Math. 467 (Fulling) 15 October 2021

Midterm Test – Solutions

1. *(Multiple choice – each 5 pts.) (Circle the correct capital letter.)*

   (a) By definition, lines are *parallel* if

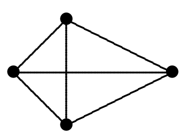
   (A) They are a constant distance apart.
   (B) They divide the plane into three "sides".
   (C) Each of them divides the plane into two "sides".
   (D) There is a unique line perpendicular to both of them.
   (E) They do not intersect.

E

   (b) Axiom C-4 authorizes us to construct an angle $\angle \mathrm{B'A'C'}$ congruent to a given angle $\angle \mathrm{BAC}$ adjacent to a given ray $\overrightarrow{\mathrm{A'B'}}$. The result is unique provided that

   (A) the *side* of the line $\overleftrightarrow{\mathrm{A'B'}}$ on which the new angle appears is prescribed.
   (B) segment AB is congruent to segment A′B′.
   (C) the Euclidean parallel postulate holds.
   (D) $\angle$BAC is a right angle.
   (E) congruence of angles is an equivalence relation.

A

   (c)  This diagram is used in the proof of

   (A) Pasch's theorem
   (B) crossbar theorem
   (C) SSS
   (D) ASA
   (E) SAS

C

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2. *(12 pts.)* Simplify $\neg\exists x\,\forall y\,[(x \le y) \;\Rightarrow\; [x = y \vee \exists z\,(x < z < y)]$.
   (Push the "¬" in as far as you can! Truth or falsity of the statement is irrelevant. In Greenberg's notation, ¬ is ∼.)

$$\forall x\,\exists y\,\neg[(x \le y) \;\Rightarrow\; [x = y \vee \exists z\,(x < z < y)].$$

$$\forall x\,\exists y\,[x \le y \wedge \neg[x = y \vee \exists z\,(x < z < y).]$$

$$\forall x\,\exists y\,[x \le y \wedge x \ne y \wedge \forall z\,(x \ge z \vee z \ge y)].$$

Or, even better,

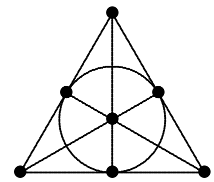
$$\forall x\,\exists y\,[x < y \wedge \forall z\,(x \ge z \vee z \ge y)].$$

Note, however, that the last subsentence can't be simplified to $x \ge z \ge y$. That would mean $x \ge z \wedge z \ge y$.

3. *(24 pts.)* State the four *betweenness* axioms.

[See Greenberg, pp. 108 and 110–111.]

4. *(8 pts.)* Explain the significance of this diagram:



It represents the Fano plane, the smallest projective plane. See Greenberg, p. 84. (You could say more, of course.)

5. *(16 pts.)* Prove **ONE** of these [(A) or (B)]. *(Extra credit for doing both is limited to 8 points.)*

   (A) Given triangles ABC and DEF with $\angle A \cong \angle D$, $\angle C \cong \angle F$, and $AC \cong DF$, then these triangles are congruent.

[See Greenberg, pp. 127 and 151–152.]

   (B) If in triangle ABC we have $AB \cong AC$, then $\angle B \cong \angle C$.

[See Greenberg, pp. 123–124.]

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6. *(Essay – 20 pts.) IMPROVEMENTS IN THE REWRITE GET ONLY HALF CREDIT. E.G., IF YOUR INITIAL SCORE IS 16, YOUR FINAL SCORE WON'T EXCEED 18.*

   The finite incidence geometries studied in Chapter 2 of Greenberg's book provide an instructive, simple analog of the independence of the parallel postulate in plane geometries with the Hilbert axioms. Explain how these finite geometries show that the parallelism property is *independent* of the three incidence axioms. (The magic word is "*model*".)