

## Topics in Applied Mathematics I

(LINEAR ALGEBRA AND VECTOR CALCULUS)

**Prerequisites:** multivariable calculus (M. 251 or equivalent); differential equations (M. 308 or equivalent) at least concurrently.

**Classes:** MWF 12:40–1:30, Blocker 156

**Instructor:** S. A. Fulling  
620H Blocker Bldg.  
845-2237  
fulling@calclab.math.tamu.edu

If I am not in my office, you can leave a note in my mailbox (in the room opposite the math department office, 6th floor of Blocker) or in the plastic pouch beside my office door.

Our class's home page on the World Wide Web is

<http://calclab.math.tamu.edu/~fulling/m311/s02/>

Tentative office hours: M 10:10–11:00, W 1:40–2:30, R 3:00–3:45

Permanent office hours will be announced later.

### Required textbook:

- S. A. Fulling, *Linearity* (World Scientific, 2000).

### Recommended books and resources:

1. M. R. Spiegel, *Vector Analysis* (Schaum's Outline Series).
2. A large amount of material from previous semesters is available from the Web page. This includes old tests with solutions and the beginnings of a student-written solutions manual for the book. Also, there is a list of "learning objectives" for each week; this may be helpful in keeping your focus in a class format where not all the material is delivered in a traditional lecture.

Other textbooks that students have found helpful:

3. S. J. Leon, *Linear Algebra with Applications*.
4. B. Kolman, *Elementary Linear Algebra* (in any of its many editions).
5. H. M. Schey, *Div, Grad, Curl, and All That*.

<b>Grading system:</b> Hour tests:	100 × 3 = 300
Final exam:	200
Homework:	≤ 130
Reviewing and class participation:	≥ <u>70</u>
Total	<u>700</u>

The "curve" will be at least as generous as the "standard" scale [i.e., 90% (= 630 pts) will guarantee an **A**, etc.].

**Schedule:** Homework will usually be due on Mondays, as follows.

Assignment	Date	Sections
1	1/18 (Fri)	1.1–1.4
2	1/28	2.1–2.3
3	2/4	2.4–3.2
4	2/11	3.3–3.5
Test on Chapters 1–3 on February 13 (Wed).		
5	2/18	4.1
6	2/25	4.2–4.4
7	3/4	4.5–5.1
8	3/18	5.2–5.4
Test on Chapters 4 and 5 on March 20 (Wed) .		
9	3/25	5.5–6.1
10	4/3 (Wed)	6.2–6.4
11	4/10 (Wed)	7.1–7.4
12	4/17 (Wed)	7.5–7.6
Test on Chapters 6 and 7 on April 19 (Fri) .		
13	4/26 (Fri)	8.1–8.2
Final exam on Monday, May 6, 10:30–12:30		

**Educational philosophy:** The procedures in this class are designed to avoid wasting honors students’ time on routine things and to facilitate some deeper learning. In part, this class is intended as a prototype of a “W course” (a future addition to the Core Curriculum in which students will get extra academic credit for writing experience). Lectures and routine homework assignments will be deemphasized. Instead, class participation, careful and focused writing, constructive criticism of others’ writing, and cooperative study will be encouraged. These activities are hard to grade by precise criteria; we will all be happier if you avoid a fixation on “points” and rest assured that the class participation and written work grades will be generous overall. **It is vital that you read the relevant sections of the textbook BEFORE coming to class, since the classes will usually be used for discussion, not lecture presentation. You should read the relevant textbook sections approximately a week before the “due date” listed above.**

In comparison with other Math. 311 sections, we will give more attention to the applications of linear algebra to differential equations, ordinary and partial. As time permits, there will be occasional brief special lectures on advanced topics, such as Gröbner bases and quantum computing.

We will be using the Internet and the World Wide Web for communications. Therefore, you will be provided with an account on the math department’s `calclab` computer system (without paying an extra lab fee). Although not strictly necessary, the capability to view DVI files (the output of `TEX`) over the Web will be helpful; if you have a DOS/Windows PC, I can provide you with software for that.

**Groups:** You'll be organized into groups of two or three. After each test the membership of the groups may be adjusted. The groups will serve two main functions, both further explained below: (1) reviewing homework papers, and (2) working problems in class.

**Homework:** As always, you are expected to work as many of the homework problems as you can, for the sake of learning. Indeed, you should try to finish the problems *well before* the “due date”, so as to benefit from class discussion and to study for tests.

However, each week you will turn in *only two* homework problems (in some weeks, only one). Each of these is to be

**a carefully written, complete, formal solution, comparable to a worked-out example in a textbook.**

In this way your class will build up its own solutions manual. For tips on good writing, please start by reading the attached “Guidelines for Reviewers (and Authors!)”. Your paper should be self-contained, so you should either copy the problem from the book, or reformulate the problem briefly in your own words at the beginning of the solution (e.g., “We are asked to find a vector that satisfies . . . ”), or (in an on-line solution) give a link to the proper “problem template” on the Web.

Problems may be individually assigned, or you may be allowed to choose one. In the latter case, roughly speaking, you should choose the hardest problem that you are confident you know how to do well.

