The Master Theorem
on the Asymptotic Behavior of Recursions
Arising from Divide-and-Conquer Algorithms

Let $a \geq 1$ and $b > 1$ be constants, let $s(n)$ be a given function, and let $f(n)$ be defined on $\mathbf{N}$ by the recursion relation

$$f(n) = af(n/b) + s(n)$$

(where n/b may be interpreted as either $\lfloor n/b \rfloor$ or $\lceil n/b \rceil$) together with suitable initial values. Then for large n:

1. If $s \in O\big(n^{\log_b a - \epsilon}\big)$ for some constant $\epsilon > 0$, then $f \in \Theta\big(n^{\log_b a}\big)$.
2. If $s \in \Theta\big(n^{\log_b a}\big)$, then $f \in \Theta\big(n^{\log_b a} \lg n\big)$.
3. If $s \in \Omega\big(n^{\log_b a + \epsilon}\big)$ for some constant $\epsilon > 0$, and if

   $$as(n/b) \leq cs(n) \quad \text{for some constant } c < 1$$

   for all sufficiently large n, then $f \in \Theta(s)$.

Source: T. H. Cormen, C. E. Leiserson, and R. L Rivest, *Introduction to Algorithms*, McGraw–Hill, New York, 1990, Secs. 4.3–4.4; notation modified for consistency with that of R. P. Grimaldi, *Discrete and Combinatorial Mathematics*.