system *except for handedness.* If $\vec{u}$ and $\vec{v}$ are ordinary ("true") vectors, then $\vec{u} \times \vec{v}$ is a *pseudovector* whose sign depends on the handedness of the coordinate system.

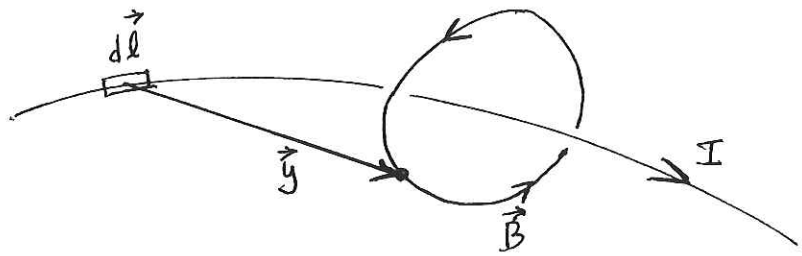
On the other hand, angular velocity, magnetic field, and so on are *already* pseudovectors, since their physical definitions involve "right-hand rules". Therefore, when such a vector appears as a factor in a cross product, the relevant statement is: If $\vec{\omega}$ is a pseudovector and $\vec{y}$ is a true vector, then $\vec{\omega} \times \vec{y}$ is a true vector. Thus in applications we often see two cross products together, one to create a pseudovector and another to undo it.

EXAMPLE: Magnetic field.

1. Biot–Savart law:

$$d\vec{B} = \frac{I\, d\vec{l} \times \vec{y}}{\|\vec{y}\|^3};$$

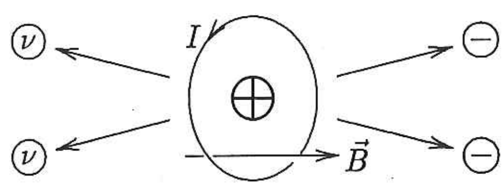
$$\vec{B} = I \oint_{\text{circuit}} \frac{d\vec{l} \times \vec{y}}{\|\vec{y}\|^3}.$$



(For a single charge, $\vec{B} = e\,\frac{\vec{v} \times \vec{y}}{\|\vec{y}\|^3} = \vec{v} \times \vec{E}$.)

2. Lorentz force law: $\vec{F} = e(\vec{E} + \vec{v} \times \vec{B})$ ($\vec{v}$ = velocity of *another* charge).

Thus the sign of $\vec{B}$ is purely a convention and cancels out of the final answer for the magnetic force between two charges. [However, in some nuclear decays, particles are emitted preferentially along the direction of an applied magnetic field. This shows that some laws of nature *do* make a distinction between left and right ("overthrow of parity" — 1956).]



From this discussion we expect that the vectors $\vec{\omega}$ associated to antisymmetric matrices via the cross product are pseudovectors, not true vectors. In other words, the isomorphism between antisymmetric operators and vectors involves an arbitrary sign convention; returning to the determinantal definition of the cross product, we see that this sign is hidden in the ordering of $\{\hat{\imath}, \hat{\jmath}, \hat{k}\}$ there.