For the time being, the limited availability of technology and lack of a standard format for electronic presentations forces us to use paper as the primary medium of presentation. However, you can earn **2 extra points** per problem by making your work *also* accessible via the World Wide Web. Acceptable formats include:

1. plain ASCII text, using “pidgin  $\text{T}_{\text{E}}\text{X}$ ” for mathematical symbols. (An explanation of this will come later.)
2. HTML
3. DVI ( $\text{T}_{\text{E}}\text{X}$  output)
4. PostScript or PDF (less satisfactory than the others because of file size)

Optical scanning of a handwritten page is discouraged (worth only one extra point). Proprietary formats such as Microsoft Word `.doc` are not acceptable for Web display; output the file as HTML instead.

You are strongly urged to keep a copy (or at least a rough draft) of your paper when you turn it in. Since your paper may be photocopied or put into a binder, *please follow these technical guidelines*: Use standard  $8.5 \times 11$ -inch paper, not too flimsy. Avoid yellow paper, blue pens, and faint pencil. Don't write too close to the edge of the page! (Leave margins of about an inch.) Use one side of the paper only. Use of word processors, especially  $\text{T}_{\text{E}}\text{X}$ , is praiseworthy. (Note that Maple files can be exported as  $\text{L}_{\text{A}}\text{T}_{\text{E}}\text{X}$  or as HTML.) Finally, the following information **must** appear at the **top of the first page** of your paper:

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Your name                                      Your calclab account name    Your attendance sheet number  
 Assignment (week) number    Problem number (3 numbers separated by periods)  
 Indication if the paper is *late* or *revised* (see below)  
 URL of the Web version of your paper (if it exists).

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(Please don't put your name, etc., on a separate cover sheet. That just wastes paper. DO NOT write your full Social Security number on papers. The attendance sheet number (available during Week 2) is optional, but it will help put the papers into alphabetical order quickly.)

Please note: Homework *authorship* in this class is *individual*. Your group membership is relevant only to *reviewing*. On the other hand, informal discussion with other students about homework problems is permitted, even encouraged. (That remark does not apply, of course, to anything announced as a take-home test.)

Getting credit for late papers will be hard — see below.

**Reviewing:** The adult world is not divided into students and teachers; it consists of diverse but comparable, fallible but mutually helpful, human beings. Similarly, a course at this level should not need a distinction between students and graders. Each week or so, your group will get all the papers turned in on one of the problems. **The papers are to be returned two class meetings later** (i.e., usually on Friday). Meanwhile, the group should do the following (with my guidance, when necessary):

1. Write a review or critique of each paper, telling the author how it should be improved. (See “Guidelines for Reviewers (and Authors!)” attached below.)
2. Choose the best paper as the one to be “published” in your class’s solutions manual (which will cover the even-numbered problems). Complications may arise here:
  - (a) The best paper may not be good enough. Return it to the author (through the grader) with suggestions for improvements. (**A successful revision earns the author another one or two points.**)
  - (b) You may choose more than one paper if they handle the problem in distinctly different ways, all of which are instructive.
  - (c) The ideal paper may be a combination of pieces or elements from more than one paper. If possible, advise the grader how to do a “cut & paste” job on the existing papers. If necessary, invite the authors to submit a joint revised effort.
3. Write a summary report, suitable for the whole class to read, describing the results of the class’s work on this problem. In particular, whenever appropriate:
  - (a) Point out what is particularly good about the paper you chose — especially if you chose more than one.
  - (b) Point out any mathematical errors that appeared more than once in the other papers. Avoid student names here. Remember that this report (but not the individual reviews) will end up on the Web!

The hope is that the summary report will be genuinely informative and useful (to the class, the grader, me, and even future classes). Therefore, its length will depend strongly on the circumstances. It might be as short as this:

All 5 papers turned in were completely correct. We chose Jason Smith's paper because it was the neatest.

Or it might be like this:

Marcia Kleberg's answer to part (a) is the best, but we thought the rest of her paper was unnecessarily long and complicated. We didn't understand Bill Petersen's reasoning in (a), but he did a good job on parts (b) and (c). So we put together Marcia's (a) and Bill's (b)–(c). We think the result is better than any of the solutions inherited from previous classes!

The other two papers we received both made a major mistake in part (c). ... [Explain what they did wrong. **In the long run, this will be the most valuable part of the report.**]

For real examples, see the reports on the Web pages from previous semesters (and try to do better, of course).

4. The reviews and summary report **must** be e-mailed by means of the Homework Review Generator program on `calclab`. See the attached instruction sheet. This program was written by a student in Math. 311 in Spring '96 and makes the whole process much more efficient for you, the course grader, and me. (This does not mean that you must read e-mail directly on the `calclab` system; it is easy to create a `.forward` file to send the mail elsewhere. Nor does it mean that you need to do all your text editing for reviews on `calclab`; see part (E) of the program instructions.)
5. The list of group members on the summary report is a *signature* certifying that all listed members took part in the discussions leading to the reviews and report. Only members who (honestly) sign the report will get credit for reviewing. Some division of labor in the early stages of reviewing is appropriate, but all members should consult on the final decision and the summary report.
6. (optional) In a separate e-mail message, make any private remarks to the course grader that you consider appropriate. You *may* suggest point scores for the papers, or rank them, but that should not be your primary concern.
7. (optional) Consult the Undergraduate University Writing Center (on Floor 2 of Evans Library) for advice on the foregoing. Be among the first to get a return on your \$8/semester investment!

