Yet another big example (with infinite domain)

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0 \qquad (0 < x < \infty, \quad 0 < y < \pi),$$

$$u(0, y) = g(y) \quad (0 < y < \pi),$$

$$u(x, 0) = 0 \quad (0 < x < \infty),$$

$$u(x, \pi) = f(x) \quad (0 < x < \infty).$$