part I.)

(D) Add up enough terms in the series S to approximate I to 2 decimal places. *Hint:* Alternating Series Estimation Theorem, p. 632.

7 (extra credit). Get a better approximation to I with less computation, using the sequence

$$b_k = \int_{k\pi}^{(k+1)\pi} \frac{\cos x}{x^2}\, dx.$$

8 (extra credit). Is the series S absolutely convergent? After deciding, make your answer to 5(A) more precise.