The grader and I will assign numerical grades to the problems (typical maximum of 10 points) and to the reviews and reports (maximum of 5 points per problem). Please note that the homework and reviewing scores will be *scaled* at the end of the semester to a reasonable distribution, with the result that a homework point will probably be worth only about half a test point.

After a homework problem has been collected and referred to a group of reviewers, late papers on that problem will be accepted only as a favor by the reviewers. If the reviewers have already finished their work, they should refuse to accept the paper, and the grader will give it minimal attention and probably only half credit.

**Class participation:** We will probably spend one day a week working example problems in class by groups. The emphasis will be on the basic types of problems that are most likely to show up on tests. This will ensure that the basic skills do not fall through the cracks of the personalized homework assignments. Transparencies and felt pens will be provided so that you can present your solutions to the class efficiently.

Pointing out mistakes and obscurities in the book (preferably by e-mail) is a way of earning extra points.

**Make-up tests:** Make-up tests are very hard to grade fairly, and they absorb a large amount of my time which would be better spent for the benefit of the whole class. Please cooperate in making these incidents as rare as possible. If you miss (or foresee that you will miss) a test, it is *your* responsibility to contact me as soon as possible to request, justify, and schedule a make-up test. (If you can't reach me directly, you can leave a message at the Math Department office, (979) 845-3261.) If the absence is not clearly excused under the Attendance section of *Student Rules*, the request may be denied.

**Calculators in exams:** Calculators are to be used only to perform *elementary operations* such as addition, multiplication, and (optionally — see remark below) evaluation of simple functions such as square roots, sines, and logarithms. Advanced facilities such as storing formulas in memory, inverting matrices, and graphing functions on the calculator display are prohibited. Violations of this rule may lead to total prohibition of calculators in exams.

The reasons for this rule should be clear:

1. Calculators differ in their sophistication. A student without a powerful calculator should not be put at an unfair disadvantage.
2. Advanced calculators can automate some of the skills being taught in the course. This would defeat the purpose of the test.

Answers should be stated in a form obtainable without a calculator. If you want to convert

$$\sqrt{2}e^{-\pi} \quad (*)$$

to a decimal fraction, that's fine, but when I grade the test I want to see (\*), too!

I encourage you to use calculators and computers on homework exercises, especially in ways that are imaginative or beyond-the-call-of-duty (such as graphics or numerical experiments). But don't become so dependent on the hardware that you fail to practice the skills you need to do well on the tests!

## Guidelines for Reviewers (and Authors!)

Write a tactful, but honest, critique of the paper. Here are some criteria to keep in mind, listed in decreasing order of importance. These goals may sometimes conflict. For example, brevity may interfere with pedagogical effectiveness, and both may conflict with completeness. The balance among them is a matter of judgment and taste.

1. **Mathematical correctness.** Any technical errors must be recognized and eliminated. Does every sentence make mathematical sense, or is the writer bluffing or showing off with mathematical terminology [s]he only partly understands?

2. **Completeness.** Has the author provided all the information needed to follow the argument or calculation from beginning to end? Has the author confused an example with an adequate treatment of the general case? (This does not mean that an example is not desirable; see 4.)

3. **Organization.** Are things said in the best order? Is the flow of the logic clear? Is the reason for each step clear? Are the main ideas in danger of being obscured by the details?

4. **Pedagogical effectiveness.** Has the author “gone the extra mile” to help the reader understand? Examples and analogies can add clarity to an already logically complete discussion. (Thus, this is primarily an opportunity to say something *positive* about the paper you’re reviewing.)

5. **Sentence structure and punctuation.** Is the paper written in sentences, and does every sentence make sense? Help stamp out sentence fragments, run-ons, and dangling participles (“Adding  $x^2$  to both sides, the answer becomes . . . ”). Has the writer left out or added a comma that unintentionally changes the meaning of the sentence (famous example: “The Republican Party opposes any tax increase[,] which would harm the economy.”)? Etc. . . .

6. **Brevity and style.** Help the writer to trim out extra words, pointless repetition, and long transitional phrases that say nothing. Is pretentious language used where simpler language would do the job at least as well? Although mathematics seldom can read like a novel, some variety in word choice and sentence structure can stave off boredom.

7. **Grammar and spelling.**

8. **Neatness.** When asked to read something, you have a right to expect it to be legible. Few people can produce a good piece of formal writing without recopying the final draft. Use of a word processor or typewriter is encouraged but not required.

It is usually *not* necessary to comment specifically on each of these matters in a review. Concentrate on the most important things you have to say about the particular piece of writing at hand. Being relatively private, the reviews and summary report may be informally written, but they should still follow the principles of good writing.